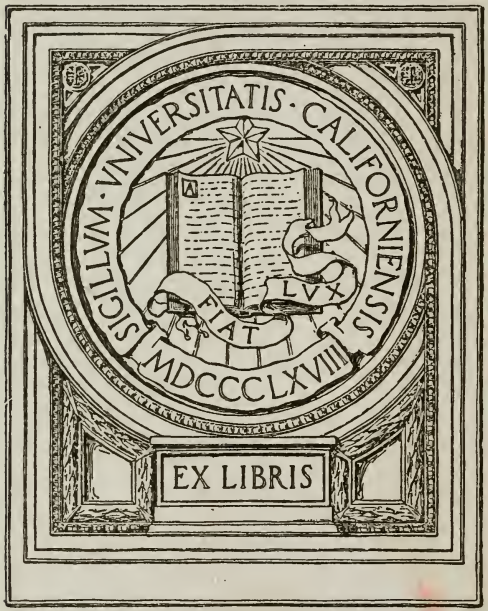

A SHILLING ARITHMETIC

LONEY AND GRENVILLE

&MCo.

Florian Cajori

A SHILLING ARITHMETIC.

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TORONTO

A SHILLING ARITHMETIC

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PREFACE.

THIS little book has been prepared in the hope of meeting the requirements of Secondary Schools, and covers the course for the Oxford and Cambridge Junior Local Examinations. The scheme recommended by the Mathematical Association has been generally followed, though it was not found advisable, or even possible, to omit entirely the subject of Recurring Decimals.

The book is written on the lines of Loney's "Arithmetic for Schools," and both its text and examples have been used when they seemed best adapted to the present purpose. Many changes and simplifications have, however, been made which a considerable experience in School Teaching has suggested.

In order to deal as fully as possible with the less elementary processes of Arithmetic, and at the same time to keep the book within a reasonable size, it is assumed that the student already knows the four "Simple" Rules and the "Compound" Rules. Twelve pages of miscellaneous questions on these are given, and it is hoped that they will be found sufficient for revision. The ordinary Tables are prefixed.

December 4th, 1905.

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ARITHMETIC.

TABLES.

MONEY.

4 Farthings	**make**	**1 Penny.**
12 Pence	**„**	**1 Shilling.**
20 Shillings	**„**	**1 Pound.**

TIME MEASURE.

60 Seconds (60″)	**make**	**1 Minute** (1′).
60 Minutes	**„**	**1 Hour.**
24 Hours	**„**	**1 Day.**
7 Days	**„**	**1 Week.**
4 Weeks	**„**	**1 Month** (Lunar).
12 Months	**„**	**1 Year.**
365 or 366 Days	**„**	**1 Year.**

[Every year, whose number is divisible by 4, is a leap year and contains 366 days, except the years A.D. 1700, 1800 and 1900, which are not leap years.]

AVOIRDUPOIS WEIGHT.

16 Drams (Drs.)	**make**	**1 Ounce** (Oz.).
16 Ounces	**„**	**1 Pound** (Lb.).
28 Pounds	**„**	**1 Quarter** (Qr.).
4 Quarters	**„**	**1 Hundredweight** (Cwt.).
20 Hundredweights	**„**	**1 Ton** (Ton).

Also 1 stone = 14 lbs.

TROY WEIGHT.

24 Grains	**make**	**1 Pennyweight** (Dwt.).
20 Pennyweights	**„**	**1 Ounce** (Oz.).
12 Ounces	**„**	**1 Pound** (Lb.).

The connection between Troy and Avoirdupois Measure is:

1 lb Avoirdupois = 7000 grains Troy,

and 1 lb. Troy = $12 \times 20 \times 24 = 5760$ grains Troy.

APOTHECARIES' WEIGHT.

20 Grains	**make**	**1 Scruple** (Scr.).
3 Scruples	„	**1 Dram** (Dr.).
8 Drams	„	**1 Ounce** (Oz.).
12 Ounces	„	**1 Pound** (Lb.).

MEASURES OF LENGTH.

LONG MEASURE.

12 Inches (In.)	**make**	**1 Foot** (Ft.).
3 Feet	„	**1 Yard** (Yd.).
$5\frac{1}{2}$ Yards	„	**1 Rod, Pole,** or **Perch** (P.).
40 Poles (or 220 Yds.)	„	**1 Furlong** (Fur.).
8 Furlongs (or 1760 Yds.)	„	**1 Mile** (Mi.).
3 Miles	„	**1 League.**

Land surveyors use a Chain (called Gunter's Chain) subdivided into 100 Links. This chain is 22 yards long. We thus have

100 Links = 1 Chain = 22 Yards.

10 Chains = 220 Yards = 1 Furlong.

Also 80 Chains make 1 Mile.

In addition the following measures are used in certain cases:

4 Inches make 1 Hand. [A horse is always spoken of as so many hands high.]

6 Feet make 1 Fathom. [The sea is often spoken of as so many fathoms deep.]

SQUARE MEASURE.

144 Square Inches (Sq. Ins.)	**make**	**1 Square Foot** (Sq. Ft.).
9 Square Feet	„	**1 Square Yard** (Sq. Yd.).
$30\frac{1}{4}$ Square Yards	„	**1 Square Pole** (Sq. Po.).
40 Square Poles	„	**1 Rood** (R.).
4 Roods	„	**1 Acre** (Ac.)
640 Acres	„	**1 Square Mile** (Sq. Mi.).

In addition, since 1 chain = 22 yards,

we have 1 square chain = 22 × 22 sq. yards = 484 sq. yards.

But 1 acre = 4 roods = 160 sq. poles

= 160 × 30¼ sq. yds. = 4840 sq. yds. = 10 sq. chains.

Hence we have 10000 sq. links = 1 sq. chain,

and 10 sq. chains = 1 acre.

CUBIC MEASURE.

1728 Cubic Inches (Cub. In.) **make 1 Cubic Foot** (Cub. Ft.).
27 Cubic Feet „ **1 Cubic Yard** (Cub. Yd.).

MEASURES OF CAPACITY.

Dry commodities, such as grain and fruit, are measured according to the following table:

2 Pints (Pts.)	**make**	**1 Quart** (Qt.).
4 Quarts	„	**1 Gallon** (Gal.).
2 Gallons	„	**1 Peck** (Pk.).
4 Pecks	„	**1 Bushel** (Bush.).
8 Bushels	„	**1 Quarter** (Qr.).
5 Quarters	„	**1 Load** (Ld.).

The first three of these measures are used for liquids also.

MEASURES OF NUMBER.

1 Dozen = 12 Units. **1 Gross = 12 Dozen = 144 Units.**
1 Score = 20 Units.
24 Sheets of Paper = 1 Quire. } **Paper Measure.**
20 Quires = 1 Ream. }

CHAPTER I.

MISCELLANEOUS EXAMPLES ON THE FOUR SIMPLE RULES. I.

1. What number must be added to 63495 to give 85934?

2. What number must be subtracted from 79384 so that the remainder may be 57896?

3. Add the sum of 549 and 637 to the difference between 983 and 794.

4. Add the difference of 1837 and 989 to the difference between 1354 and 1179.

5. A man whose age is now 37 years was 26 years old when his daughter was born; how old is she now?

6. The sum of two numbers is 1379 and one of them is 684; what is the other?

7. In a box there were 3487 coloured balls; 546 were white, 1364 were red, 987 were blue, and the rest were black. How many black balls were there?

8. In 1891 the population of London was 4211743, that of the county of Durham was 1024369, that of Gloucestershire was 548886, and that of Devonshire 636225; by how many did the population of London exceed the total population of these three counties?

9. Add together the numbers 3708, 619, 13462, and 8755, and multiply the result by 39.

10. Multiply fifty-nine million five hundred thousand one hundred and ninety-eight by five thousand and forty-two; and to the product add three hundred and one thousand six hundred and eighty-four.

11. On board a ship there were 9634 boxes of oranges, and each box contained 637 oranges; how many oranges were there on board?

12. A train runs 347 miles in each day; how many miles would it run in a year of 365 days?

13. One sovereign is equal to 240 pence; how many pence are 573 sovereigns equal to?

14. In a street there are 139 houses; each house contains 4 families and each family 5 persons; how many people live in the street?

15. A book has 465 pages; on each page there are 37 lines and in each line 43 letters; how many letters are there in the book?

16. There are 57 boxes of rice, each containing eight hundred and nine thousand three hundred and nine grains, and 76 other boxes, each containing seven hundred and nineteen thousand two hundred and ninety-four grains. Write in words how many grains of rice there are altogether.

17. How many railway carriages are required to carry 901 children if 53 children are put into each carriage?

18. In a field there are 24541 cabbages arranged in 97 equal rows; how many cabbages are there in a row?

19. A boy has 3845 counters; how many heaps, each containing 97 counters, can he make, and how many has he left?

20. 208008 oranges are to be packed into boxes, each of which will hold 856 oranges; how many boxes are required?

21. What number multiplied by 837 will give 496341?

22. What number divided by 796 will give 804 as quotient?

23. What number multiplied by 23 will give the same product as 391 multiplied by 37?

24. The product of two numbers is 102823 and the greater of them is 397; find the other.

25. What number divided by 293 will give 79 as quotient and 237 as remainder?

26. What number divided into 34034 will give 97 as quotient and 278 as remainder?

27. The quotient arising from the division of a number by 78 is 342 and the remainder is 47; what is the number?

28. The quotient arising from the division of 163713 by a certain number is 384 and the remainder is 129; find the divisor.

29. An army consisted of 22977 men in all; each regiment contained 950 men and 49 officers: how many regiments were there in the army?

30. Divide five million by 1170; what is the nearest whole number to five million which is exactly divisible by 1170?

II.

1. In a school there were 127 boys, 113 girls, and 89 infants; how many children were there in all?

2. A man whose age is 45 years was 27 years old when his son was born; how old is his son now?

3. The sum of two numbers is 216, and one is 79; find the other.

4. In a school of 347 pupils there were 128 girls and 97 infants; how many boys were there?

5. Multiply the sum of 359 and 331 by the difference between them.

6. How many letters are there in a book which contains 324 pages, each page containing 23 lines, and each line 47 letters?

7. A boy has 987 counters; how many heaps of 79 counters each can he make, and how many counters has he left?

8. From a ship full of oranges 123 vans are loaded; if each van contains 58 boxes, and each box contains 144 oranges, how many oranges did the ship contain?

9. To three times a certain number I add 59, and thus obtain 2426; what is the number?

10. How many times must 43 be added to 1649 to give 4186?

11. It is said that there are in the Old Testament twenty-three thousand two hundred and fourteen verses, and in the New Testament seven thousand nine hundred and fifty-nine; write in words by how many the former exceeds the latter.

12. What number multiplied by 57 will give the same product as 247 multiplied by 21?

13. What number divided by 367 will give 59 as the quotient and 126 as the remainder?

14. A and B play at marbles; A had 347 to begin with, and B had 135; B wins 106 from A: how many has each now?

15. If a bicyclist travel 57 miles a day, how far will he go in 17 days, and how long will he take to go 741 miles?

16. By what number must 123 be multiplied so that when the product is added to 1349 the final result is 8360?

17. In a train containing 310 passengers, the number of first and third-class passengers was 220, that of the second and third-class 265; how many passengers were there in each class?

18. At a game of cricket three boys, A, B, and C, score between them 222 runs; A and C make between them 193 runs; B and C make 159; how many runs does each boy score?

19. An army consists of 23161 men; each regiment consists of 960 men and 47 officers: how many regiments are there in the army?

20. If a train travel from London to Exeter in 342 minutes, and it goes 880 yards in each minute, find the distance between London and Exeter in yards.

21. In a street there are 154 houses. Of these, 23 have three families, 14 have four, and the remainder two residing in them. If each family contains five persons, what is the population of the street?

22. A book has 676 pages. Of these 17 have only pictures, and 23 have only one column with 58 letters in each line. The rest have two columns each with 27 letters in each line. Each column contains 44 lines; how many letters are there in the book?

23. The product of two numbers is 67267, and the least of them is 137; find the sum of the numbers.

24. The quotient arising from the division of a number by 53 is 29 and the remainder is 23; what is this number?

25. A man was 26 years of age when his daughter was born; how old would she be when he is 50 years of age, and what age would he be when she is 45 years of age?

26. A man died in 1892 aged 75 years, and his son died in 1884 aged 41; how old was the father when the son was born?

27. What is the smallest number that must be subtracted from 34657 so that it may be exactly divided by 129?

28. The remainder of a division is 91, the quotient is 502, and the dividend 63252 more than the sum of both; what is the divisor?

29. How many horses at £21 each are worth 56 cows at £15 each?

30. A man gave 104 cows for 52 horses valued at £24 each; how much did he get for each cow?

31. A man's yearly income is £1200; he pays for house rent and rates and taxes, £124; for food and servants, £260; for clothes, £94; for education, £45; for holidays, £74; and for other expenses, £123: how much has he left of his income?

COMPOUND QUANTITIES.—MONEY. EXAMPLES III.

1. Reduce £64. 17*s.* 6*d.*, £817. 5*s.*, £737. 15*s.*, and £2437. 2*s.* 6*d.* (1) to half-crowns and (2) to three-pences.

2. How many farthings are there altogether in 7 sovereigns, 7 half-crowns, 7 florins, and 7 sixpences ?

3. How many letters, paying penny postage, require stamps to the value of £2387. 11*s.* 10*d.* ?

Reduce to guineas, shillings, pence, etc. :

4. 5436*d.*, 8324*d.*, and 79384*d.* **5.** 83462*f.*, 72375*f.*, and 8457*f.*

6. 5932, 6381, and 14372 half-pence.

Reduce to half-crowns :

7. 9960*f.* and 13680*f.* **8.** 26760*f.* and 1872*f.*

9. 11540, 14220, and 17340 half-pence.

10. By how much is £3498. 6*s.* $1\frac{3}{4}d.$ greater than £2943. 13*s.* $7\frac{1}{2}d.$?

11. By how much is £7348. 17*s.* $3\frac{3}{4}d.$ less than £13649. 3*s.* $5\frac{1}{2}d.$?

12. Add together £34. 6*s.* $3\frac{1}{2}d.$, £64. 11*s.* $5\frac{1}{4}d.$, £205. 4*s.* $7\frac{1}{2}d.$, and £29. 0*s.* $1\frac{3}{4}d.$, and subtract the sum from £759. 17*s.* $3\frac{1}{4}d.$

13. Subtract the sum of £73. 11*s.* 7*d.*, £42. 15*s.* $3\frac{3}{4}d.$, £27. 16*s.* $1\frac{1}{2}d.$, and £94. 14*s.* $5\frac{1}{4}d.$ from £315. 13*s.* $6\frac{3}{4}d.$

14. A lady has £20 in her pocket and pays bills to the amount of £2. 3*s.* $5\frac{1}{2}d.$, £7. 4*s.* $3\frac{3}{4}d.$, £1. 18*s.* $3\frac{1}{4}d.$, £1. 17*s.* $2\frac{1}{2}d.$, and £5. 16*s.* $4\frac{1}{4}d.$; what money has she left ?

15. From the sum of £133. 11*s.* 6*d.* and £47. 5*s.* $3\frac{1}{2}d.$ take away the difference between £57. 6*s.* $9\frac{1}{4}d.$ and £13. 18*s.* $10\frac{1}{2}d.$

16. To the difference between £734. 11*s.* $3\frac{1}{2}d.$ and £943. 17*s.* $6\frac{1}{4}d.$ add the difference between £463. 14*s.* 6*d.* and £344. 11*s.* $8\frac{1}{4}d.$

17. From the difference between £234. 11*s.* 7*d.* and £69. 14*s.* $2\frac{1}{2}d.$ subtract the difference between £342. 14*s.* $7\frac{1}{2}d.$ and £265. 17*s.* $5\frac{1}{4}d.$

18. Find the value of

£34. 11*s.* 7*d.* − £1. 3*s.* 4*d.* + £14. 13*s.* $6\frac{1}{2}d.$ + £46. 5*s.* $2\frac{1}{4}d.$ − £73. 16*s.* $5\frac{1}{2}d.$

19. If out of £52 I pay away £25. 16*s.* $1\frac{3}{4}d.$, £2. 17*s.* $11\frac{1}{2}d.$, £9. 5*s.* $3\frac{1}{2}d.$, and 7*s.* $7\frac{1}{2}d.$, how much shall I have left ?

20. Add together £7. 19*s.* $6\frac{3}{4}d.$, £11. 0*s.* 10*d.*, £28. 3*s.* $4\frac{1}{2}d.$, and £16. 8*s.* $0\frac{1}{4}d.$, and subtract the sum from £100.

21. What sum will be left when the amounts £5. 10*s.* $7\frac{1}{2}d.$, £12. 18*s.* $6\frac{3}{4}d.$, £9. 4*s.* $6\frac{1}{4}d.$, and £3. 2*s.* $4\frac{1}{2}d.$ of four bills have been paid out of £35 ?

22. 10 dozen sheep at £2. 3*s.* 6*d.* each.

23. 17 gross of shoes at 15*s.* 6*d.* each.

24. 785 lbs. of tea at 1*s.* 8*d.* per lb.

25. 23 tons of coal at £1. 3*s.* 6*d.* per ton.

26. Multiply twenty-nine pounds nineteen shillings and tenpence three farthings by a hundred and ninety-two.

27. Subtract the sum of £17. 18*s.* 2½*d.* and £247. 13*s.* 11¼*d.* from 550 guineas.

28. Out of a sum of money 17 men received 3*s.* 4½*d.* each, and there remained 2*s.* 5¾*d.* : how much was the original sum?

29. What is the total cost of a city dinner at £4. 13*s.* 9*d.* each, the number of guests being 312?

30. A grocer bought 300 lbs. of tea at 2*s.* 4½*d.* a pound, but 60 lbs. of it being damaged the price for this was reduced to 1*s.* 9*d.* a pound: what was the cost of the whole?

31. Find the whole cost of 20 dozen boxes of fruit at 14*s.* 7½*d.* per box, 40 dozen at 13*s.* 9¼*d.*, and 60 dozen at 12*s.* 8*d.*

32. A man buys 24 calves at £2. 5*s.* 6*d.* each ; 27 pigs at 15 shillings each ; 59 sheep at £1. 18*s.* each ; and 4 horses at £16. 10*s.* each : how much in all does he pay?

33. The charge for dinner was 12*s.* 6*d.* a head with a reduction of 5*s.* if no wine was taken. Of 368 people who dined 120 took no wine. What was the total bill for the dinner?

34. At a certain wool sale 14575 bales were sold. Taking each bale as weighing 300 lbs., and the price of the wool at 11*d.* per lb., what amount of money was realized?

35. Seven persons spend between them £14. 3*s.* 7½*d.* ; how much is that per person?

36. If £319. 9*s.* 2½*d.* be equally divided amongst 59 men, what is the share of each?

37. If in 28 days a man's wages amount to £4. 0*s.* 6*d.*, what does he earn per day?

38. If 69 sheep be bought for £143. 3*s.* 6*d.*, what is the cost of each sheep?

39. A sum of £254. 9*s.* 8½*d.* is equally divided among 47 men ; how much does each man get?

40. The following sums were given in aid of a charity: £17. 3*s.* 2*d.*, £21. 17*s.* 11*d.*, £12. 19*s.* 5½*d.*, and £24. 12*s.* 10*d.*, and the whole was then divided between 47 poor persons ; how much did each receive?

41. How many persons may receive a pension of fifty-five guineas out of an annual income of five thousand five hundred pounds? How much will be left after they have been paid?

42. I have due to me six sums of £4. 9*s.* 3¼*d.* each, seven of £1. 2*s.* 6¾*d.*, and one of £3. 0*s.* 11¾*d.* ; how long will the money last me at £1. 7*s.* 3*d.* per week?

43. How many persons can be paid 11*s.* 6*d.* each out of a sum of £170. 10*s.* 2*d.*? Also if the balance be distributed equally amongst them, how much more will each receive?

44. To how many persons can the sum of £19. 19*s.* 11*d.* be paid out of an estate worth £10000? How much each may 25 other persons receive from the residue?

45. How many times can £17. 5*s.* 9½*d.* be subtracted from £100, and what will be the last remainder?

46. How many times can £31. 4*s.* 7½*d.* be subtracted from £500, and what will be the last remainder?

47. Into how many parcels of £1. 4*s.* 3¾*d.* each can £700 be divided? How much more would be required to make up one parcel more?

48. A man bought 37 pieces of cloth for £220. 3*s.* at 5*s.* 8*d.* per yard; how many yards were there in each piece?

49. How many dozen articles can be bought for £232. 18*s.*, supposing that each article costs 2*s.* 10*d.*?

50. How many patients can be maintained by a hospital whose revenue is £31076 if the cost of each patient be £22. 13*s.* 4*d.*?

51. A sum of money amounting to £904. 11*s.* 4*d.* was shared amongst a number of people. If each received £56. 10*s.* 8½*d.*, how many people were there?

Reduce

52. £97. 17*s.* 6*d.* to half-crowns.

53. £693 to guineas.

54. 376 guineas to florins.

55. 560 guineas to pounds.

56. £368. 5*s.* to guineas and shillings.

57. 2462 half-crowns to fourpences.

58. 1071 half-crowns to half guineas.

59. £66. 10*s.* to francs of 9½*d.* each.

60. £123. 15*s.* to dollars of 4*s.* 1½*d.* each.

MISCELLANEOUS EXAMPLES IV.

Compound Rules. Money.

1. Add together £175. 17*s.* 6*d.*, five hundred guineas, eighty-seven half-crowns, and 1143 threepenny pieces.

2. Add together two million three hundred thousand and forty pence, eight hundred and eight thousand sixpences, and ten million one hundred and thirty-five thousand nine hundred and fifty shillings.

3. On the first day of the year a boy has £1. 1*s.* 4*d.*, and in each month he spends one-quarter of the money that he has at the commencement of that month. How much does he spend in the first five months, and how much has he left on June 1st?

4. After paying out of £100, £20. 13*s.* 4¾*d.* to my butcher, £30. 15*s.* 6½*d.* to my tailor, and £46. 13*s.* 11¼*d.* to my grocer, I divided the remainder among 27 persons; find the share of each person.

5. Two horses and a carriage cost together £250; one of the horses and the carriage cost £181. 13*s.* 7*d.*; the other horse and the carriage cost £155. 6*s.* 5*d.*: what was the separate cost of each of the three?

6. What sum will remain when four bills, amounting to £5. 17*s.* 4½*d.*, £13. 4*s.* 7¾*d.*, £2. 15*s.* 1*d.*, and £10. 13*s.* 2¼*d.* respectively, have been paid out of £37?

7. A man starts with £10 in his pocket, and pays four bills of amounts £1. 17*s.* 5½*d.*, £2. 13*s.* 2*d.*, £1. 4*s.* 8½*d.*, and £4. 3*s.* 11*d.*; how much has he left?

8. What sum added to £43. 14*s.* $3\frac{3}{4}d.$ will make 48000 farthings?

9. A has £57. 8*s.* $9\frac{1}{4}d.$ and B has 57891 farthings; if A receives from B 11111 farthings, and B receives from A £11. 1*s.* $1\frac{1}{4}d.$, which has the larger sum finally, and by how much?

10. What is the difference in the value of two farms, one being worth £23 per acre, and containing 215 acres, whilst the other is worth £31 per acre, but only contains 155 acres?

11. A has 27 ponies, each worth 22 guineas, and B has 16 horses, worth £20. 15*s.* each; supposing they exchange their property, which man should give the other money, and how much?

12. At a fair a farmer sells 453 sheep at £2. 15*s.* a sheep; and he buys 47 bullocks, 12 at £30. 10*s.* each, 14 at £24. 3*s.* each, and the remainder at £18. 17*s.* 6*d.* each. Does he receive or spend more money, and how much?

13. If I am accustomed to pay 25*s.* a ton for coal, and the same is offered to me at 19*s.* 6*d.* a ton, with carriage at the rate of 3*s.* 2*d.* a ton, what shall I save by accepting the offer and purchasing $12\frac{1}{2}$ tons?

14. How much is spent in 13 years by a person who spends £723. 15*s.* 11*d.* each year? How much does he save during these years if his yearly income be £1034. 17*s.* 5*d.*?

15. A man gives two ten-pound notes to pay for his board during the month of May, which has 31 days, at the rate of 6*s.* 9*d.* per day; what change should he receive?

16. A house and its furniture are worth £2435. 2*s.* 4*d.*, and the house is worth three times its furniture; what is the value of each?

17. A safe and the money it contains are worth £119. 7*s.* 8*d.*, and the money in the safe is worth 57 times the value of the safe; find the value of the latter.

18. How many yards of cloth at 3*s.* $7\frac{1}{2}d.$ a yard should be given in exchange for $935\frac{1}{2}$ yards of velvet worth 18*s.* $1\frac{1}{2}d.$ a yard?

19. How many times may the sum of £3. 2*s.* 4*d.* be paid out of £4325 17*s.* 6*d.*? How many threepenny pieces may the remainder be changed for? What will then be left?

20. A person changed a £10 note, and received an equal number of half-crowns, florins, shillings, sixpences, and threepences; how many were there of each?

21. A person laid out a part of £116. 9*s.* 2*d.* in the purchase of 684 yards of linen at 3*s.* $1\frac{1}{2}d.$ per yard, and the rest in calico at $11\frac{1}{2}d.$ a yard; how many yards of calico did he buy?

22. A man laid out £12. 18*s.* 8*d.* in gloves at 2*s.* 8*d.* the pair; some of them faded and were unsaleable; the rest he sold for £14. 13*s.* 10*d.* at 3*s.* 5*d.* the pair. How many pairs faded?

23. If I spend each week £1. 18*s.* 6*d.* for rent and taxes, £5. 3*s.* 0*d.* for house-keeping bills, £1. 15*s.* for clothes, and £2. 3*s.* 6*d.* for sundries, for how many weeks will £407 last me?

24. If I owe a sum of £6. 12*s.* $8\frac{1}{2}d.$, how soon may I pay it, supposing I can spare only a half-crown, a shilling, a threepenny piece, and a halfpenny per week?

25. The soldiers of a regiment received as prize-money £6720; to afford each soldier 15*s.* extra the prize-money should be £7440. How many soldiers are there in the regiment?

26. A postman whose pay for a week is 15*s.* is fined 1*s.* 6*d.* if he comes in late, and at the end of 13 weeks he receives £8. 15*s.* 6*d.*; how often was he late?

27. A clerk, whose payment is at the rate of £2. 10*s.* per week, is fined 1*s.* for each day that he is late; if in 5 weeks he received £12. 3*s.*, how often was he late?

28. An equal number of men, women, and boys earn £111. 7*s.* 6*d.* in five weeks; if each man earns 3*s.* 9*d.*, each woman 2*s.* 9*d.*, and each boy 1*s.* 9*d.* per day, how many were there of each?

COMPOUND QUANTITIES. WEIGHTS AND MEASURES.

EXAMPLES V.

1. An engine raises a weight of 25 tons 17 cwt. 23 lbs.: how many lbs. is this?

2. How many drams are there in 37 tons 19 cwt. 3 qrs.?

3. Find the number of grains in 25 lbs. 9 oz. 13 dwt. of silver.

4. A gold-digger finds a nugget weighing 16 lbs. 7 oz. 11 grs.: how many grains are there in it?

5. The streets of a town are 14 mi. 2 fur. 5 yds. in length: what is this in inches?

6. How many foot-rules will be required to extend over a distance of 5 poles 3 yards?

7. A piece of matting contains 3 sq. yds. 5 sq. ft.: what is this in sq. ins.?

8. Part of an estate is 15 ac. 2 r. 29 sq. po. 22 sq. yds. 7 sq. ft. 121 sq. ins. in extent: find its area in sq. ins.

9. How many cub. ins. are there in

(1) 31 cub. yds. 21 cub. ft., (2) 437 cub. yds. 17 cub. ft. 1347 cub. ins.?

10. Find the number of pints in

(1) 5 pks. 2 qts. 1 pt., (2) 15 qrs. 5 bush. 3 pks. 1 gal. 3 qts.

11. Supposing an infant's pulse to beat 110 times in a minute, how old is the child after five million and fifty beats of its pulse?

12. Find to the nearest day how long it would take to count a hundred million, supposing one to count at the rate of 150 a minute for 8 hours a day and 6 days a week.

13. How many tons, cwts., etc., are there in 849768 drs. of iron?

14. 3147062 letter weights of 1 oz. each have to be made: find how many tons, cwt., etc., of brass are required.

15. Find in lbs., oz., etc., the weight of 698437 grs. of gold.

16. Find in lbs., oz., etc., the weight of 54687 grs. (Apoth.) of bismuth.

17. Reduce 6437846 yds. and 100000000 in. to miles, furs., etc.

18. Find the number of ac., ro., etc., in 857946 sq. in., and in 41700163 sq. in.

19. Two pits contain 4684 cub. ft. and 793482 cub. in. respectively: find their cubic contents in cub. yds., etc.

20. 2672768 half pints to gallons, quarts, etc.

Perform the following additions:

	d.	hrs.	mins.	secs.
21.	6	17	37	44
	10	23	32	51
	6	7	8	9
	11	5	22	6

	d.	hrs.	mins.	secs.
22.	22	7	16	43
	13	9	14	45
	16	23	37	41
	15	19	49	22
	7	5	18	17

	lbs.	oz.	drs.
23.	24	13	7
	34	8	9
	27	5	4
	33	11	12
	46	0	13

	tons	cwts.	qrs.	lbs.
24.	2	11	1	2
	5	3	0	6
		17	2	9
	18	13	3	1
	13	10	1	9

	oz.	dwt.	grs.
25.	7	13	20
	8	15	18
	5	12	11
	6	16	13
	4	13	21

	lbs.	oz.	dwt.	grs.
26.	8	9	10	11
	9	7	12	15
	8	11	17	19
	7	10	15	9
	6	9	4	3

	yds.	ft.	in.
27.	43	0	10
	21	2	9
	65	1	7
	34	0	11
	16	2	4

	mi.	fur.	yds.
28.	63	6	123
	35	7	25
	82	5	63
	97	1	46
	3	3	97

	sq. yds.	sq. ft.	sq. in.
29.	11	7	37
	17	8	59
	14	7	103
	29	5	97
	34	6	112

	ac.	r.	sq. po.	sq. yds.
30.	5	3	31	22
	18	1	24	18
	17	0	37	17
	8	2	25	25
	4	1	0	28

	qrs.	bush.	pks.	gals.
31.	18	6	0	1
	13	7	2	1
	17	5	1	1
	2	1	3	1
	1	2	3	1

Subtract

32. 22 hrs. 23 mins. 45 secs. from 37 hrs. 43 mins. 36 secs.

33. 2 yrs. 45 days 17 hrs. 23 mins. from 3 yrs. 23 days 12 hrs. 17 mins.

34. 17 lbs. 9 oz. 10 drs. from 37 lbs. 7 oz. 9 drs.

35. 10 oz. 15 dwt. 22 grs. from 13 oz. 12 dwt. 10 grs.

36. 37 oz. 9 dwt. 15 grs. from 40 oz. 11 dwt. 3 grs.

37. 32 po. 4 yds. 2 ft. from 37 po. 3 yds. 1 ft.

38. 10 mi. 7 fur. 136 yds. from 13 mi. 6 fur. 113 yds.

39. 536 ac. 3 ro. 17 sq. po. 9 sq. yds. from 648 ac. 2 ro. 13 sq. po. 7 sq. yds.

40. 16 bush. 3 pks. 1 gal. from 21 bush. 2 pks.

Multiply

41. 7 days 15 hrs. 13 mins. 37 secs. by 10, 70, and 349.

42. 5 tons 15 cwt. 2 qrs. 17 lbs. by 137 and 2560.

43. 8 oz. 11 dwt. 15 grs. by 42 and 257.

44. 4 mi. 5 fur. 17 po. 4 yds. by 84 and 235.

45. 17 sq. yds. 8 sq. ft. 27 sq. in. by 6, 10, and 35.

46. 7 qrs. 6 bush. 3 pks. 1 gal. by 245 and 1000.

47. What is the net weight of 29 barrels, each weighing 2 cwt. 3 qrs. 21 lbs., allowing $5\frac{3}{4}$ lbs. per barrel for waste?

48. Find the weight of 5 dozen spoons each weighing 2 oz. 4 dwt.

49. Soldiers in marching take 75 steps a minute, and in quick marching 108 steps; if a step be taken at 2 ft. 8 in., how far would a company of soldiers go in 3 hours, the last quarter of an hour being at quick march?

50. 400 calendar years are 146097 days; what would be the length of a calendar year in seconds, supposing all such years to be of the same length?

51. Twenty railway wheels together weigh 2 tons 12 cwt. 0 qr. 16 lbs.; what is the weight of each?

52. A loaded truck weighs 15 tons 6 cwt. 84 lbs., and the truck itself weighs 2 tons 5 cwt.; it contains 448 equal packages: find the weight of each package.

53. If 6 dozen table-spoons weigh 11 lbs. 6 oz. 12 dwt., and the same number of tea-spoons weigh 3 lbs. 7 oz. 10 dwt., what is the difference in weight between a table-spoon and a tea-spoon?

54. Three fields measuring respectively 6 ac. 2 r. 14 po., 11 ac. 1 r. 19 po., and 13 ac. 1 r. 12 po. are allotted in equal portions to 35 labourers; what does each get, and how much more land would be required to extend the same gift to 82 labourers?

MISCELLANEOUS EXAMPLES VI.

Weights and Measures.

1. How many seconds are there in 100 ordinary years?

2. How many seconds are there in the month of June?

3. Find the number of seconds between 6.30 p.m. on 1st October, 1901, and 12 noon on 20th June, 1904.

4. How many hours were there between A.D. 1800 and A.D. 1810, the former inclusive and the latter exclusive, allowing for all the leap years?

5. How many grains of gold are there in a cup weighing 8 oz. 4 dwts.?

6. Prove that there are the same number of farthings in £59. 12*s.* $1\frac{3}{4}$*d.* as there are pounds in 25 tons 10 cwt. 3 qrs. 19 lbs.

7. A boy was born in the year 1901; how many days will he have lived on the anniversary of his birthday in the year 1908?

8. How many times does the bell of a clock sound, when striking the hour, in 8 years, two of which will, of course, be leap years?

9. How many days, hours, etc., are there in a lunar month which contains 2551443 seconds?

10. Light travels at the rate of 186330 miles per second, and it takes 8 min. 20 secs. to come from the sun; how far off is the sun?

How many days are there between the following dates, one day only of those named being counted?

11. March 16th, 1905, and May 31st, 1905.

12. February 7th, 1904, and March 4th, 1905.

13. A ship lands at a port a cargo of 480 tons of coal; how many trucks carrying 145 cwt. each can be loaded from it, and how many cwts. remain when they have been filled?

14. How long will 20 tons of coal last for 5 fires, each fire consuming 18 lbs. per day?

15. A bar of metal, whose weight is 53 lbs. 5 dwt., is made into coins, each coin weighing 1 oz. 5 dwt. each; find how many coins were obtained.

16. How many rings, each weighing 4 dwt. 18 grs., can a goldsmith make from a mixture of 1 lb. 10 oz. 1 dwt. 18 grs. of pure gold with 7 oz. 7 dwt. 6 grs. of alloy?

17. A silver tankard, which weighs 1 lb. 10 oz. 10 dwt., cost £2. 12*s.* 6*d.*; find the cost of the silver per ounce.

18. How many fields, each containing 9 ac. 22 po. 12 sq. yds. are there in 201 ac. 12 po. 22 sq. yds.?

19. How many pieces of string, each 4 ft. 5 in. long, can be cut from a length of 52 yds.?

20. How many pieces of ribbon, each 14 ins. long, can be cut from a roll of length 100 yds., and how much is left over?

21. The bars of an iron railing are 5 ft. 10 in. long. If laid down end to end they would stretch to a distance of 23 mi. 610 yds. How many rails were there?

22. How many florins in a continuous straight line would reach from Cambridge to London, a distance of 57 miles, the width of a florin being one inch, and what would be their total value in pounds?

23. A railway is 1572 miles long; find the number of bolts required to fix the chairs for the rails upon the sleepers, there being 2 bolts to each chair, 2 chairs to each sleeper, and the sleepers being 11 inches apart.

24. The distance between two telegraph stations is 18 mi. 1 fur. 44 yds.; how many posts, 8 rods apart, are there between them?

25. On the occasion of the thanksgiving for the recovery of the Prince of Wales it was estimated that the length of the route was 7 miles, and that there were 20 rows of people on each side of the route, and that each person presented a front of 15 inches; how many spectators were there?

26. The wheels of a waggon go round 6600 and 3520 times respectively in a certain journey. One of the larger wheels is 13½ feet in circumference. Find (1) the distance in miles, and (2) the circumference of one of the smaller wheels.

27. A wheel makes 1028 revolutions in passing over 2 miles 4 furlongs 9 poles 5 yards 6 inches ; what is the circumference of the wheel ?

28. The circuit of a racing-path is 137½ yards, how many times must a bicyclist go round to complete 5 miles ?

29. A wheel makes 1542 revolutions in passing over 2 miles 249 yards 1 foot ; what is its circumference ?

30. The circumferences of the large and small wheels of a bicycle are 96 and 88 inches respectively ; how many more turns will the latter have made than the former in going a distance of 15 miles ?

31. A carriage wheel is 8 feet 3 inches in circumference ; how many turns does it make in driving 7 miles 1331 yards ?

32. An estate of 1416 ac. 2 r. 16 po. was divided into allotments each 4 ac. 3 r. 27 po. in area ; how many allotments were made ?

33. A field containing 18 acres 36 poles is divided into allotments each containing 1 rood 14 poles ; how many of them are there ?

34. How many trucks are there in a coal-train which carries 101½ tons if each truck holds 7 tons 6 cwt. 3½ qrs. ?

35. Into how many lots can 57 tons 5 cwt. 1 qr. 7 lbs. be divided so that each lot contains 1 ton 12 cwt. 2 qrs. 25 lbs. ?

36. Assuming that a sovereign weighs 123 grains, find how many sovereigns can be coined from 10 lbs. (Av.) of gold.

37. Twenty pounds of copper are drawn out into a fine copper wire which is such that 60 yards of it weigh 48 oz. ; how many yards of the copper wire are produced ?

38. If 1*s.* 6*d.* a week be laid by out of a salary of £10 per year of 52 weeks, how much per week is left to be spent ?

39. A man bought 7 pieces of cloth, each containing 27 yards, for £55. 12*s.*, and he sold 56 yards at 5*s.* 3½*d.* per yard ; at what price per yard must he sell the rest in order to gain £3. 11*s.* on the whole ?

40. If a man spends £20. 6*s.* 8*d.* per week, and in 9 years saves £3814. 4*s.*, what was his income per day taking a year as 52 weeks ?

41. A sovereign weighs nearly 123¼ grains ; how many lbs., etc., will 10000 sovereigns weigh ?

42. How much silk at 5*s.* 6*d.* a yard can be bought for 22 guineas ?

CHAPTER II.

MEASURES AND MULTIPLES OF NUMBERS.

GREATEST COMMON MEASURE AND LEAST COMMON MULTIPLE.

1. One number is said to be *divisible* by another when, if the first number be divided by the second, there is no remainder. Thus 12 is divisible by 2, since $12 \div 2 = 6$.

57 „ 3, „ $57 \div 3 = 19$.

2. If one number divide another exactly, that is, without a remainder, the first number is said to be a **measure** of the second. Thus, in the example of the previous article 2 is a measure of 12 and 3 a measure of 57.

3. If two or more numbers on being multiplied together give a third number, each of these two, or more, numbers is a **factor** of the third number.

Thus $2 \times 3 = 6$; hence both 2 and 3 are factors of 6. Again, $4 \times 7 \times 11 = 308$; hence 4, 7, and 11 are all factors of 308.

4. Even and Odd. Definition. A number is said to be even if it be divisible by 2. If it be not divisible by 2 it is called an odd number.

The even numbers are thus 2, 4, 6, 8, 10, 12, 14, 16, ...

The odd numbers are 1, 3, 5, 7, 9, 11, 13, 15, ...

Hence odd and even numbers occur alternately.

5. Prime Numbers. Definition. A prime number is a number which is divisible by no other number except unity. Numbers which are not prime, and of which therefore factors can be found, are called composite numbers.

Thus the prime numbers, which are less than 50, are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47. For it would be found that none of these numbers could be divided without a remainder by any other number.

No even number is a prime number except 2. For every other even number is divisible by 2, and therefore is not prime.

6. Two numbers are said to be **prime to one another** when there is no number, except unity, which will divide them both.

Thus 8 and 9 are prime to one another. [For the only numbers, except unity, which divide 8 are 2 and 4; and the only number that divides 9 is 3. There is therefore no number which divides both 8 and 9.] It will be noted, however, that neither eight nor nine are prime numbers.

7. It will be found that any number is divisible by

2, if the figure in the units place be even or zero;
3, if the sum of its digits be divisible by 3;
4, if the number formed by its last two digits be divisible by 4;
5, if the figure in the units place be 5 or 0;
8, if the number formed by its last three digits be divisible by 8;
9, if the sum of its digits be divisible by 9;
10, if the figure in the units place be 0.

8. *Consider the number* 210.

It is divisible by 2 (rule), and we see that

$$210 = 2 \times 105.$$

Again 105 is divisible by 3 (rule), and we have

$$210 = 2 \times 3 \times 35.$$

Also 35 is divisible by 5, and we get

$$210 = 2 \times 3 \times 5 \times 7.$$

Instead of 210 we now have the four prime numbers 2, 3, 5, 7 multiplied together. These are called the **Prime Factors** of 210, and 210 itself is called a **Composite Number** because it is *composed* of prime numbers multiplied together.

If we take another composite number, say 3276, we shall find in the same way that

$$3276 = 2 \times 2 \times 3 \times 3 \times 7 \times 13 = 2^2 . 3^2 . 7 . 13.$$

All composite numbers can be split up in this manner into their prime factors. We therefore get the following definition: *The prime numbers whose product makes up a composite number are called the* **Prime Factors** *of that number.*

9. Useful Rule for finding the Prime Factors of any numbers. Divide by 2 as often as you can, then by 3 as often as you can, then by 5 as often as you can, and so on, *beginning with the lowest possible prime numbers.*

Ex. *Find the Prime Factors of* 138600.
The process is as annexed;

$$\therefore\ 138600 = 2^3 . 3^2 . 5^2 . 7 . 11.$$

2	138600
2	69300
2	34650
3	17325
3	5775
5	1925
5	385
7	77
	11

10. It is thus found that in breaking a number into Prime Factors the simplest method is to determine whether it is divisible by the successive prime numbers. The prime numbers from 1 to 50 have been given in Art. 5.

EXAMPLES VII.

In the cases of the following numbers obtain, where possible, the prime factors which are less than 12:

1. 63, 119, 169.
2. 216, 343, 235.
3. 1001, 1211, 985, and 1793.
4. 52104, 24066, and 67914.
5. Write down the prime numbers between 200 and 250.
6. Which of the following numbers are prime:
67, 87, 99, 113, 129, 633, 451, and 237?

Break up into prime factors the numbers:

7. 35, 48, 66, 75, 88, 96.
8. 108, 135, 168, 176, 196, 224.
9. 236, 248, 296, 306, 342, 624.
10. 464, 567, 736, 812, 864, 1376.
11. 1518, 1408, 2532.
12. 2528, 6111, 8118.
13. 5754, 4364, 4874.
14. 9702, 5775.
15. 2156, 32175.

11. Common Measure. Any number which divides two or more given numbers is called a Common Measure of the given numbers.

Thus 3 is a common measure of 24 and 30, because it divides both 24 and 30. Also, 7 is a common measure of 42, 56, 84, and 98, because it divides these four numbers.

12. The *greatest* number which divides two or more given numbers is called their **Greatest Common Measure.** It is often also called the Highest Common Factor.

The expression Greatest Common Measure is, for brevity, generally written G.C.M. Similarly the expression Highest Common Factor is shortened to H.C.F.

13. On breaking into factors the numbers 648 and 1584 we have $648 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$ .....(1)
and $1584 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 11$ .....(2)

Thus the only prime factors common to 648 and 1584 are 2's and 3's.

The greatest number of 2's in either is 3, and the greatest number of 3's in either is 2.

Hence the G.C.M. or the H.C.F.

$$= 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2 = 72.$$

One method of finding the G.C.M. of two or more numbers is therefore to break them up into their component factors.

EXAMPLES VIII.

Find the G.C.M. of

1. 16 and 24. **2.** 15 and 25. **3.** 21 and 49. **4.** 22 and 66.
5. 32 and 72. **6.** 45 and 81. **7.** 54 and 90. **8.** 68 and 85.
9. 84 and 126. **10.** 98 and 119. **11.** 108 and 172. **12.** 196 and 294.
13. 224 and 296. **14.** 264 and 594. **15.** 287 and 451. **16.** 486 and 738.
17. 1128 and 1464. **18.** 1419 and 2277. **19.** 72, 132, and 156.
20. 240, 336, and 528. **21.** 154, 385, 539, and 847. **22.** 1001, 1573, and 1859.

14. If, however, the numbers be large, or if the prime factors be themselves large, the process of finding the prime factors is in general rather long. The G.C.M. of two numbers may always, however, be found by the following rule:

Divide the smaller of the two numbers into the greater and determine the remainder; divide this remainder into the first divisor and determine the second remainder; divide the second remainder into the second divisor, and so on until there is no remainder. The last divisor is the G.C.M. *required. If this last divisor is* 1 *the two numbers have no measure except unity.*

The proof of the rule is beyond the student at this stage.

Ex. *Find the* G.C.M. *of* 1363 *and* 1034.

```
1034 ) 1363 ( 1
       1034
       ----
        329 ) 1034 ( 3
              987
              ---
               47 ) 329 ( 7
                    329
                    ...
```

G.C.M. = 47.

```
3 ) 1034 | 1363 ( 1
    987  | 1034
    ---  |
     47  |  329 ( 7
         |  329
         |  ...
```

Notice that, in the second method, which saves space, each quotient is written in a line with the corresponding dividend and on the same side as it.

15. When we have to find the G.C.M. of more than two numbers we may either break the numbers into their prime

factors and then pick out the G.C.M., or we may find the G.C.M. of two and then the G.C.M. of the result and the third; then the G.C.M. of this result and the fourth, and so on.

EXAMPLES IX.

Find the G.C.M. of

1. 851 and 943. **2.** 667 and 1073. **3.** 2067 and 2691.
4. 2419 and 5617. **5.** 4199 and 5083. **6.** 1209 and 1457.
7. 2343 and 11076. **8.** 2262 and 2951. **9.** 5146 and 12201.
10. 7733 and 7999. **11.** 66429 and 169037. **12.** 15463 and 554470.
13. 61140 and 556206. **14.** 571428 and 999999.
15. 3780, 4410, and 3423. **16.** 31464, 50616, and 58824.

Are the following numbers prime to one another; if not what is their G.C.M.?

17. 187, 209. **18.** 221, 357. **19.** 247, 414. **20.** 685, 346.

Least Common Multiple.

16. Multiple. A multiple of any given number is any number which is divisible by the given number.

Thus the multiples of 3 are 6, 9, 12, 15, ... (For 3 divides each of these numbers.) So the multiples of 7 are 14, 21, 28, 35, ...

17. Common Multiple. Any number which is a multiple of each of two, or more, given numbers is a common multiple of these given numbers, and the least of such common multiples is called the **Least Common Multiple** of the given numbers. The expression Least Common Multiple is generally abbreviated into L.C.M.

Thus the multiples of 2 are 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, ..., and the multiples of 3 are 6, 9, 12, 15, 18, 21, 24, ...

The common multiples of 2 and 3 are therefore 6, 12, 18, 24, ..., and their L.C.M. is clearly 6.

18. The L.C.M. of two numbers which are prime to one another is their product.

For consider the two numbers 9 and 8, which are prime to one another, and therefore have no common factor. Any common multiple must be divisible by both 9 and 8, and the least number divisible by 9 and by 8 is 72, that is, 9×8.

19. If the two numbers be not prime to one another (that is, if they have a common factor), their least common multiple is not their product.

Consider the two numbers 8 and 12, which are not prime to one another. Their product is 96; this number, though

a common multiple, is not the *least* common multiple; for the numbers 24, 48, and 72 are all multiples of 8 and 12, and are all less than 96.

20. When the numbers whose least common multiple is required can be easily broken up into factors, we can at once write down the L.C.M.

Ex. *Find the* L.C.M. *of* 27, 24, *and* 15.

We have $27 = 3 \times 9 = 3 \times 3 \times 3 = 3^3$, ...(1)

$24 = 3 \times 8 = 3 \times 2 \times 2 \times 2 = 3 \times 2^3$, ...(2)

and $15 = 3 \times 5$.....(3)

Since the L.C.M. must be divisible by 27, *i.e.* 3^3,

$\therefore$ $3 \times 3 \times 3$ must be a factor of it.

Again, it must be divisible by 24, *i.e.* $2^3 \times 3$;

$\therefore$ $2 \times 2 \times 2 \times 3$ must be a factor.

Lastly, it must be divisible by 15, *i.e.* 3×5;

$\therefore$ 3×5 must be a factor.

Selecting the least number of these prime factors which will ensure that 27, 24, and 15 shall divide their product, we obtain

$$2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5;$$

$$\therefore \text{ L.C.M.} = 2^3 . 3^3 \times 5 = 8 \times 27 \times 5 = 1080.$$

Ex. *Find the* L.C.M. *of* 30, 42, 245, *and* 63.

Here $30 = 2 \times 15 = 2 \times 3 \times 5$, $42 = 2 \times 21 = 2 \times 3 \times 7$,

$245 = 5 \times 49 = 5 \times 7^2$, and $63 = 7 \times 9 = 7 \times 3^2$.

The highest powers of 2, 3, 5, and 7 that occur are therefore 2, 3^2, 5, and 7^2.

Thus the required L.C.M. $= 2 \times 3^2 \times 5 \times 7^2 = 2 \times 9 \times 5 \times 49 = 4410$.

The rule for finding the L.C.M. may be stated as follows: *Break up each number into its prime factors and powers of prime factors; select the highest power of each prime factor that occurs in these products; multiply together these highest powers; the result is the required* L.C.M.

21. In many cases the work may be more expeditiously performed as follows:

Set down the numbers in a line and separate them by commas. Divide them by any prime number which is a factor of two, or more of them.

In our case the first two are clearly divisible by 2. Divide this 2 into the numbers into which it is divisible, set down the corresponding quotients and also the other two numbers, 245 and 63, unaltered.

2) 30, 42, 245, 63

3) 15, 21, 245, 63

5) 5, 7, 245, 21

7) 1, 7, 49, 21

1, 1, 7, 3

We see that the next highest prime number, 3, divides three of the numbers in the second line. Perform the division by 3, set down the quotients and also the unaltered number 245.

The next prime number, 5, divides two of the numbers in the third line, and hence, as before, we obtain the fourth line.

Finally, the next prime number, 7, divides three of the numbers in the fourth line, and hence we obtain the fifth line.

The L.C.M. required is equal to the product of the numbers in the last line and of the numbers by which we have divided. Thus the required answer $= 2 \times 3 \times 5 \times 7 \times 7 \times 3 = 4410$.

22. If any of the numbers given are seen to be divisible into any of the other numbers given, they may be at once left out, and thus the process is somewhat shortened.

Ex. *Find the* L.C.M. *of* 9, 77, 11, 117, 63, 847, 1089, 13.

$$\begin{array}{r|l} 3 & \cancel{9},\ \cancel{77},\ \cancel{11},\ 117,\ 63,\ 847,\ 1089,\ \cancel{13} \\ \hline 3 & 39,\ 21,\ 847,\ 363 \\ \hline & 13,\ \cancel{7},\ 847,\ \cancel{121} \end{array}$$

Set down the numbers in a line with commas between. We may omit the number 9 because any number which is divisible by 63 is also divisible by 9. We therefore cross it through. Again 77 and 11 both divide 847 and 13 divides 117. We cross them through also. We now choose the *smallest* prime factor of *two* or *more* of the remaining numbers and divide all we can by it. If a number cannot be divided exactly we write it down again. Doing this, we get in the second line 39, 21, 847, 363. Repeating the operation, we get in the third line 13, 7, 847, 121. Since 7 and 121 divide 847, we cross them through. We cannot continue the process, and the L.C.M. will be the product of the numbers in the last line and of the divisors;

$$\therefore \text{ L.C.M.} = 3 \times 3 \times 13 \times 847 = 99099.$$

In some cases it is difficult to see any prime factors of the quantities whose L.C.M. is required. In such cases we find the G.C.M. of two quantities, and this G.C.M. will be the largest number that will divide both.

EXAMPLES X.

Find the L.C.M. of

1. 6 and 10. **2.** 8 and 18. **3.** 18 and 45. **4.** 36 and 54.

5. 63 and 105. **6.** 25 and 35. **7.** 21 and 28. **8.** 39 and 52.

9. 57 and 76. **10.** 65 and 91. **11.** 82 and 123. **12.** 3, 4, 5, and 6.

13. 12, 18, 27, and 54. **14.** 11, 30, 42, 66, and 90.

15. 44, 48, 52, and 96. **16.** 39, 50, 75, 90, 117, and 145.

17. 7, 8, 9, 54, 72, 81, and 99. **18.** 15, 18, 24, 27, 32, and 36.

19. 299 and 351.

20. 391 and 493.

21. 144, 192, 324, 360, and 576.

22. 364, 2520, and 5265.

Resolve into prime factors, and hence deduce the G.C.M. and L.C.M of

23. 210, 429, and 504.

24. 1020, 5445, and 9350.

Find the G.C.M. and L.C.M. of

25. 3864 and 3969.

26. 3024, 4752, and 7488.

27. 7560, 27720, and 108108.

23. We subjoin some miscellaneous examples.

Ex. *Find the greatest number which will divide* 5731 *and* 4612 *and leave as remainders* 19 *and* 34 *respectively.*

If a number divide 5731 and leave a remainder 19, the same number must divide 5731 − 19, that is 5712, and *leave a remainder* 0, *i.e.* it must divide 5712 exactly.

So any number which divides 4612 and leaves a remainder 34 must divide 4612 − 34, *i.e.* 4578 exactly.

We therefore require the largest number which will divide 5712 and 4578 exactly. This will be their G.C.M., and is found to be 42.

$$
\begin{array}{r|l}
4578 & 5712 \,(\, 1 \\
 & \underline{4578} \\
 & 1134 \\
\hline
1134 & 4578 \,(\, 4 \\
 & \underline{4536} \\
 & 42 \\
\hline
42 & 1134 \,(\, 27 \\
 & \underline{84} \\
 & 294 \\
 & \underline{294} \\
 & \cdots
\end{array}
$$

MISCELLANEOUS EXAMPLES XI.

G.C.M. and L.C.M.

1. Find the greatest number which will divide 2000 and 2708, and leave remainders 11 and 17 respectively.

2. Find the greatest number by which 3912 and 92832 must be divided so as to leave as remainders 40 and 25 respectively.

3. A number between 6000 and 7000 when divided by 7, 9, 11, or 15 always leaves the same remainder 4; find the number.

4. Find the greatest weight, in grains, that will measure both a lb. Troy and a lb. Av.

5. Three cisterns contain respectively 2310, 819, and 4998 gallons of water. Find the vessel of greatest capacity by which they can be emptied.

6. What is the largest sum of money that can be paid an exact number of times by each of two people, who have respectively £18. 18*s.* 3*d.* and £32. 2*s.* 9*d.* in their possession?

7. What is the length of the longest tape that will exactly measure two fields, whose lengths are 817 and 989 yards?

8. What is the least number which, when divided by 7, 8, or 11, always gives the remainder 2?

9. What is the largest number that divides both 2397 and 2491 without a remainder?

What is the smallest number that can be divided by either of these numbers without a remainder?

10. Find the least number which when divisible by 2, 20, 7, 9, 63, 45, 11, or 117 will leave a remainder 1.

11. Find the least number that can be divided exactly by all numbers up to 12 inclusive.

12. Find the least sum of money which contains 14*s.*, 32*s.* 8*d.*, and 21*s.* each an exact number of times. Find also the greatest common measure of these three sums.

13. Find the smallest sum of which 14*s.* 7*d.*, £1. 11*s.* 6*d.*, and £3. 15*s.* are exact parts.

14. Taking a kilometre as 39369 inches, find the shortest distance that can be expressed as an exact number of miles, and also as an exact number of kilometres.

15. What is the least length which is a multiple of 11 ft., 14 ft. 8 in., and 16 ft. 6 in., these being the lengths of three iron bars?

Also what is the least number of bars of the same length into which the three bars can be divided?

16. Find the least sum of money that could be paid in either florins, half-crowns, or guineas.

17. A franc being taken as $9\frac{1}{2}d.$ and a dollar as 4*s.* $1\frac{1}{2}d.$, find the least debt in dollars that can be paid by an exact number of francs.

18. Write down all the numbers between 100 and 1000 that have 137 as their G.C.M.

19. There are four bells, each of which strikes at intervals of 3, 7, 12, and 14 seconds. The four begin to strike at 12 o'clock; when will they next strike together, and how often will they strike in unison in 7 minutes?

20. The circumference of the front wheel of a carriage is $6\frac{1}{4}$ feet, and that of the back wheel is $10\frac{3}{4}$ feet; how far must the carriage go before both wheels finish complete revolutions together?

21. Four persons walk continuously in a circle starting at the same time from the same point and in the same direction. They complete the circle in $2\frac{1}{2}$, 3, $3\frac{1}{2}$, and 4 minutes respectively. After how long will they be again at the starting point, and how many times will each have been round?

22. What is the shortest length of a race-course that can be measured exactly by tapes whose lengths are 30 yds. and 1540 ft.?

23. If on a straight road stones be placed at intervals of 50 feet, and a measuring tape of 15 yards be used to measure the road, commencing at one of the stones, how often will the other end of the tape coincide with a stone in the distance of one mile?

CHAPTER III.

VULGAR FRACTIONS.

24. The numbers which we have been using up till now contain an exact number of units, and are called **Whole Numbers** or **Integers.**

Suppose, however, that any unit (for example a rod one yard long) is divided into 5 equal parts, then each of these parts is called one-fifth ($=\frac{1}{5}$) of the whole rod; two of these parts together make two-fifths ($=\frac{2}{5}$); three of the parts make three-fifths ($=\frac{3}{5}$), and so on. Each of these quantities, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$ ..., is called a Fraction or Broken Number. [The word Fraction is derived from the Latin word *Fractus*, which means Broken.]

We see that three-fifths is printed thus, $\frac{3}{5}$. Hence the lower number tells us into how many parts the whole rod is divided and is called the **Denominator**; whilst the upper number tells us how many of these parts are taken and is called the **Numerator.** Thus $\frac{5}{8}$ of £1 would mean that £1 is to be divided into 8 equal parts and 5 of them are to be taken.

25. Fractions, such as $\frac{5}{8}$, $\frac{7}{16}$, and $\frac{35}{17}$, are called **Vulgar,** that is, **Common** Fractions.

When the denominators consist of tens, or powers of tens, the corresponding fractions are often written in another manner, as we shall see later on, and are then called **Decimal** Fractions.

EXAMPLES XII.

Express in figures the following fractions:

1. Two thirds, five ninths, and six thirteenths.

2. Seventeen thirty-thirds, forty-four eighty-fifths, and one hundred and three two hundred and seventieths.

Write down in words the values of

3. $\frac{5}{8}$, $\frac{7}{9}$, $\frac{8}{13}$. **4.** $\frac{7}{20}$, $\frac{13}{23}$, $\frac{15}{47}$. **5.** $\frac{37}{105}$, $\frac{43}{897}$.

What is the value of

6. $\frac{2}{3}$ of 1*s.*, $\frac{1}{4}$ of £1, $\frac{1}{8}$ of £1. **7.** $\frac{15}{16}$ of £1, $\frac{5}{8}$ of 11*s.*, $\frac{11}{16}$ of 12*s.*

8. $\frac{2}{7}$ of a guinea, $\frac{3}{8}$ of 2*s.* 6*d.* **9.** $\frac{1}{7}$ of a cwt. **10.** $\frac{11}{32}$ of a mile.

26. Proper and Improper Fractions. A fraction like $\frac{7}{12}$, whose numerator is less than its denominator, is called a *Proper Fraction.* Such a fraction is clearly less than unity, since the whole is divided into a certain number of parts and a less number taken to form the fraction.

A fraction whose numerator is equal to, or greater than, its denominator is termed an *Improper Fraction*, *e.g.* $\frac{7}{7}$, $\frac{15}{11}$.

A fraction like $\frac{12}{12}$ is clearly equal to unity, since the whole is divided into a certain number of parts and all are taken.

$$\therefore \quad 1 = \tfrac{12}{12} = \tfrac{5}{5} = \tfrac{29}{29} \ldots .$$

Fractions like $\frac{19}{12}$ may at first puzzle the student. "How can unity be divided into 12 parts and 19 of these be taken?" he may ask. He must remember that in such a case *more units than one are divided.* Thus, if 2 units are each divided into 12 parts, we have altogether 24 parts, and of these 19 may be taken, *e.g.* 2 ft. = 24 in., of which we may take 19 in., *i.e.* $\frac{19}{12}$ of a ft.

It follows that $2 = \frac{24}{12} = \frac{14}{7} = \frac{18}{9} \ldots$, according as we divide each of our units into 12, 7, or 9 parts and take all of them in each case.

Similarly $5 = \frac{5}{1} = \frac{35}{7} = \frac{40}{8} = \frac{60}{12}$.

Thus, if we wish to express any whole number as a vulgar fraction with a given denominator, we multiply the whole number by the given denominator, and the product will be the required numerator.

27. Mixed Numbers. We now see that

$$\frac{41}{12} = \frac{36+5}{12} = \frac{36}{12} + \frac{5}{12} = 3 + \frac{5}{12}.$$

This last result is written $3\frac{5}{12}$, and is read thus: Three and five-twelfths. Such quantities as $3\frac{5}{12}$, consisting of a whole number and a fraction, are called Mixed Numbers.

28. *To express a given mixed number as an improper fraction.* Take as an example the mixed number $5\frac{7}{8}$.

By Art. 26 we can express 5 as a fraction with denominator 8. Multiply 8 by 5 and we have 40.

Thus $5 = \frac{40}{8}$. Hence $5\frac{7}{8} = 5 + \frac{7}{8} = \frac{40}{8} + \frac{7}{8} = \frac{47}{8}$.

29. *To reduce an improper fraction to a whole or mixed number.* If we divide the numerator by the denominator the quotient will be the whole part of the mixed number, and the remainder will be the numerator of the fractional part.

Ex. 1. $\frac{21}{7} = \frac{21+4}{7} = \frac{21}{7} + \frac{4}{7} = 3 + \frac{4}{7} = 3\frac{4}{7}$.

For $\frac{21}{7}$ denotes that the whole is to be divided into 7 parts and 21 of them taken. But 7 of these parts making up the unit, 14 of them make up twice the unit, and 21 of them make three times the unit, that is, $\frac{21}{7} = 3$ times unity $= 3$.

Ex. 2. $\frac{77}{9} = 8\frac{5}{9}$; $\frac{40}{5} = 8$; $\frac{7}{6} = 1\frac{1}{6}$, etc.

Ex. 3. $\frac{3845}{247} = 15\frac{140}{247}$. For, on dividing 3845 by 247, the quotient is 15 and the remainder 140.

EXAMPLES XIII.

Express the whole numbers

1. 3, 4, 7, 9, and 13 as fractions with denominator 5.

2. 8, 11, 17, 23, 34, and 46 ,, ,, 7.

3. 14, 17, 22, 29, 37, 59, and 83 ,, ,, 11.

Express as improper fractions the following mixed numbers:

4. $1\frac{1}{4}$, $2\frac{1}{2}$, $3\frac{1}{3}$. **5.** $4\frac{1}{5}$, $3\frac{2}{3}$, $2\frac{1}{6}$. **6.** $1\frac{3}{5}$, $2\frac{4}{5}$, $1\frac{7}{8}$. **7.** $5\frac{2}{5}$, $6\frac{3}{7}$, $7\frac{4}{9}$.

8. $4\frac{3}{4}$, $6\frac{3}{8}$, $8\frac{5}{9}$. **9.** $7\frac{3}{10}$, $8\frac{9}{11}$, $6\frac{3}{5}$. **10.** $2\frac{9}{100}$, $4\frac{101}{1000}$, $7\frac{7087}{10000}$.

11. $11\frac{3}{4}$, $13\frac{5}{6}$, $15\frac{6}{7}$. **12.** $17\frac{3}{8}$, $19\frac{5}{9}$, $21\frac{7}{11}$. **13.** $2\frac{3}{13}$, $3\frac{14}{15}$, $4\frac{5}{17}$.

Reduce to whole or mixed numbers the following improper fractions:

14. $\frac{8}{2}$, $\frac{12}{4}$, $\frac{6}{3}$. **15.** $\frac{18}{3}$, $\frac{28}{7}$, $\frac{56}{8}$. **16.** $\frac{17}{2}$, $\frac{13}{4}$, $\frac{11}{3}$.

17. $\frac{37}{6}$, $\frac{37}{9}$, $\frac{43}{11}$. **18.** $\frac{57}{10}$, $\frac{47}{11}$, $\frac{53}{13}$. **19.** $\frac{65}{9}$, $\frac{67}{14}$, $\frac{69}{17}$.

20. $\frac{39}{16}$, $\frac{53}{18}$, $\frac{67}{15}$. **21.** $\frac{346}{19}$, $\frac{853}{36}$. **22.** $\frac{9847}{47}$, $\frac{2856}{93}$.

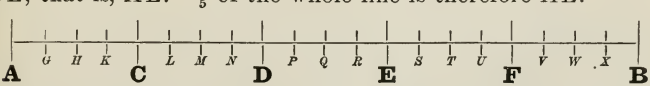
23. The thirteenth part of nineteen.

24. The sixty-third part of four thousand and five.

30. *Any simple vulgar fraction is unaltered if its numerator and denominator be multiplied by the same number.*

Consider the fraction $\frac{3}{5}$, and suppose that its numerator and denominator are both multiplied by 4. The fraction then becomes $\frac{12}{20}$.

Take a line AB and divide it into five equal parts AC, CD, DE, EF, and FB. Then each of these parts is $\frac{1}{5}$ of the whole; thus $\frac{3}{5}$ of the whole is 3 of these parts, that is, AC, CD, and DE, that is, AE. $\frac{3}{5}$ of the whole line is therefore AE.



Now divide each of the lines AC, CD, DE, EF, and FB into four equal parts, so that altogether we have the twenty equal

parts AG, GH, HK, KC, ... XB. Each of these is $\frac{1}{20}$ of the whole. Now AE is equal (as will be seen on counting) to 12 of these, that is, to $\frac{12}{20}$ of the whole.

Thus AE is equal both to $\frac{3}{5}$ of AB and also to $\frac{12}{20}$ of AB.

Therefore $$\frac{3}{5}=\frac{12}{20}=\frac{3\times 4}{5\times 4}.$$

In exactly the same way any other case may be considered. We thus see the truth of the statement at the beginning of this article.

31. Again, since $\frac{3}{5}=\frac{12}{20}$, therefore $\frac{12}{20}=\frac{3}{5}$, and $\frac{3}{5}$ is derived from $\frac{12}{20}$ by dividing both numerator and denominator by 4.

Hence it is clear that a fraction is unaltered if its numerator and denominator be divided by the same number.

Thus $\frac{138}{207}=\frac{126}{189}$ (by dividing both numerator and denominator by 11)
$=\frac{14}{21}$ (by dividing both by 9)
$=\frac{2}{3}$ (by dividing both by 7).

32. The smaller the numerator and denominator of a fraction are, the more manageable it becomes. The numerator and denominator of a fraction should therefore be always divided by any common factor that they contain. When all the common factors have been divided out the fraction is said to be in its *lowest terms*, and the operation of dividing out these common factors is known as *reducing the fraction to its lowest terms*.

Examples of fractions reduced to their lowest terms:

Ex. 1. $\frac{28}{21}=\frac{7\times 4}{7\times 3}=\frac{4}{3}$.

Ex. 2. $\frac{1683}{2431}=\frac{11\times 153}{11\times 221}=\frac{153}{221}=\frac{17\times 9}{17\times 13}=\frac{9}{13}$.

In finding whether the numerator and denominator have a common factor, the student should try as divisors the successive prime numbers 2, 3, 5, 7, 11, 13, 17, 23, ...

33. It will probably be more convenient, if none of the prime numbers up to 13 are factors, to at once find the G.C.M. of the numerator and denominator. For, if the student cannot guess a factor which will divide both the numerator and denominator of a fraction, he can always determine such a factor, if it exist, by finding their G.C.M.

EXAMPLES XIV.

Reduce to their lowest terms the following fractions:

1. $\frac{8}{10}, \frac{9}{15}, \frac{16}{24}$. **2.** $\frac{7}{14}, \frac{8}{16}, \frac{10}{30}$. **3.** $\frac{5}{15}, \frac{11}{33}, \frac{12}{48}$.

4. $\frac{5}{25}, \frac{7}{49}, \frac{9}{24}$. **5.** $\frac{16}{24}, \frac{18}{32}, \frac{20}{36}$. **6.** $\frac{4}{12}, \frac{3}{18}, \frac{6}{16}$.

7. $\frac{13}{52}, \frac{14}{84}, \frac{15}{75}$. **8.** $\frac{165}{30}, \frac{144}{60}, \frac{168}{28}$. **9.** $\frac{175}{50}, \frac{185}{15}, \frac{196}{35}$.

10. $\frac{69}{123}, \frac{64}{148}, \frac{60}{318}$. **11.** $\frac{37}{111}, \frac{43}{215}, \frac{59}{236}$. **12.** $\frac{115}{155}, \frac{255}{285}$.

13. $\frac{144}{648}, \frac{231}{1056}, \frac{729}{1296}$. **14.** $\frac{744}{2328}$. **15.** $\frac{949}{1387}$. **16.** $\frac{1081}{1311}$.

17. $\frac{447}{4461}$. **18.** $\frac{4531}{5819}$. **19.** $\frac{10998}{320058}$. **20.** $\frac{35649}{86317}$.

34. *To compare the values of different fractions.*

Suppose we were asked to compare the values of $\frac{3}{5}$ and $\frac{4}{7}$, *i.e.* to find out which is the greater fraction. The student might possibly think $\frac{4}{7}$ greater than $\frac{3}{5}$, since 4 is greater than 3 and 7 is greater than 5.

Let us, however, consider a rod 35 inches long as our unit,

$\frac{1}{5}$ of it = 7 in.; $\therefore$ $\frac{3}{5}$ of it = 21 in. = $\frac{21}{35}$ of it.

Again $\frac{1}{7}$ of it = 5 in.; $\therefore$ $\frac{4}{7}$ of it = 20 in. = $\frac{20}{35}$ of it.

But $\frac{21}{35}$ is greater than $\frac{20}{35}$; $\therefore$ $\frac{3}{5}$ is greater than $\frac{4}{7}$.

This shows us that if we want to compare fractions, we should make their denominators the same.

Now $\frac{3}{5} = \frac{3 \times 7}{5 \times 7} = \frac{21}{35}$, and $\frac{4}{7} = \frac{4 \times 5}{7 \times 5} = \frac{20}{35}$.

Here 35 is called the Least Common Denominator (L.C.D.) of the fractions. It will naturally be the L.C.M. of the denominators. *To find the multiplier for any fraction we divide its denominator into the* L.C.D.

Ex. *Arrange in order of magnitude the fractions* $\frac{7}{15}$, $\frac{17}{35}$, *and* $\frac{10}{21}$.

Since $15 = 3 \times 5$, $35 = 5 \times 7$, and $21 = 3 \times 7$, the L.C.M. of the denominators $= 3 \times 5 \times 7 = 105$.

Hence $\frac{7}{15} = \frac{7 \times 7}{15 \times 7} = \frac{49}{105}$, $\frac{17}{35} = \frac{17 \times 3}{35 \times 3} = \frac{51}{105}$, and $\frac{10}{21} = \frac{10 \times 5}{21 \times 5} = \frac{50}{105}$.

The first of these fractions is therefore the least, and the second is the greatest.

In the previous example we need not necessarily have arranged the fractions so that they all had a denominator equal to the *least* common denominator of the given fractions. Any common denominator would have done, but there is generally a considerable saving of work if we reduce the fractions to their *least* common denominator.

EXAMPLES XV.

Reduce the following fractions to equivalent fractions with the lowest common denominator:

1. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. **2.** $\frac{2}{3}, \frac{3}{5}, \frac{1}{6}$. **3.** $\frac{5}{12}, \frac{17}{80}, \frac{19}{120}$.

Arrange in order of magnitude the fractions:

4. $\frac{1}{3}, \frac{7}{20}, \frac{3}{10}$. **5.** $\frac{1}{3}, \frac{2}{7}, \frac{8}{21}$. **6.** $\frac{11}{60}, \frac{8}{45}, \frac{7}{36}$.

7. $\frac{1}{2}, \frac{2}{3}, \frac{3}{5}$. **8.** $\frac{3}{4}, \frac{4}{5}, \frac{2}{3}$. **9.** $\frac{4}{7}, \frac{2}{5}, \frac{1}{4}, \frac{5}{8}$.

10. $\frac{11}{28}, \frac{3}{8}, \frac{8}{21}$. **11.** $\frac{3}{4}, \frac{7}{12}, \frac{2}{3}, \frac{1}{6}, \frac{1}{2}$. **12.** $\frac{16}{165}, \frac{4}{45}, \frac{1}{11}$.

13. $\frac{11}{12}, \frac{29}{30}, \frac{17}{18}, \frac{7}{16}, \frac{47}{48}$. **14.** $\frac{3}{62}, \frac{7}{124}, \frac{4}{93}$.

ADDITION OF SIMPLE FRACTIONS.

35. When two or more fractions have the same denominator their sum is obtained by adding the numerators.

Thus $\frac{3}{5}+\frac{4}{5}=3 \text{ fifths}+4 \text{ fifths}=7 \text{ fifths}=\frac{7}{5}=\frac{3+4}{5}$.

So $$\frac{7}{8}+\frac{3}{8}+\frac{5}{8}+\frac{1}{8}=\frac{7+3+5+1}{8}=\frac{16}{8}=2.$$

If they have not the same denominator, reduce them to their L.C.D.

Ex. 1. *Add together* $\frac{1}{2}$, $\frac{1}{3}$, *and* $\frac{1}{5}$.

Their L.C.D. is 30. Consider the fraction $\frac{1}{2}$: since $30\div 2=15$,

$$\therefore \frac{1}{2}=\frac{15}{30}. \quad \text{So } \frac{1}{3}=\frac{10}{30}, \text{ and } \frac{1}{5}=\frac{6}{30}.$$

$$\therefore \frac{1}{2}+\frac{1}{3}+\frac{1}{5}=\frac{15+10+6}{30}=\frac{31}{30}=1\tfrac{1}{30}.$$

Ex. 2. *Find the value of* $3\frac{5}{6}+\frac{7}{15}+2\frac{9}{35}+4\frac{8}{14}$.

$$3\tfrac{5}{6}+\tfrac{7}{15}+2\tfrac{9}{35}+4\tfrac{8}{14}$$
$$=(3+2+4)+\tfrac{5}{6}+\tfrac{7}{15}+\tfrac{9}{35}+\tfrac{8}{14}, \text{ on rearranging,}$$
$$=9+\tfrac{5}{6}+\tfrac{7}{15}+\tfrac{9}{35}+\tfrac{4}{7}, \text{ on reducing to lowest terms.}$$
$$=9+\tfrac{175}{210}+\tfrac{98}{210}+\tfrac{54}{210}+\tfrac{120}{210}, \text{ on bringing to the L.C.D.,}$$
$$=9+\frac{175+98+54+120}{210}=9+\tfrac{447}{210}=9+2\tfrac{27}{210}=11\tfrac{9}{70}.$$

36. The rule for the addition of simple fractions is therefore: *Find the* L.C.M. *of all the denominators and change all the fractions into fractions all having this* L.C.M. *as denominator; add all the numerators of the resulting fractions for a new numerator and keep the same denominator; finally, reduce the result, if possible, to lower terms or to a mixed number, if necessary.*

N.B.—Before applying the above rule any fraction which is not in its lowest terms should be reduced to its lowest terms; any improper fractions should be changed to mixed numbers; and, in the addition of the mixed numbers, the whole numbers should be added first and then the fractions.

EXAMPLES XVI.

Add together the following:

1. $\frac{3}{4}, \frac{5}{4}$. **2.** $\frac{1}{5}, \frac{3}{5}, \frac{4}{5}$. **3.** $\frac{2}{7}, \frac{3}{7}, \frac{2}{7}$. **4.** $\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}$.

5. $\frac{4}{111}, \frac{26}{111}, \frac{7}{111}$. **6.** $\frac{1}{2}, \frac{1}{3}$. **7.** $\frac{1}{3}, \frac{1}{4}$. **8.** $\frac{1}{4}, \frac{1}{5}$. **9.** $\frac{2}{3}, \frac{3}{4}$.

10. $\frac{4}{5}, \frac{5}{6}$. **11.** $\frac{1}{8}, \frac{1}{9}$. **12.** $\frac{7}{8}, \frac{8}{9}$. **13.** $\frac{9}{10}, \frac{10}{11}$. **14.** $\frac{2}{3}, \frac{3}{4}, \frac{3}{8}$.

15. $\frac{3}{5}, \frac{5}{6}, \frac{3}{10}$. **16.** $\frac{4}{5}, \frac{5}{8}, \frac{3}{20}$. **17.** $\frac{3}{4}, \frac{5}{6}, \frac{11}{12}$. **18.** $\frac{5}{8}, \frac{7}{12}, \frac{11}{24}$.

19. $\frac{1}{6}, \frac{7}{9}, \frac{13}{18}$. **20.** $\frac{7}{5}, \frac{7}{10}, \frac{7}{20}$. **21.** $\frac{3}{4}, \frac{3}{8}, \frac{3}{16}$. **22.** $\frac{5}{7}, \frac{3}{8}$.

23. $\frac{3}{5}, \frac{7}{15}, \frac{8}{35}$. **24.** $\frac{2}{3}, \frac{5}{9}, \frac{13}{18}$. **25.** $\frac{3}{5}, \frac{7}{9}, \frac{11}{15}, \frac{2}{3}$. **26.** $\frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}$.

27. $\frac{1}{3}, \frac{5}{6}, \frac{7}{12}, \frac{11}{18}$. **28.** $\frac{2}{9}, \frac{5}{18}, \frac{1}{6}$. **29.** $\frac{13}{55}, \frac{15}{77}, \frac{8}{35}$. **30.** $\frac{5}{7}, \frac{3}{5}, \frac{7}{15}, \frac{23}{35}$.

31. $\frac{3}{7}, \frac{5}{8}, \frac{9}{14}, \frac{13}{56}$. **32.** $\frac{2}{3}, \frac{8}{15}, \frac{17}{35}, \frac{5}{7}$. **33.** $\frac{5}{11}, \frac{19}{33}, \frac{23}{55}, \frac{8}{15}$.

34. $\frac{11}{19}, \frac{13}{57}, \frac{18}{95}$. **35.** $\frac{5}{13}, \frac{8}{65}, \frac{25}{91}$. **36.** $\frac{8}{17}, \frac{15}{34}, \frac{20}{51}, \frac{5}{6}$.

37. $3\frac{1}{2}, 4\frac{2}{3}$. **38.** $3\frac{5}{6}, 7\frac{3}{4}$. **39.** $8\frac{3}{5}, 9\frac{5}{8}$. **40.** $4\frac{1}{5}, 6\frac{3}{7}, 8\frac{11}{35}$.

41. $5\frac{1}{3}, \frac{6}{7}, 3\frac{1}{2}, \frac{16}{17}$. **42.** $5\frac{2}{9}, 2\frac{5}{6}, 1\frac{11}{27}, 3\frac{1}{2}$. **43.** $6\frac{11}{24}, 5\frac{13}{56}, 8\frac{5}{48}, 11\frac{23}{84}$.

44. $7\frac{1}{4}, \frac{7}{8}, 4\frac{8}{9}, \frac{13}{30}$. **45.** $\frac{53}{8}, \frac{79}{10}, \frac{44}{5}$. **46.** $\frac{71}{6}, \frac{62}{7}, \frac{129}{14}$.

47. $\frac{28}{9}, \frac{67}{12}, \frac{69}{8}$. **48.** $\frac{8}{3}, \frac{15}{4}, \frac{24}{5}, \frac{35}{6}, \frac{48}{7}$. **49.** $\frac{5}{18}, \frac{7}{45}, \frac{57}{25}, \frac{97}{30}$.

50. $\frac{47}{14}, \frac{17}{42}, \frac{297}{56}, \frac{83}{24}$.

SUBTRACTION OF SIMPLE FRACTIONS.

37. When two fractions have the same denominator we subtract one from the other by subtracting their numerators.

Thus $\frac{7}{8}-\frac{5}{8}=7$ eighths -5 eighths $=2$ eighths $=\frac{2}{8}=\frac{7-5}{8}$.

So $\frac{11}{12}-\frac{7}{12}=\frac{11-7}{12}=\frac{4}{12}=\frac{1}{3}$.

When the two fractions have not the same denominator, they must be reduced to fractions having the same denominator (for simplicity, the least common denominator), and then the difference of their numerators must be taken.

Ex. 1. *Subtract* $\frac{1}{3}$ *from* $\frac{1}{2}$.

$$\frac{1}{2}-\frac{1}{3}=\frac{3}{6}-\frac{2}{6}=\frac{3-2}{6}=\frac{1}{6}.$$

Ex. 2. *Find the value of* $\frac{7}{8} - \frac{1}{6} - \frac{7}{15}$.

The L.C.M. of 8, 6, and 15 is easily found to be 120;

therefore $\frac{7}{8} - \frac{1}{6} - \frac{7}{15} = \frac{7 \times 15}{8 \times 15} - \frac{20}{6 \times 20} - \frac{7 \times 8}{15 \times 8} = \frac{105}{120} - \frac{20}{120} - \frac{56}{120}$

$$= \frac{105 - 20 - 56}{120} = \frac{105 - 76}{120} = \frac{29}{120}.$$

Ex. 3. *Subtract* $22\frac{1}{3}$ *from* $63\frac{3}{8}$.

$$63\tfrac{3}{8} - 22\tfrac{1}{3} = 63 - 22 + \frac{3}{8} - \frac{1}{3} = 41 + \frac{9}{24} - \frac{8}{24} = 41 + \frac{9-8}{24} = 41 + \frac{1}{24} = 41\tfrac{1}{24}.$$

When we subtract a mixed number from a mixed number we thus subtract separately the whole numbers and the fractions.

EXAMPLES XVII.

Find the value of

1. $\frac{3}{4} - \frac{1}{4}$.	**2.** $\frac{5}{6} - \frac{1}{6}$.	**3.** $\frac{6}{7} - \frac{5}{7}$.	**4.** $\frac{7}{12} - \frac{5}{12}$.	**5.** $\frac{6}{11} - \frac{4}{11}$.
6. $\frac{5}{6} - \frac{2}{3}$.	**7.** $\frac{2}{3} - \frac{5}{12}$.	**8.** $\frac{2}{5} - \frac{3}{10}$.	**9.** $\frac{1}{3} - \frac{1}{4}$.	**10.** $\frac{1}{5} - \frac{1}{6}$.
11. $\frac{1}{9} - \frac{1}{10}$.	**12.** $\frac{5}{7} - \frac{3}{5}$.	**13.** $\frac{7}{8} - \frac{5}{6}$.	**14.** $\frac{9}{10} - \frac{5}{6}$.	
15. $\frac{8}{9} - \frac{7}{8}$.	**16.** $\frac{21}{23} - \frac{43}{69}$.	**17.** $\frac{20}{33} - \frac{35}{77}$.	**18.** $\frac{19}{28} - \frac{22}{35}$.	
19. $\frac{22}{35} - \frac{26}{45}$.	**20.** $\frac{11}{34} - \frac{8}{51}$.	**21.** $\frac{17}{38} - \frac{13}{57}$.	**22.** $\frac{11}{52} - \frac{9}{65}$.	
23. $\frac{27}{58} - \frac{13}{87}$.	**24.** $\frac{37}{68} - \frac{23}{85}$.	**25.** $\frac{29}{64} - \frac{13}{72}$.	**26.** $\frac{49}{54} - \frac{25}{36}$.	
27. $4\frac{1}{2} - 3\frac{1}{3}$.	**28.** $18\frac{3}{4} - 14\frac{5}{8}$.	**29.** $3\frac{13}{14} - 2\frac{3}{7}$.	**30.** $7\frac{23}{96} - 5\frac{13}{32}$.	

38. In some cases the fractional part of the number to be subtracted is greater than the fractional part of the first number. In such cases we proceed as follows:

Ex. 1. *Subtract* $12\frac{20}{21}$ *from* $23\frac{1}{77}$.

Here $\frac{20}{21}$ cannot be subtracted from $\frac{1}{77}$, because the former fraction is the larger. We therefore take a unit from the 23 and add on to the $\frac{1}{77}$, so that we obtain a fraction greater than unity from which $\frac{20}{21}$ can be subtracted.

Thus $23\frac{1}{77} = 22 + 1\frac{1}{77} = 22 + \frac{78}{77}$; $\therefore\ 23\frac{1}{77} - 12\frac{20}{21} = 22 + \frac{78}{77} - 12 - \frac{20}{21}$

$= 10 + \frac{78}{77} - \frac{20}{21} = 10 + \frac{234}{231} - \frac{220}{231}$, since 231 is the L.C.M. of 77 and 21,

$$= 10 + \frac{234 - 220}{231} = 10 + \frac{14}{231} = 10\tfrac{2}{33}.$$

Before the application of the rule fractions should generally be reduced to their lowest terms, and improper fractions to mixed numbers. In the case of mixed numbers it is best to subtract the integers from the integers and the fractions from the fractions, as in the above example.

Ex. 2. *Find the value of* $10\frac{1}{21} - 3\frac{2}{15} - 1\frac{39}{70} + 11\frac{16}{210}$.

The least common denominator is 210.

Hence the expression $= 10 - 3 - 1 + 11 + \frac{1}{21} - \frac{2}{15} - \frac{39}{70} + \frac{16}{210}$

$$= 17 + \frac{10}{210} - \frac{2\times14}{15\times14} - \frac{39\times3}{70\times3} + \frac{16}{210}$$

$$= 17 + \frac{10 - 28 - 117 + 16}{210} = 17 + \frac{26 - 145}{210}$$

$$= 16 + 1 + \frac{26 - 145}{210} = 16 + \frac{210 + 26 - 145}{210}$$

$$= 16 + \frac{236 - 145}{210} = 16 + \frac{91}{210} = 16 + \frac{7\times13}{7\times30} = 16\frac{13}{30}.$$

EXAMPLES XVIII.

Find the value of

1. $1 - \frac{5}{6}$. **2.** $1 - \frac{11}{12}$. **3.** $3 - \frac{3}{7}$. **4.** $8 - \frac{13}{14}$. **5.** $7 - \frac{4}{11}$.

6. $4 - \frac{11}{18}$. **7.** $8 - 5\frac{1}{2}$. **8.** $17 - 4\frac{3}{4}$. **9.** $15 - 7\frac{5}{8}$. **10.** $11 - 3\frac{3}{4}$.

11. $23 - 7\frac{11}{12}$. **12.** $28 - 18\frac{15}{17}$. **13.** $3\frac{2}{7} - 2\frac{5}{7}$. **14.** $6\frac{1}{4} - 4\frac{3}{4}$.

15. $7\frac{3}{8} - 3\frac{5}{8}$. **16.** $11\frac{4}{9} - 6\frac{8}{9}$. **17.** $11\frac{3}{14} - 7\frac{11}{14}$. **18.** $15\frac{12}{19} - 10\frac{18}{19}$.

19. $4\frac{1}{4} - 3\frac{1}{2}$. **20.** $7\frac{1}{3} - 5\frac{3}{4}$. **21.** $11\frac{1}{5} - 8\frac{2}{3}$. **22.** $14\frac{1}{11\,2} - 13\frac{1}{14}$.

23. $16\frac{15}{32} - 13\frac{33}{48}$. **24.** $23\frac{13}{29} - 19\frac{37}{58}$. **25.** $7\frac{9}{50} - 3\frac{37}{60}$.

26. $4\frac{1}{52} - 2\frac{1}{13}$. **27.** $25\frac{8}{44} - 21\frac{27}{55}$. **28.** $11\frac{1}{42} - 5\frac{27}{56}$.

29. $16\frac{23}{36} - 7\frac{41}{45}$. **30.** $10\frac{3}{14} - 8\frac{7}{9}$.

Subtract

31. $4\frac{33}{55}$ from $\frac{1}{6} + 2\frac{1}{7} + 13\frac{3}{10} + \frac{4}{21}$. **32.** $\frac{6}{7} + \frac{4}{5} + \frac{2}{3}$ from $\frac{7}{8} + \frac{5}{6} + \frac{3}{4}$.

33. $\frac{4}{3} + \frac{6}{5} + \frac{8}{7}$ from $\frac{3}{2} + \frac{5}{4} + \frac{7}{6}$.

Find the value of

34. $\frac{2}{5} - \frac{3}{8} + \frac{1}{24}$. **35.** $\frac{13}{60} - \frac{13}{42} + \frac{13}{90}$.

36. $3\frac{7}{48} - 11\frac{5}{8} + 12\frac{7}{24} - \frac{7}{136}$. **37.** $3\frac{1}{3} - 4\frac{3}{4} + 5\frac{4}{5} - 7\frac{5}{6} + 8\frac{6}{7}$.

38. $2\frac{3}{4} + \frac{13}{8} + 2\frac{8}{10} - 3\frac{5}{24} + 1\frac{8}{15}$. **39.** $13\frac{2}{15} - 5\frac{5}{27} - 6\frac{3}{45} + 5\frac{1}{12}$.

40. What number subtracted from $115\frac{7}{11}$ will leave $37\frac{3}{4}$?

41. What number added to $\frac{7}{9} + \frac{5}{7}$ will give $1\frac{13}{21}$?

42. From the sum of $\frac{1}{2}$ and $\frac{1}{3}$ take the difference between $\frac{1}{5}$ and $\frac{1}{6}$.

43. By how much is the sum of $5\frac{3}{8}$ and $4\frac{5}{7}$ less than the difference of $20\frac{7}{9}$ and $3\frac{11}{12}$?

44. Find the least fraction which added to the sum of $4\frac{3}{4}$, $3\frac{4}{5}$, and $6\frac{7}{8}$ will make the result a whole number.

45. By how much does the difference of $21\frac{3}{7}$ and $4\frac{5}{8}$ exceed the sum of $\frac{5}{7}$, $3\frac{3}{4}$, and $4\frac{2}{3}$?

46. *A*, *B*, *C*, and *D* respectively can do $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{6}$, and $\frac{2}{9}$ of a piece of work in a day; how much would *A* and *D* working together for a day do less than *B* and *C* working together for a day?

MULTIPLICATION OF FRACTIONS.

I. When the multiplier is an integer.

39. **Ex.** *Multiply* $\frac{2}{11}$ *by* 3.

Suppose we take the length of a cricket pitch, 22 yards, as our unit.

$\frac{2}{11}$ of it = 4 yds.

$\therefore$ 3 times $\frac{2}{11}$ of it = 12 yds. = $\frac{12}{22}$ of it = $\frac{6}{11}$ of it.

$$\therefore \frac{2}{11} \times 3 = \frac{6}{11} = \frac{2 \times 3}{11}.$$

This shows us that if we have to multiply a fraction by a whole number, we must multiply the numerator by it.

N.B. $\frac{6}{11} = \frac{1}{11} \times 6 = 6 \times \frac{1}{11} = \frac{1}{11}$ of 6.

$\therefore \frac{6}{11} = 6 \div 11.$

EXAMPLES XIX.

Multiply

1. $\frac{1}{3}$ by 5 and 8. **2.** $\frac{2}{7}$ by 3 and 4. **3.** $\frac{2}{9}$ by 7 and 5.

4. $\frac{4}{7}$ by 6 and 9. **5.** $\frac{6}{11}$ by 8 and 3. **6.** $\frac{13}{15}$ by 8 and 3.

7. $\frac{5}{11}$ by 6 and 7. **8.** $\frac{13}{14}$ by 7 and 18. **9.** $6\frac{1}{6}$ by 9 and 15.

10. $4\frac{8}{13}$ by 11 and 7. **11.** $5\frac{3}{16}$ by 3 and 14. **12.** $9\frac{3}{11}$ by 7 and 2.

13. $1\frac{3}{16}$ by 36 and 28. **14.** $3\frac{3}{11}$ by 45 and 37. **15.** $2\frac{7}{12}$ by 56 and 100.

16. $4\frac{3}{4}$ by 68 and 44. **17.** $2\frac{15}{17}$ by 37 and 51.

18. $3\frac{13}{18}$ by 85 and 144. **19.** $20\frac{17}{21}$ by 102 and 147.

II. When the multiplier is a fraction.

40. **Ex. 1.** *Multiply* $\frac{7}{9}$ *by* $\frac{5}{8}$.

Consider the length 2 yards to be our unit.

Then $\frac{7}{9}$ of it = 56 in.

But we have to find $\frac{5}{8}$ of this.

Now $\frac{5}{8}$ of 56 in. = 35 in. = $\frac{35}{72}$ of our unit;

$$\therefore \frac{7}{9} \times \frac{5}{8} = \frac{35}{72} = \frac{7 \times 5}{9 \times 8}.$$

Thus we have, in the case of fractions like $\frac{7}{9}$ and $\frac{5}{8}$, the following rule:

Multiply the numerators together for a new numerator and the denominators together for a new denominator.

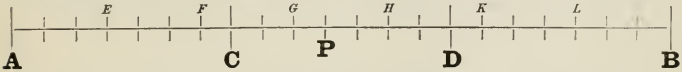
Ex. 2. *Multiply* $2\frac{2}{5}$ *by* $1\frac{6}{7}$.

$2\frac{2}{5} \times 1\frac{6}{7} = \frac{12}{5} \times \frac{13}{7}$, on reducing mixed numbers to improper fractions,

$$= \frac{12 \times 13}{5 \times 7} = \frac{156}{35} = 4\frac{16}{35}.$$

41. The rule for the multiplication of two fractions may be also shown to the eye.

We shall show by this method that $\frac{2}{3} \times \frac{5}{7} = \frac{5}{7} \times \frac{2}{3} = \frac{10}{21}$.



Let the unit be a straight line, AB, and let it be divided into three equal parts at the points C and D. Also let each of the parts AC, CD, and DB be divided into seven equal parts, so that each of the smaller subdivisions is $\frac{1}{21}$ of AB.

Also let AB be divided into seven equal parts at the points E, F, G, H, K, and L, so that each of the parts AE, EF, FG, GH, HK, and KL contains three of these subdivisions.

Then $\frac{2}{3}$ of the unit $AB = AD = 14$ subdivisions,

and therefore $\frac{2}{3} \times \frac{5}{7} = AD \times \frac{5}{7} = \frac{5}{7}$ of 14 subdivisions

$= 10$ subdivisions $= AP = \frac{10}{21}$.

Also $\frac{5}{7}$ of the unit $= 5$ times $AE = AK = 15$ subdivisions;

therefore $\frac{5}{7} \times \frac{2}{3} = AK \times \frac{2}{3} = \frac{2}{3}$ of 15 subdivisions

$= 10$ subdivisions $= AP = \frac{10}{21}$.

Since $\frac{5}{7} \times \frac{2}{3} = \frac{2}{3} \times \frac{5}{7}$, it follows that the order of multiplication of two fractions is immaterial.

42. When we have more than two fractions to multiply, the same rule applies. For example:

$$\frac{3}{5} \times \frac{7}{9} \times \frac{11}{13} = \frac{3 \times 7}{5 \times 9} \times \frac{11}{13} = \frac{21}{45} \times \frac{11}{13} = \frac{21 \times 11}{45 \times 13} = \frac{3 \times 7 \times 11}{5 \times 9 \times 13}.$$

To multiply any number of fractions together we therefore *multiply all the numerators for a new numerator, and all the denominators for a new denominator.*

43. Cancelling. We can in many cases simplify the work of multiplying fractions together by crossing out common factors, that is, by dividing the numerator and denominator by the same number or numbers. (Art. 31.)

Ex. *Multiply together the fractions* $\frac{8}{15}$, $\frac{21}{22}$, *and* $\frac{11}{14}$.

$$\text{The product} = \frac{\overset{2}{\cancel{8}} \times \overset{1}{\cancel{21}} \times \overset{1}{\cancel{11}}}{\underset{5}{\cancel{15}} \times \underset{2}{\cancel{22}} \times \underset{2}{\cancel{14}}} = \frac{2}{5}.$$

In the product the 21 of the numerator and the 15 of the denominator both contain 3 as a factor; divide each therefore by 3; cross the 21 lightly through and write 7 above it, and cross the 15 through and write 5 under it.

Again, the 14 of the denominator contains 7 as a factor; cross it through and write 2 under it, and cross out the 7 of the numerator.

Similarly, since 11 is contained in 22 cross out the 11 and cross through the 22 and put 2 below it.

Finally, the two 2's in the denominator make 4, and 4 goes into 8 twice; cross out the 8 and put 2 over it, and cross out the two 2's.

We have finally left $\frac{2}{5}$, which is the answer.

EXAMPLES XX.

Multiply

1. $\frac{1}{2}$ by $\frac{1}{3}$. **2.** $\frac{1}{7}$ by $\frac{1}{11}$. **3.** $\frac{3}{8}$ by $\frac{1}{4}$. **4.** $\frac{3}{4}$ by $\frac{1}{3}$. **5.** $\frac{7}{9}$ by $\frac{4}{5}$.

6. $\frac{3}{4}$ by $\frac{4}{7}$. **7.** $\frac{6}{7}$ by $\frac{7}{11}$. **8.** $\frac{8}{25}$ by $\frac{5}{4}$. **9.** $\frac{3}{28}$ by $\frac{2}{3}$. **10.** $\frac{16}{21}$ by $\frac{7}{8}$.

11. $\frac{5}{24}$ by $\frac{8}{15}$. **12.** $\frac{8}{27}$ by $\frac{3}{16}$. **13.** $\frac{32}{33}$ by $\frac{11}{16}$. **14.** $1\frac{3}{4}$ by $\frac{2}{7}$.

15. $1\frac{5}{6}$ by $\frac{3}{11}$. **16.** $4\frac{2}{7}$ by $\frac{5}{6}$. **17.** $3\frac{3}{5}$ by $\frac{7}{9}$. **18.** $4\frac{4}{9}$ by $2\frac{1}{4}$.

19. $6\frac{3}{8}$ by $2\frac{2}{3}$.

Multiply together

20. $\frac{5}{7}$, $4\frac{2}{3}$, and $5\frac{2}{5}$. **21.** $\frac{45}{56}$, $1\frac{5}{9}$, and $7\frac{1}{5}$. **22.** $\frac{2}{21}$, $1\frac{3}{4}$, $\frac{6}{17}$, and $\frac{34}{63}$.

23. $\frac{13}{144}$, $\frac{17}{42}$, $1\frac{1}{13}$, and $\frac{1}{51}$. **24.** $11\frac{1}{12}$, $1\frac{5}{7}$, $\frac{1}{19}$, and 3.

25. $8\frac{3}{7}$, $1\frac{1}{11}$, $1\frac{32}{59}$, and $\frac{11}{26}$. **26.** $\frac{180}{525}$, $\frac{735}{264}$, and $\frac{66}{315}$.

27. The sum of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ by the sum of $\frac{3}{2}$, $\frac{4}{3}$, and $\frac{5}{4}$.

28. Find the difference between $1\frac{7}{9}\times 3\frac{1}{8}$ and $3\frac{7}{8}\times 2\frac{5}{9}$.

29. Find the difference between $9\frac{6}{7}\times 5\frac{3}{4}$ and $53\frac{5}{7}-10\frac{2}{3}$.

DIVISION OF SIMPLE FRACTIONS.

I. When the divisor is an integer.

44. **Ex.** *Divide $\frac{3}{4}$ by 5.*

Take one minute as our unit.

Then $\frac{3}{4}$ of one minute $=\frac{3}{4}$ of 60 secs. $=45$ secs.

$\therefore$ $\frac{3}{4}$ of one minute $\div 5 = 9$ secs.

$=9$ times $\frac{1}{60}$ of a minute

$=\frac{9}{60}$ of a minute

$=\frac{3}{20}$ of a minute.

$$\therefore \frac{3}{4}\div 5=\frac{3}{20}=\frac{3}{4\times 5}.$$

Similarly we have $$\frac{8}{7}\div 4=\frac{8}{28}=\frac{2}{7}=\frac{8\div 4}{7}.$$

Hence: *To divide a fraction by an integer either divide the numerator by the integer, or multiply the denominator by the integer.*

It is more simple to divide the numerator, whenever this is possible.

EXAMPLES XXI.

Divide

1. 3 by 4. **2.** 11 by 12. **3.** 6 by 9. **4.** 35 by 21. **5.** 48 by 36.

6. $\frac{2}{3}$ by 5. **7.** $\frac{3}{4}$ by 7. **8.** $\frac{5}{6}$ by 10. **9.** $\frac{3}{5}$ by 7. **10.** $\frac{4}{7}$ by 8.

11. $\frac{3}{3}\frac{2}{7}$ by 8. **12.** $\frac{3}{4}\frac{6}{3}$ by 9. **13.** $\frac{4}{5}\frac{8}{3}$ by 15. **14.** $\frac{5}{7}\frac{7}{3}$ by 19. **15.** $\frac{5}{6}\frac{5}{3}$ by 11.

16. $\frac{5}{6}\frac{2}{9}$ by 13. **17.** $3\frac{2}{11}$ by 7. **18.** $3\frac{3}{8}$ by 9. **19.** $12\frac{4}{7}$ by 11. **20.** $4\frac{4}{13}$ by 8.

21. $6\frac{9}{10}$ by 3. **22.** $5\frac{5}{14}$ by 15. **23.** $6\frac{8}{15}$ by 14. **24.** $3\frac{7}{16}$ by 11.

25. $4\frac{2}{17}$ by 10. **26.** $18\frac{1}{3}$ by 5. **27.** $5\frac{5}{8}$ by 7. **28.** $15\frac{1}{5}$ by 19.

29. $23\frac{3}{4}$ by 19. **30.** $43\frac{1}{2}$ by 29.

II. When the divisor is a fraction.

45. Ex. *Divide* $\frac{3}{7}$ *by* $\frac{2}{5}$.

Suppose we were to divide $\frac{3}{7}$ by 2: then by Art. 44, $\frac{3}{7} \div 2 = \frac{3}{14}$.

But we have to divide $\frac{3}{7}$ by $\frac{2}{5}$, a quantity 5 *times as small as* 2: therefore our answer will be 5 *times as great* as when we divide it by 2.

$$\therefore \frac{3}{7} \div \frac{2}{5} = \frac{3}{14} \times 5 = \frac{15}{14} = \frac{3 \times 5}{7 \times 2}.$$

Hence we have the rule:

To divide one simple fraction by another, invert (that is, interchange the numerator and denominator of) the divisor, and multiply.

Ex. 1. $5 \div \frac{2}{3} = 5 \times \frac{3}{2} = \frac{15}{2} = 7\frac{1}{2}$. **Ex. 2.** $\frac{3}{4} \div \frac{5}{6} = \frac{3}{4} \times \frac{6}{5} = \frac{18}{20} = \frac{9}{10}$.

Ex. 3. $\frac{15}{28} \div \frac{3}{49} = \frac{\overset{5}{\cancel{15}}}{\underset{4}{\cancel{28}}} \times \frac{\overset{7}{\cancel{49}}}{\cancel{3}} = \frac{35}{4} = 8\frac{3}{4}$.

Ex. 4. $4\frac{4}{13} \div 9\frac{1}{3} = \frac{56}{13} \div \frac{28}{3} = \frac{\overset{2}{\cancel{56}}}{13} \times \frac{3}{\cancel{28}} = \frac{6}{13}$.

Thus when the divisor, or dividend, is a mixed number it must be reduced to a fraction before the rule is applied.

EXAMPLES XXII.

Divide

1. 1 by $\frac{3}{4}$. **2.** 1 by $\frac{5}{7}$. **3.** 1 by $\frac{11}{13}$. **4.** 2 by $\frac{14}{17}$. **5.** 5 by $\frac{25}{36}$.

6. $\frac{4}{5}$ by $\frac{3}{5}$. **7.** $\frac{7}{9}$ by $\frac{2}{3}$. **8.** $\frac{1}{2}$ by $\frac{1}{3}$. **9.** $\frac{2}{3}$ by $\frac{3}{2}$. **10.** $\frac{3}{4}$ by $\frac{1}{2}$.

11. 5 by $3\frac{4}{7}$. **12.** 11 by $2\frac{4}{9}$. **13.** $2\frac{1}{2}$ by $3\frac{1}{3}$. **14.** $1\frac{1}{4}$ by $\frac{5}{7}$.

15. $1\frac{3}{4}$ by $\frac{7}{12}$. **16.** $5\frac{5}{8}$ by $2\frac{1}{4}$. **17.** $3\frac{1}{7}$ by $1\frac{1}{14}$. **18.** $3\frac{1}{3}$ by $4\frac{2}{7}$.

19. $2\frac{2}{5}$ by $3\frac{3}{7}$. **20.** $10\frac{5}{13}$ by $3\frac{3}{8}$. **21.** $9\frac{5}{21}$ by $2\frac{23}{37}$. **22.** $5\frac{5}{31}$ by $1\frac{37}{43}$.

23. $4\frac{20}{87}$ by $2\frac{26}{79}$.

24. Divide the difference between $9\frac{3}{4}$ and $7\frac{3}{5}$ by the product of $8\frac{1}{6}$ and $\frac{5}{7}$.

25. Divide the sum of $3\frac{7}{11}$ and $2\frac{5}{8}$ by the difference between $8\frac{1}{8}$ and $5\frac{2}{11}$.

26. Divide $\frac{16}{11}$ by the difference between $\frac{56}{143}$ and $\frac{72}{187}$.

27. Find the value of $\frac{2}{15} - \frac{3}{28} + \frac{1}{42}$ divided by $\frac{1}{75} + \frac{3}{40} - \frac{1}{12}$.

28. What number multiplied by $3\frac{4}{7}$ will give $6\frac{3}{5}$?

29. What number divided by $6\frac{3}{5}$ will give $10\frac{3}{8}$?

46. A Compound Fraction is a fraction of a fraction. Thus $\frac{2}{3}$ of $\frac{4}{5}$ and $\frac{6}{5}$ of $\frac{11}{13}$ of $7\frac{1}{2}$ are both compound fractions.

Now $\frac{4}{5}$ of £1 = $\frac{4}{5}$ of 240 pence = 4 times 48 pence = 192 pence.

$\therefore$ $\frac{2}{3}$ of $\frac{4}{5}$ of £1 = $\frac{2}{3}$ of 192 pence = 2 times 64 pence = 128 pence.

Also $\frac{2}{3} \times \frac{4}{5}$ of £1 = $\frac{8}{15}$ of 240 pence = 8 times 16 pence = 128 pence.

$$\therefore \frac{2}{3} \text{ of } \frac{4}{5} = \frac{2}{3} \times \frac{4}{5}.$$

Thus the word "of" has the same meaning as "multiplied by," but the last sentence of Art. 48 must be noted.

EXAMPLES XXIII.

Find the value of

1. $\frac{2}{3}$ of $\frac{3}{4}$. **2.** $\frac{4}{5}$ of $\frac{5}{9}$. **3.** $\frac{3}{7}$ of $1\frac{14}{15}$. **4.** $1\frac{3}{4}$ of $\frac{5}{7}$. **5.** $\frac{5}{11}$ of $\frac{36}{49}$ of $\frac{77}{90}$.

6. $\frac{7}{13}$ of $\frac{52}{119}$ of $\frac{39}{75}$. **7.** $3\frac{1}{8}$ of $\frac{4}{5}$. **8.** $4\frac{1}{8}$ of $\frac{5}{11}$. **9.** $3\frac{1}{4}$ of $3\frac{1}{7}$.

Divide

10. $\frac{7}{8}$ of $\frac{9}{10}$ by $\frac{3}{5}$ of $\frac{11}{4}$. **11.** $3\frac{13}{56}$ by $\frac{7}{9}$ of $\frac{5}{8}$. **12.** $\frac{8}{9}$ of $\frac{11}{18}$ by $\frac{77}{45}$ of $\frac{11}{72}$.

13. $\frac{7}{10}$ of $3\frac{3}{5}$ by $8\frac{1}{4}$ of $3\frac{1}{11}$. **14.** $\frac{3}{5}$ of $\frac{6}{7}$ of $\frac{4}{9}$ of $\frac{11}{16}$ by $\frac{9}{10}$ of $\frac{5}{21}$ of $\frac{7}{12}$.

47. Brackets. It is found convenient to introduce symbols called brackets to join, or bracket together, two or more quantities. These are of various shapes, thus: ( ), { }, [ ]. The first is the one in common use.

Thus $(\frac{1}{2} + \frac{1}{3}) \div 4$ means that the $\frac{1}{2}$ and $\frac{1}{3}$ are to be added together and then treated as one quantity and divided by 4.

Again $(\frac{3}{5} - \frac{2}{7}) \div (\frac{3}{7} + \frac{2}{5})$ means that the quantity which is obtained by subtracting $\frac{2}{7}$ from $\frac{3}{5}$ is to be divided by the quantity which is obtained by adding $\frac{3}{7}$ to $\frac{2}{5}$. This expression thus $= (\frac{21}{35} - \frac{10}{35}) \div (\frac{15}{35} + \frac{14}{35}) = (\frac{11}{35}) \div (\frac{29}{35}) = \frac{11}{35} \times \frac{35}{29} = \frac{11}{29}$.

EXAMPLES XXIV.

Find the value of

1. $12+(19-5)$. **2.** $14-(11-7)$. **3.** $22-(8-5)$.

4. $(11-6)\times(8-5)$. **5.** $(12-3)\times(10-7)$. **6.** $4\times(11-5)+8\times(6-3)$.

7. $8(13-5)-4(18-5)$. **8.** $6-\{3-4(5-2)\}$. **9.** $9-\{3(6-2)-5(9-7)\}$.

10. $3\frac{1}{4}+(1\frac{3}{4}-\frac{7}{8})$. **11.** $17\frac{1}{2}-(5\frac{3}{4}-3\frac{1}{2})$. **12.** $7\frac{3}{4}-(5\frac{2}{5}-1\frac{3}{5})$.

13. $3\frac{3}{7}+(8\frac{1}{2}-5\frac{3}{4}+2\frac{1}{8})$. **14.** $8\frac{1}{2}-3\times(6\frac{1}{2}-4\frac{1}{3})$.

15. $5(6\frac{3}{5}-4\frac{2}{3})\div 5\frac{1}{2}$. **16.** $\frac{7}{29}$ of $(5\frac{1}{3}-3\frac{2}{5})$. **17.** $(\frac{5}{2}-\frac{2}{5})\div(\frac{4}{3}-\frac{3}{4})$.

18. $(\frac{1}{2}+\frac{3}{4}+\frac{5}{6})\div(\frac{2}{3}+\frac{1}{4}+\frac{7}{6})$. **19.** $(3\frac{1}{4}\times 4\frac{1}{3})\div(2\frac{1}{2}-\frac{1}{3})\times(3\frac{1}{2}-\frac{1}{4})$.

48. We shall now give some examples in which both additions and subtractions and multiplications or divisions occur. In such simplifications there is one rule to which the student must give his careful attention; this is, that *Multiplications and Divisions must be performed before Additions and Subtractions.*

Thus the expression $\frac{1}{2}\times\frac{3}{4}+\frac{1}{8}$ means that $\frac{1}{2}$ is to be multiplied by $\frac{3}{4}$ and the result added to $\frac{1}{8}$. It does *not* mean that $\frac{3}{4}$ is to be added to $\frac{1}{8}$ and the result multiplied by $\frac{1}{2}$. This would be denoted by $\frac{1}{2}\times(\frac{3}{4}+\frac{1}{8})$.

Thus $\frac{1}{2}\times\frac{3}{4}+\frac{1}{8}=\frac{3}{8}+\frac{1}{8}=\frac{4}{8}=\frac{1}{2}$.

Ex. 1. *Simplify* $3\frac{1}{7}$ of $\frac{7}{8}-\frac{3}{5}\times 3\frac{3}{4}+\frac{7}{8}$ of $1\frac{1}{7}$.

The expression $=\frac{22}{\cancel{7}}\times\frac{\cancel{7}}{8}-\frac{3}{\cancel{5}}\times\frac{\overset{3}{\cancel{15}}}{4}+\frac{\cancel{7}}{\cancel{8}}\times\frac{\cancel{8}}{\cancel{7}}=\frac{22}{8}-\frac{9}{4}+1=\frac{11}{4}-\frac{9}{4}+1=1\frac{1}{2}$.

Ex. 2. *Find the value of* $1\frac{3}{4}\div 1\frac{2}{5}-3\frac{1}{9}\times\frac{3}{14}+1\frac{2}{5}\div\frac{8}{35}$.

The expression $=\frac{7}{4}\div\frac{7}{5}-\frac{28}{9}\times\frac{3}{14}+\frac{7}{5}\div\frac{8}{35}$

$=\frac{\cancel{7}}{4}\times\frac{5}{\cancel{7}}-\frac{\overset{2}{\cancel{28}}}{\underset{3}{\cancel{9}}}\times\frac{\cancel{3}}{\cancel{14}}+\frac{7}{\cancel{5}}\times\frac{\overset{7}{\cancel{35}}}{8}=\frac{5}{4}-\frac{2}{3}+6\frac{1}{8}=6+\frac{30-16+3}{24}=6\frac{17}{24}$.

Ex. 3. *Find the values of* $\frac{3}{4}$ *of* $\frac{5}{6}-\frac{1}{3}$ *and* $\frac{3}{4}$ *of* $(\frac{5}{6}-\frac{1}{3})$.

$\frac{3}{4}$ of $\frac{5}{6}-\frac{1}{3}=\frac{3}{4}\times\frac{5}{6}-\frac{1}{3}=\frac{15}{24}-\frac{8}{24}=\frac{7}{24}$.

Also $\frac{3}{4}$ of $(\frac{5}{6}-\frac{1}{3})=\frac{3}{4}$ of $(\frac{5}{6}-\frac{2}{6})=\frac{3}{4}$ of $\frac{3}{6}=\frac{3}{4}\times\frac{1}{2}=\frac{3}{8}$.

The word "of" is often used where a bracket would be more suitable, and the operation denoted by it must be performed before the operations denoted by $\times$ and $\div$. Thus $\frac{7}{8}\div\frac{3}{5}$ of $\frac{11}{13}$ means $\frac{7}{8}\div(\frac{3}{5}$ of $\frac{11}{13})$ and $=\frac{7}{8}\div\frac{3\times 11}{5\times 13}=\frac{7}{8}\times\frac{5}{3}\times\frac{13}{11}$. It is not the same as $\frac{7}{8}\div\frac{3}{5}\times\frac{11}{13}$, which $=\frac{7}{8}\times\frac{5}{3}\times\frac{11}{13}$.

EXAMPLES XXV.

Find the value of

1. $3\frac{1}{5} \times 6\frac{1}{4} + 5\frac{1}{3} \times 1\frac{1}{2}$.
2. $8\frac{1}{2} - \frac{3}{4} \times 4\frac{1}{2}$.
3. $(8\frac{1}{2} - \frac{3}{4}) \times 4\frac{1}{2}$.
4. $(7\frac{5}{6} - \frac{7}{8}) \times 3\frac{3}{4} - 2\frac{1}{2}$.
5. $(7\frac{5}{6} - \frac{7}{8}) \times (3\frac{3}{4} - 2\frac{1}{2})$.
6. $7\frac{5}{6} - \frac{7}{8} \times 3\frac{3}{4} - 2\frac{1}{2}$.
7. $7\frac{1}{5} \div 5\frac{1}{7} + 3\frac{2}{5} \div 2\frac{3}{7}$.
8. $2\frac{1}{2} \div 3\frac{1}{4} + 4\frac{5}{13} \div 2\frac{5}{7}$.
9. $\frac{4}{7}$ of $\frac{5}{8}$ of $\frac{17}{20} - \frac{1}{4}$ of $\frac{5}{14}$ of 2.
10. $5\frac{2}{5} + 3\frac{1}{5} \div 1\frac{1}{3} - 4 \div 1\frac{9}{11}$.
11. $3\frac{2}{5} \times 5\frac{1}{2} \times \frac{7}{9} - \frac{1}{3} \times \frac{5}{12}$.
12. $5\frac{7}{8} \times 1\frac{3}{47} - 3\frac{3}{8} \div 2\frac{4}{7} - \frac{13}{16}$.
13. $3\frac{1}{125}$ of $3\frac{4}{7} \div (\frac{47}{345}$ of 9).
14. $1\frac{5}{12}$ of $2\frac{1}{7} - \frac{1}{2}$ of $1\frac{1}{4} \div 17\frac{1}{2}$.
15. $\frac{18}{17}$ of $(1 - \frac{64}{81}) + \frac{8}{11} \times \frac{1}{6}(\frac{1}{2} + \frac{5}{12})$.
16. $(9\frac{1}{9} + 8\frac{1}{8} - 7\frac{1}{7}) \div (9\frac{1}{9} - 6\frac{9}{56} + 7\frac{1}{7})$.
17. $(\frac{3}{11} - \frac{7}{1685} - \frac{1}{1005}) \div (15\frac{2}{5} - 4\frac{1}{2} \times 2\frac{1}{3} - \frac{2}{7})$.
18. $(13\frac{5}{6}$ of $5\frac{4}{7}$ of $2\frac{3}{5}) \div (14\frac{3}{10}$ of $3\frac{5}{7}$ of $3\frac{7}{12})$.
19. $\frac{1}{6} \div \frac{37}{49} - \frac{2}{13}$ of $1\frac{2}{37} + \frac{2}{3} \div \frac{37}{17}$.
20. $\frac{5}{7}$ of $(4 + 3\frac{1}{5} - 5\frac{1}{10}) - \frac{1}{9}$ of $\frac{26}{5}$ of $(6 - 4\frac{11}{13})$.
21. $4 \times (\frac{2}{3} + \frac{4}{15}$ of $\frac{5}{28} - \frac{7}{24}) \div (\frac{17}{41} \div \frac{51}{82})$.
22. $(4\frac{1}{3}$ of $1\frac{2}{19} - 3\frac{3}{4}$ of $1\frac{1}{5}) \div 1\frac{17}{38}$.
23. $(4\frac{1}{4} + 6\frac{1}{2} - 7\frac{5}{8}) \div (6\frac{2}{3} - 2\frac{1}{4}$ of $2\frac{1}{8})$.
24. Find the sum of $\frac{3}{5}$ of $\frac{2}{3} + \frac{1}{6}$ and $\frac{3}{5}$ of $(\frac{2}{3} + \frac{1}{6})$.
25. Find the difference between $\frac{2}{3}$ of $\frac{3}{5} + \frac{1}{4}$ and $\frac{2}{3}$ of $(\frac{3}{5} + \frac{1}{4})$.

49. A **Complex Fraction** is one in which the numerator, or the denominator, or both, are fractions.

Examples are $\frac{\frac{2}{3}}{4}$, $\frac{4}{\frac{5}{6}}$, $\frac{\frac{5}{7}}{\frac{9}{10}}$, $\frac{4\frac{5}{7}}{\frac{9}{10} \times 5}$, etc.

Just as in Art. 30 it can easily be shown that $\frac{3}{11} = 3 \div 11$, that is, that to divide a unit into 11 equal parts and take 3 of them is the same thing as to divide 3 units into 11 equal parts. A fraction may thus be looked upon as the quotient of the numerator by the denominator.

Hence the complex fraction $\frac{\frac{5}{7}}{\frac{9}{10}} = \frac{5}{7} \div \frac{9}{10} = \frac{5}{7} \times \frac{10}{9} = \frac{50}{63}$.

So $\frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{3} + \frac{1}{4}} = \frac{\frac{3}{6} + \frac{2}{6}}{\frac{4}{12} + \frac{3}{12}} = \frac{\frac{5}{6}}{\frac{7}{12}} = \frac{5}{6} \div \frac{7}{12} = \frac{5}{\cancel{6}} \times \frac{\cancel{12}^{2}}{7} = \frac{10}{7} = 1\frac{3}{7}$.

To simplify any complex fraction we reduce the numerator and denominator each to a simple fraction, and then divide the one by the other.

Ex. 1. *Find the value of* $\dfrac{\frac{2}{3}\times\frac{4}{5}+\frac{3}{4}}{\frac{3}{4}\div\left(\frac{1}{2}+\frac{2}{5}\right)}$.

$$\text{It} = \frac{\frac{8}{15}+\frac{3}{4}}{\frac{3}{4}\div\frac{9}{10}} = \frac{\frac{32}{60}+\frac{45}{60}}{\frac{3}{4}\times\frac{10}{9}} = \frac{\frac{77}{60}}{\frac{5}{6}}$$

$$= \frac{77}{60}\div\frac{5}{6} = \frac{77}{60}\times\frac{6}{5} = \frac{77}{50} = 1\frac{27}{50}.$$

Ex. 2. *Simplify* $\dfrac{6}{3-\dfrac{2}{5+\frac{3}{4}}}$.

Begin at the lowest line; $5+\frac{3}{4}=\frac{23}{4}$.

$$\therefore\ 3-\frac{2}{5+\frac{3}{4}} = 3-\frac{2}{\frac{23}{4}} = 3-2\times\frac{4}{23} = 3-\frac{8}{23} = \frac{61}{23}.$$

$$\therefore\ \text{the expression} = \frac{6}{\frac{61}{23}} = 6\times\frac{23}{61} = \frac{138}{61} = 2\frac{16}{61}.$$

EXAMPLES XXVI.

Find the values of

1. $\dfrac{\frac{3}{5}}{\frac{4}{}}$. **2.** $\dfrac{\frac{4}{5}}{\frac{7}{}}$. **3.** $\dfrac{\frac{7}{8}}{\frac{9}{}}$. **4.** $\dfrac{3\frac{1}{4}}{5}$. **5.** $\dfrac{6\frac{1}{2}}{11}$. **6.** $\dfrac{\frac{3}{}}{\frac{4}{5}}$. **7.** $\dfrac{\frac{4}{}}{4\frac{5}{7}}$. **8.** $\dfrac{\frac{7}{}}{\frac{5}{8}}$.

9. $\dfrac{10}{\frac{7}{9}}$. **10.** $\dfrac{4}{4\frac{1}{2}}$. **11.** $\dfrac{3}{3\frac{1}{5}}$. **12.** $\dfrac{\frac{3}{4}}{\frac{7}{8}}$. **13.** $\dfrac{\frac{4}{5}}{1\frac{7}{12}}$. **14.** $\dfrac{4\frac{3}{4}}{15\frac{1}{5}}$. **15.** $\dfrac{\frac{7}{8}-\frac{1}{3}}{\frac{3}{4}}$.

16. $\dfrac{\frac{3}{5}-\frac{1}{2}}{\frac{4}{5}}$. **17.** $\dfrac{\frac{6}{11}-\frac{1}{3}}{\frac{7}{9}}$. **18.** $\dfrac{\frac{5}{8}}{3\frac{1}{2}-1\frac{1}{4}}$. **19.** $\dfrac{\frac{7}{11}}{2\frac{2}{3}-1\frac{1}{2}}$. **20.** $\dfrac{\frac{5}{8}-\frac{1}{3}}{\frac{5}{8}+\frac{1}{3}}$.

21. $\dfrac{5\frac{4}{9}}{2\frac{1}{3}+5\frac{1}{3}}$. **22.** $\dfrac{\frac{3}{4}+\frac{1}{2}}{\frac{3}{4}-\frac{1}{3}}$. **23.** $\dfrac{3\frac{7}{8}-2\frac{3}{4}}{4\frac{5}{8}-3\frac{1}{4}}$. **24.** $\dfrac{\frac{5}{7}+\frac{4}{5}}{\frac{4}{5}-\frac{5}{7}}$.

25. $\dfrac{3\frac{1}{5}\text{ of }5\frac{1}{4}}{5\frac{1}{4}\div3\frac{1}{3}}$. **26.** $\dfrac{6\frac{1}{2}\div2\frac{3}{5}}{5\frac{3}{8}+7\frac{1}{4}}$. **27.** $\dfrac{\frac{5}{7}-\frac{3}{5}}{\frac{5}{7}+\frac{3}{5}}$. **28.** $\dfrac{8\frac{1}{2}\text{ of }9\frac{3}{4}\text{ of }5}{12\frac{1}{7}\text{ of }1\frac{1}{11}\text{ of }30\frac{4}{5}}$.

29. $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{1}{4}+\frac{1}{5}+\frac{1}{6}}$. **30.** $\dfrac{\frac{1}{2}+\frac{3}{4}+\frac{5}{6}}{\frac{2}{3}+\frac{1}{4}+\frac{7}{6}}$. **31.** $\dfrac{5\frac{2}{5}\text{ of }7\frac{2}{9}}{8\frac{7}{24}-3\frac{5}{12}}$.

32. $\dfrac{\frac{2}{3}+\frac{3}{4}-\frac{5}{6}}{\frac{2}{3}\text{ of }\frac{3}{4}\text{ of }\frac{5}{6}}$. **33.** $\dfrac{1}{2+\frac{1}{3}}$. **34.** $\dfrac{2}{3-\frac{1}{4}}$. **35.** $\dfrac{2}{4+\frac{5}{6\frac{1}{4}}}$.

36. $\dfrac{3\frac{1}{2}}{7-\frac{8}{5\frac{1}{7}}}$. **37.** $\dfrac{9}{6-\frac{3}{1-\frac{1}{4}}}$. **38.** $\dfrac{6}{5+\frac{4}{3+\frac{1}{2}}}$. **39.** $\dfrac{5}{3-\frac{4}{7-\frac{1}{7}}}$.

FRACTIONS OF CONCRETE QUANTITIES.

50. **Ex. 1.** *Find the value of* $\frac{8}{15}$ *of* £17. 6*s*. 3*d*.

Since $\frac{8}{15}$ of a quantity $= \frac{1}{15}$ of 8 times the quantity
or $= 8$ times $\frac{1}{15}$ of the quantity,

we see that we must multiply £17. 6*s*. 3*d*. by 8 and divide the result by 15, or we must divide it by 15 and multiply the result by 8.

£.	*s*.	*d*.	
3)17	6	3	
5)5	15	5	$= \frac{1}{3}$ of £17. 6*s*. 3*d*.
1	3	1	$= \frac{1}{15}$ „ „
		8	
£9	4	8	$= \frac{8}{15}$ „ „

Ex. 2. *Find the value of* $6\frac{3}{7}$ of £14. 7*s*. $10\frac{1}{2}d$.

Since $6\frac{3}{7} = \frac{45}{7}$ we may obtain the value by multiplying the given sum by 45 and then dividing the result by 7. Or we may proceed thus:

£.	*s*.	*d*.	
7)14	7	$10\frac{1}{2}$	
2	1	$1\frac{1}{2}$	$= \frac{1}{7}$ of £14. 7*s*. $10\frac{1}{2}d$.
		3	
6	3	$4\frac{1}{2}$	$= \frac{2}{7}$ „ „
86	7	3	$= 6 \times$ £14. 7*s*. $10\frac{1}{2}d$.
£92	10	$7\frac{1}{2}$	$= 6\frac{3}{7}$ „ „

The fifth line is obtained by multiplying the first line by 6.

EXAMPLES XXVII.

Find the values of

1. $\frac{2}{3}$ and $\frac{7}{8}$ of 1*s*.
2. $\frac{3}{4}$ and $\frac{1}{8}$ of 1*s*.
3. $\frac{3}{5}$ and $\frac{2}{3}$ of £1.
4. $\frac{5}{8}$ and $\frac{11}{16}$ of £1.
5. $\frac{3}{5}$ of 12*s*. 6*d*.
6. $\frac{2}{3}$ and $3\frac{1}{2}$ of 2*s*. 6*d*.
7. $\frac{3}{7}$ of a guinea.
8. $\frac{3}{7}$ of £2. 12*s*. 6*d*.
9. $\frac{5}{9}$ of 31*s*. 6*d*.
10. $\frac{7}{11}$ of £3. 0*s*. 8*d*.
11. $\frac{6}{7}$ of £12. 13*s*. $5\frac{1}{2}d$.
12. $2\frac{2}{5}$ of £3. 15*s*.
13. $3\frac{5}{7}$ of £12. 6*s*. 9*d*.
14. $\frac{9}{11}$ of £6. 3*s*. 7*d*.
15. $\frac{5}{9}$ of £5. 13*s*. $4\frac{1}{4}d$.
16. £3. 4*s*. $6\frac{3}{7}d. \times 8$.
17. £4. 13*s*. $6\frac{3}{8}d. \times 17$.
18. $\frac{2}{3}$ of $\frac{3}{7}$ of £1.
19. $\frac{5}{3}$ of $4\frac{1}{5}$ of £2. 5*s*.
20. $\frac{9}{14} \times \frac{13}{18}$ of 21*s*.
21. $\frac{3}{8}$ of $\frac{7}{10}$ of £3. 8*s*. 6*d*.
22. $£3\frac{4}{9} + 4\frac{5}{8}s.$
23. $\frac{7}{13}$ of £3. 18*s*. $+ \frac{5}{11}$ of £4. 19*s*. 11*d*. $+ \frac{3}{17}$ of 18*s*. 5*d*.
24. $5\frac{1}{12}$ of 2*s*. 6*d*. $- \frac{1}{40}$ of 50 guineas $+ \frac{7}{11}$ of £9. 8*s*. 10*d*.
25. $\frac{9}{16}$ of £5. 17*s*. $- \frac{3}{7}$ of £8. 2*s*. 9*d*. $+ \frac{10}{13}$ of £1. 7*s*. 1*d*.
26. $\frac{2}{5}$ of £8. 14*s*. 7*d*. $- \frac{7}{9}$ of £3. 5*s*. 9*d*. $- \frac{5}{12}$ of £4. 6*s*. 5*d*.
27. $\left(\frac{2}{3} + \frac{3}{7} + \frac{4}{9} + 3\right)$ of £1260.
28. $\frac{2}{5}$ of £4. 2*s*. 6*d*. $+ \frac{3}{7}$ of $5\frac{2}{3}$ guineas.
29. $\frac{7}{8}$ of $\frac{2}{3}$ of a guinea $- \frac{2}{5}$ of $7\frac{2}{3}$ of a half-crown.

30. $\frac{4}{11}$ of 1*s.* 9*d.* + $\frac{2}{11}$ of 5*s.* 3*d.* − $\frac{2}{11}$ of 8*s.* 9*d.*

31. $\frac{5}{6}$ of £2. 9*s.* 4*d.* − $\frac{3}{5}$ of £4 + $2\frac{1}{2}$ of $\frac{3}{2}$ of a guinea.

32. $\frac{5}{9}$ of a guinea + $\frac{3}{16}$ of £1 + $\frac{7}{10}$ of a crown + $\frac{5}{8}$ of a shilling.

33. Subtract $\frac{7}{17}$ of £119 from $\frac{1}{2}\frac{3}{3}$ of £178. 5*s.*

Find the values of

34. $\frac{3}{5}$ of 20 lbs. **35.** $\frac{5}{8}$ of 1 furlong. **36.** $\frac{1}{121}$ of an acre.

37. $\frac{3}{11}$ of a mile. **38.** $\frac{7}{9}$ of a yard. **39.** $\frac{7}{20}$ of a day.

40. $9\frac{3}{8}$ of a lb. Av. **41.** $5\frac{4}{5}$ of a lb. Troy. **42.** $5\frac{3}{7}$ of a gallon.

43. $\frac{1}{22}$ of 1 ac. 33 sq. po. 22 sq. yds. **44.** $\frac{5}{9}$ of 5 tons 3 cwt. 2 qrs.

45. $\frac{5}{16}$ of a ton + $\frac{7}{12}$ cwt. − $\frac{1}{3}$ lb. **46.** $1\frac{3}{11}$ mile − $2\frac{3}{4}$ fur. + $\frac{7}{11}$ po.

47. $15\frac{3}{4}$ of 27 lbs. 6 oz. − $\frac{5}{7}$ of 3 cwt. 2 qrs. 7 lbs.

48. $\frac{1}{11}$ of 14 ac. 3 ro. 36 po. $2\frac{3}{4}$ sq. yds. − $\frac{3}{5}$ of 1 ac. 2 ro. 17 po. $25\frac{1}{3}$ yds.

51. *To express one given quantity as the fraction of a second given quantity of the same kind, that is, to find by what fraction the second quantity must be multiplied in order to give the first.*

Ex. 1. *What fraction is 5d. of one shilling?*

One penny = $\frac{1}{12}$th of 12 pence = $\frac{1}{12}$th of 1*s.* ;

∴ 5 pence = $\frac{5}{12}$ths of 12 pence = $\frac{5}{12}$ths of 1*s.* ;

thus the required fraction has as numerator the number giving the first quantity and as denominator the number giving the second quantity.

Ex. 2. *Reduce 15s. 4d. to the fraction of £2.*

Before we can compare these two sums we must reduce them both to a common denomination, say pence.

Now 15*s.* 4*d.* = 184 pence, and £2 = 480 pence.

Now 1 penny = $\frac{1}{480}$ of 480 pence = $\frac{1}{480}$ of £2 ;

∴ 184 pence = $\frac{184}{480}$ of 480 pence = $\frac{184}{480}$ of £2.

Hence the required fraction = $\frac{184}{480} = \frac{23}{60}$.

The rule is therefore: *Reduce both quantities to the same denomination; the required fraction has as numerator the number expressing the first quantity, and as denominator the number expressing the second quantity.*

Ex. 3. *Reduce $\frac{3}{7}$ of £8. 4s. 2½d. to the fraction of $\frac{5}{11}$ of £7. 16s. 3½d.*

$\frac{3}{7}$ of £8. 4*s.* 2½*d.* = 3 × £1. 3*s.* 5½*d.* = £3. 10*s.* 4½*d.*

= 70*s* 4½*d.* = 844½*d.* = 1689 halfpence.

$\frac{5}{11}$ of £7. 16*s.* 3½*d.* = 5 × 14*s.* 2½*d.* = £3. 11*s.* 0½*d.*

= 71*s.* 0½*d.* = 852½*d.* = 1705 halfpence.

∴ required fraction = $\frac{1689}{1705}$.

EXAMPLES XXVIII.

1. What fractions of 1*s.* are 3*d.*, 4*d.*, 7*d.*, 9*d.*, $4\frac{1}{2}d.$, $7\frac{1}{2}d.$, $9\frac{3}{4}d.$, and $10\frac{1}{2}d.$?

2. What fractions of £1 are 5*s.*, 7*s.* 6*d.*, 12*s.* 6*d.*, 15*s.*, 17*s.* 6*d.*, and 18*s.* 8*d.*?

3. What fractions of 12*s.* 6*d.* are 2*s.* 6*d.*, 5*s.*, 7*s.* 6*d.*, and 10*s.* 6*d.*?

What fraction is

4. 3*s.* 4*d.* of £7?

5. 4*s.* 9*d.* of £2. 9*s.* 1*d.*?

6. 2*s.* $2\frac{1}{2}d.$ of £1. 17*s.* 6*d.*?

7. 3*s.* $7\frac{1}{2}d.$ of £10. 17*s.* 6*d.*?

8. 13*s.* $1\frac{1}{2}d.$ of £4. 11*s.* $10\frac{1}{2}d.$?

9. A sixpence of 17*s.* 4*d.*?

10. 3*s.* of £17. 17*s.*?

11. 2*s.* $5\frac{1}{2}d.$ of £59?

What fraction of £1 is

12. $\frac{3}{16}$ of a guinea + $1\frac{1}{6}$ of a crown − $\frac{2}{15}$ of half-a-crown?

13. $\frac{2}{3}$ of $\frac{1}{4}$ of $2\frac{2}{7}$ of £3. 17*s.* + $\frac{1}{18}$ of £40. 17*s.* 4*d.*?

What fraction of a guinea is

14. $1\frac{1}{4}$ of £3. 18*s.* 2*d.*?

15. $\frac{3}{5}$ of £3. 7*s.* 6*d.* + $\frac{2}{3}$ of $\frac{1}{4}$ of $4\frac{1}{2}$ guineas?

What fraction is

16. $2\frac{1}{2}$ guineas of £$1\frac{3}{4}$?

17. $\frac{6}{7}$ of a sixpence of £3. 7*s.* 6*d.*?

18. $\frac{3}{5}\frac{1}{7}$ of £2. 2*s.* 9*d.* of £2. 2*s.* $7\frac{1}{2}d.$?

19. $\frac{7}{24}$ of 5 guineas of £9?

20. ($\frac{5}{9}$ of a guinea − $\frac{1}{6}$ of £1) of half-a-guinea?

21. Express the difference between $\frac{1}{12}$ of £1 and $\frac{1}{14}$ of a guinea as a fraction of half-a-crown.

22. From $2\frac{3}{4}$ of 11*s.* 8*d.* take $1\frac{1}{2}$ of 17*s.* $2\frac{1}{2}d.$, and express the result as a fraction of 13*s.* $5\frac{1}{4}d.$

23. What fraction of £7. 13*s.* 5*d.* must be added to $\frac{3}{11}$ of $\frac{5}{7}$ of £5. 15*s.* 6*d.* so that the sum may be £2. 4*s.* 5*d.*?

24. What fraction of £1. 12*s.* 6*d.*, together with $7\frac{1}{2}$ guineas, will equal £8. 5*s.*?

25. If $\frac{7}{19}$ of a guinea be taken from $\frac{3}{7}$ of 27*s.*, what fraction of $\frac{5}{11}$ of 15*s.* will be left?

26. Find what fraction of a guinea is equal to the difference between $\frac{2}{7}$ of a crown and $\frac{13}{11}$ of a shilling.

What fraction is

27. $2\frac{1}{2}$ ozs. of 1 lb. Av.?

28. $3\frac{1}{2}$ mins. of 1 hr.?

29. $7\frac{1}{2}$ yds. of 1 mile?

30. 121 sq. yds. of 1 acre?

31. $17\frac{1}{2}$ oz. of 5 cwt.?

32. 1 lb. Troy of 1 lb. Av.?

33. $3\frac{1}{2}$ sq. poles of 14 acres?

34. 1 cwt. 4 st. 10 lbs. of 1 ton?

35. 1540 yds. 2 ft. 9 in. of 1 mile?

36. 2 qrs. $2\frac{2}{3}$ lbs. of 1 cwt.?

37. 3 qrs. 11 lbs. of 19 cwt.?

38. 2 dwt. 15 grs. of 3 oz.?

39. 4 sq. po. of 3 ac. 2 r. 36 sq. po.?

40. 3 lbs. 8 oz. of 1 cwt. 3 qrs. 14 lbs.?

41. 3 qrs. 14 lbs. 7 oz. of 1 cwt. 6 lbs. 2 oz.?

42. 319 days of a year of $365\frac{1}{4}$ days?

43. 1 fur. 5 yds. of $22\frac{1}{2}$ miles? **44.** 3 qrs. 18 lbs. $10\frac{2}{3}$ oz. of a cwt.?

45. 16 ft. 6 in. of 1 furlong? **46.** 3 cwt. 3 qrs. 20 lbs. of 1 ton?

47. 3 qrs. 11 lbs. $3\frac{1}{5}$ oz. of 140 lbs. Troy (7000 grs. Troy = 1 lb. Av.)?

48. $1\frac{1}{8}$ pint of 27 bushels? **49.** 5 lbs. 9 oz. of 4 tons 9 cwt.?

50. 3 tons 4 cwt. 1 qr. $2\frac{4}{5}$ lbs. of 5 tons 7 cwt. 14 lbs.?

What fraction of

51. $\frac{8}{9}$ of 2 cwt. 1 qr. 4 lbs. 8 oz. is $\frac{2}{7}$ of 5 cwt. 10 lbs.?

52. $\frac{3}{4}$ of $\frac{5}{9}$ of a league is $\frac{2}{5}$ of $\frac{5}{7}$ of a mile?

53. What weight is the same fractional part of 12 cwt. 3 qrs. 21 lbs. that £4. 6*s.* $5\frac{3}{4}$*d.* is of £6. 15*s.* $10\frac{3}{4}$*d.*?

52. We shall close the present chapter with some miscellaneous problems involving vulgar fractions.

Ex. 1. *A man walked* $\frac{2}{5}$ *of a certain journey and then rode* 9 *miles to finish the journey; what was the length of his journey?*

Since he walked $\frac{2}{5}$ of the journey, he must have ridden $1 - \frac{2}{5}$, *i.e.* $\frac{3}{5}$, of it.

$\therefore \frac{3}{5}$ of the journey = 9 miles,

i.e. $\frac{1}{5}$ of the journey = 3 miles;

$\therefore$ the whole journey = 15 miles.

Ex. 2. *A man who owned* $\frac{5}{18}$ *of a ship sold* $\frac{3}{7}$ *of his share for* £2500; *what is the value of the ship?*

The share he sold $= \frac{5}{18} \times \frac{3}{7} = \frac{5}{42}$ of the whole;

$\therefore \frac{5}{42} \times$ whole value of the ship = £2500;

$\therefore \frac{1}{42} \times$ whole value of the ship = £2500 ÷ 5 = £500;

$\therefore$ whole value of the ship = £500 × 42 = £21000.

Ex. 3. *A person spends* $\frac{2}{3}$ *of his money, then* $\frac{3}{5}$ *of the remainder, and then has* £4; *what had he to commence with?*

After spending $\frac{2}{3}$ of his money he has $\frac{1}{3}$ left. He then spends $\frac{3}{5}$ of this $\frac{1}{3}$, that is $\frac{3}{5} \times \frac{1}{3}$ of his original money, that is $\frac{1}{5}$.

He then has left $\frac{1}{3} - \frac{1}{5}$, that is $\frac{5-3}{15}$, that is $\frac{2}{15}$ of his original money.

But this is £4; $\therefore \frac{2}{15} \times$ his original money = £4;

$\therefore$ his original money = £4 ÷ $\frac{2}{15}$ = £4 × $\frac{15}{2}$ = £30.

MISCELLANEOUS EXAMPLES XXIX.

Vulgar Fractions.

1. A man owned $\frac{7}{12}$ of a lottery ticket which drew a prize of £500; what was his share worth?

2. A man owes £14 to each of three creditors; to one he pays $\frac{1}{3}$, to another $\frac{2}{7}$, and to the third $\frac{2}{5}$ of his debt; what does he still owe?

3. Gunpowder is composed of $\frac{1}{10}$th sulphur, $\frac{3}{20}$ths charcoal, and $\frac{3}{4}$ saltpetre; how much of each material is required to make up $1\frac{1}{8}$ ton?

4. If gunpowder contain $\frac{3}{4}$ of its weight of saltpetre, and saltpetre cost 5*s.* $7\frac{3}{4}$*d.* a cwt., what is the cost of the saltpetre in 16 tons 12 cwt. of gunpowder?

5. 500 boys are distributed in 3 houses; the smallest house contains $\frac{7}{25}$ of the whole number, and the largest house contains $\frac{19}{14}$ of the smallest; what is the number in each?

6. In a chest of mixed tea weighing 1 cwt. 1 qr. 14 lbs., $\frac{11}{14}$ of the whole was Congou, worth 1*s.* 7*d.* per lb., $\frac{2}{5}$ of the remainder was Pekoe, worth 2*s.* 4*d.* per lb., and the rest was Assam, worth 2*s.* 3*d.* per lb.; what was the value of the chest?

7. The value of unwrought silver being 2*s.* 5*d.* per oz., and the workmanship costing $\frac{4}{5}$ the value of the metal, what is the whole cost of a salver weighing 45 oz.?

8. Three times a number and the seventh part of it when added together give 374; what is the number?

9. The ninth part of a certain number exceeds its eleventh part by 2; what is the number?

10. The fifteenth part, the twelfth part, and the seventh part of a certain number when added together give 1353; what is the number?

11. From a number is taken the sum of its fifth, seventh, and ninth parts, and the remainder is 1892; what is the number?

12. The difference of two numbers is 228, and their sum is equal to 14 times the smaller; what are the numbers?

13. By what fraction of itself does $\frac{1}{17}$ exceed $\frac{5}{88}$?

14. In a cricket-match $\frac{19}{20}$ of the runs are scored by the batsmen and the "extras" amount to 6; find the total score.

15. In a chess-match where the time is limited it is found after 35 minutes' play that $\frac{7}{12}$ of the whole time is left; what was the time allowed?

16. Four men between them paid a bill; the first paid $\frac{1}{4}$, the second $\frac{1}{5}$, and the third $\frac{1}{6}$; what fraction did the fourth pay?

17. $\frac{1}{4}$ and $\frac{3}{7}$ of a sum of money are paid away; what fraction is left?

18. $\frac{5}{6}$ of a certain distance is 6 miles; what is the distance?

19. Of what length of road are 150 yds. 2 ft. the $\frac{4}{17}$ part?

20. How many pieces of string, each $\frac{3}{5}$ of a foot in length, can be cut from a piece $4\frac{3}{4}$ yards long, and how long is the portion left over?

21. How many pieces of wood, each $\frac{4}{7}$ of an inch in length, can be cut from a rod $5\frac{7}{8}$ feet long, and what length will be left over?

22. A vessel holds $4\frac{4}{5}$ quarts; how many times can it be filled from a vat which contains $367\frac{2}{3}$ gallons, and how much will then be left in the vat?

23. If $\frac{3}{7}$ of a ship be worth £6324, what share of it is worth £1844. 10*s.*?

24. If $\frac{7}{1\,3}$ of a ton be worth £17. 10*s.*, what is $\frac{4}{6}\frac{9}{5}$ of a ton worth?

25. If $2\frac{3}{4} - 1\frac{5}{6}$ of an estate cost £440, what will $2\frac{1}{2}$ of $6\frac{5}{1\,2}$ of it cost?

26. Convert 41 oz. Troy into Avoirdupois weight.

27. After $\frac{1}{4}$ of the books in a library have been dusted, and then $\frac{2}{9}$ of the remainder, it is found that 770 are left; how many have been dusted?

28. If I pay away one-third of my money, then one-quarter of what is left, and finally one-fifth of the remainder, what fraction of my original money have I left?

29. A post has one-fifth of its length in the mud, one-half in the water, and the rest of it, 9 feet in length, is out of the water; what is the whole length of the post?

30. In an orchard one-half of the trees are apple trees, one-fourth pear trees, one-sixth plum trees, and the remainder, 50 in number, are cherry trees; how many are there in all?

31. A boy, after giving $\frac{1}{4}$ of his pocket-money to one boy, and $\frac{7}{3}$ of the remainder to another, has 9*d.* left; how much had he at first?

32. A person who paid away two-thirds of the contents of his purse, and was robbed of two-sevenths of the remainder, found that he had still £3. 15*s.* left; how much had he at first?

33. For one-third of a mile a submarine cable was laid overland, $\frac{3}{1\,3}$ of it was suspended in the water, and $\frac{1\,3}{1\,7}$ rested on the bed of the sea; what was the length of the cable?

34. A man spent $\frac{5}{1\,6}$ of his money, then $\frac{2}{3}$ of the remainder, and then found he had £1485 left; how much had he at first?

35. A man pays away half his money to *A*, a third of what he has left to *B*, and a fifth part of what he has still left to *C*; if, after these payments, he has 12*s.* 8*d.* left, how much had he at first?

36. A person spends $\frac{2}{3}$ of his money, then $\frac{1}{3}$ of the remainder, and then $\frac{1}{8}$ of what he still has left. Finally he had £7 in hand; how much had he at first?

37. A man after spending $\frac{2}{5}$ and $\frac{1}{9}$ of his money finds he has £33 left; how much had he at first?

38. If $\frac{9}{1\,7}$ of a man's money be multiplied by 5 and £83 be taken from the result the remainder is £34; how much money has the man?

39. A person dies leaving a third of his property to his wife, a fourth to each of his two sons, and the rest to his daughter, who gets £1666. 13*s.* 4*d.*; what was his property worth, and how much did the wife and each son receive respectively?

40. In a subscription list one-half of the subscriptions are a guinea each, one-third a half guinea each, and the 5*s.* subscriptions, which complete the list, amount to £12; find the whole amount subscribed.

CHAPTER IV.

DECIMAL FRACTIONS.

53. A Decimal Fraction is a fraction whose denominator is 10, or some power of 10, such as 100, 1000, etc.

In the last chapter we have often had to reduce fractions to a common denominator, and one great advantage of decimal fractions is that in their case the reduction is easy.

54. Notation for Decimal Fractions. The fraction $\frac{4}{10}$ is written ·4, the fraction $\frac{4}{100}$ is written ·04, the fraction $\frac{4}{1000}$ as ·004, and so on.

Thus ·863 means $\frac{8}{10}+\frac{6}{100}+\frac{3}{1000}$, that is, $\frac{863}{1000}$, and ·00347 means $\frac{0}{10}+\frac{0}{100}+\frac{3}{1000}+\frac{4}{10000}+\frac{7}{100000}$, that is, $\frac{347}{100000}$.

So 74·39 means $74+\frac{3}{10}+\frac{9}{100}$, that is, $74\frac{39}{100}$.

The point used in the decimal notation is called "the decimal point," or "the point" for brevity.

The number "8·6" is read as "*eight point six.*" So 3·00473 is read as "*three point nought nought four seven three.*"

55. We may deduce the Decimal Notation in another way. We know that 333 means 3 hundreds + 3 tens + 3 'ones' (or units), and that therefore each figure denotes a quantity 10 times as small as that of the figure immediately to the left of it. It is found very useful to have figures *to the right of the units digit*, still keeping to the rule that each figure is 10 times as small as the same figure if this were placed immediately on its left.

In order to shew which is the units digit a point is placed immediately after it. Thus in 333·33 the 3 is the units digit. This would read "three hundred and thirty-three point three, three." Now the first '3' to the right of the point has, by our rule, $\frac{1}{10}$th of the value of the '3' immediately before it, so that it $=\frac{1}{10}$ of 3 units or $\frac{3}{10}$. So the second '3' to the right of the point $=\frac{3}{100}$.

Thus 333·33 = 3 hundreds + 3 tens + 3 units + $\frac{3}{10}+\frac{3}{100}$.

So ·69245 $=\frac{6}{10}+\frac{9}{100}+\frac{2}{1000}+\frac{4}{10000}+\frac{5}{100000}=\frac{69245}{100000}$, and ·0043 $=\frac{4}{1000}+\frac{3}{10000}=\frac{43}{10000}$.

From these cases we deduce the rule:

Any decimal fraction (or whole number and decimal fraction) is equal to a vulgar fraction whose numerator consists of the figures of

the decimal (or whole number and decimal) with the dot removed, and whose denominator consists of a 1 *followed by as many zeros as there were figures following the decimal point in the original decimal fraction.*

EXAMPLES XXX.

Express as decimal fractions :

1. $\frac{1}{10}$, $\frac{7}{10}$, $\frac{9}{10}$. **2.** $\frac{37}{100}$, $\frac{7}{1000}$. **3.** $\frac{4}{100}$, $\frac{9}{10000}$.

4. $\frac{34}{100000}$, $\frac{837}{1000}$. **5.** $\frac{349}{100000}$, $\frac{6034}{1000000}$. **6.** $5\frac{5}{1000}$, $9\frac{107}{1000000}$.

7. $\frac{18}{100}$, $8\frac{325}{100000000}$. **8.** $\frac{5}{1000}$, $\frac{673491}{100000000}$.

9. $\frac{83497}{10000}$, $\frac{638975}{10000}$. **10.** $\frac{1578}{100}$, $\frac{845}{100000000}$.

11. Three tenths ; four and seven tenths.

12. Six and eight tenths ; fifty-three hundredths.

13. Eleven and seventeen hundredths ; sixty-five thousandths.

14. Eight and nineteen thousandths ; eighty-five ten-thousandths.

15. Nineteen hundred thousandths ; five millionths.

Express the following decimals as vulgar fractions in their lowest terms:

16. ·6, 3·2, 7·8. **17.** ·15, ·48, 3·72. **18.** ·36, ·864, ·05.

19. ·625, ·007, 11·256. **20.** ·216, ·1296, 5·6485.

21. ·00256, ·0008125. **22.** ·050005, ·32846. **23.** 37·000625.

24. 36·0705. **25.** ·015625. **26.** ·016125.

27. ·0625 × ·0032. **28.** ·016 ÷ 64.

Express in words the following decimals :

29. ·7, ·04, ·37. **30.** ·93, ·008, ·00017, ·00063, ·0000345.

56. *A decimal fraction is multiplied by ten if its decimal point be moved one place to the right, by a hundred if it be moved two places to the right, by a thousand if three places to the right, and so on.*

Consider the decimals ·4627, 4·627, 46·27, 462·7, ... Now

$\cdot 4627 = \frac{4627}{10000}$, by the last article,

$4\cdot 627 = \frac{4627}{1000} = 10 \times \frac{4627}{10000} = 10 \times \cdot 4627$,

$46\cdot 27 = \frac{4627}{100} = 100 \times \frac{4627}{10000} = 100 \times \cdot 4627$,

$462\cdot 7 = \frac{4627}{10} = 1000 \times \frac{4627}{10000} = 1000 \times \cdot 4627$, and so on.

Similarly a decimal fraction is divided by 10, 100, 1000, ... if its decimal point be moved one, two, three, ... places towards the left.

Thus ·004395 = ·04395 ÷ 10.

For ·004395 $= \frac{4395}{1000000} = \frac{4395}{100000} \div 10 = \cdot 04395 \div 10$.

So ·038457 = 3·8457 ÷ 100.

For ·038457 $= \frac{38457}{1000000} = \frac{38457}{10000} \div 100 = 3\cdot 8457 \div 100$.

Similarly $\cdot 3859732 = 385\cdot 9732 \div 1000$.

57. *A decimal fraction is unaltered by adding any number of cyphers at the right hand of the fraction.*

Thus ·0427 and ·0427000 mean the same.

For (by Art. 55) $\cdot 0427000 = \frac{4270000}{10000000} = \frac{4270}{10000} = \cdot 0427$.

This proof always holds; for each 0 to the right hand of the decimal requires, when we change the decimal to a vulgar fraction, an extra 0 in the denominator, and these 0's therefore cancel.

So $5 = 5 \cdot 0000 \ldots$.

58. *A decimal is divided by* 10 *if a cypher be placed at its left so as to immediately follow the decimal point.*

Thus $\cdot 0324 = \cdot 324 \div 10$.

For $\cdot 0324 = \frac{324}{10000}$ (by Art. 55) $= \frac{324}{1000} \div 10 = \cdot 324 \div 10$.

Similarly for any other cases.

We therefore see that any decimal fraction whose point is already at its extreme left may be divided by 10, 100, 1000, ... by introducing one, two, three, ... additional cyphers immediately to the right of the decimal point.

Thus $\cdot 638 \div 1000 = \cdot 000638$, $\cdot 00729 \div 100 = \cdot 0000729$,

and $17 \cdot 3845 \div 10000 = \cdot 00173845$.

EXAMPLES XXXI.

Multiply by 10 the following decimals:

1. 4·6, 8·9, 6·43. **2.** 7·09, 18·434, 27·8453.

Multiply by 100:

3. 45·9, 8·6, 7·38, 4·57. **4.** 9·804, 11·653, 12·057, 8·004.

Multiply by 10000:

5. 6·34, 7·005, ·0003. **6.** ·00103, 13·06084, ·000009.

Divide by 10:

7. 3, 9, 6·4, ·05, 11·16. **8.** 10·004, ·0009, ·00802.

Divide by 1000:

9. 8, 11, 5, ·07, ·9. **10.** 8·03, 5·79, ·007, ·0008.

Write down the value of the following:

11. $8 \cdot 45 \times 100$, $9 \cdot 27 \times 1000$, $\cdot 0034 \times 100$.

12. $\cdot 076 \times 10$, $\cdot 089 \times 100$, $9 \cdot 37 \times 10000$.

13. $\cdot 6 \div 10$, $\cdot 08 \div 100$, $\cdot 009 \div 1000$.

14. $89 \cdot 03 \div 100$, $46 \cdot 03 \div 1000$, $46 \cdot 03 \times 100$.

15. $89 \cdot 03 \times 1000$, $5 \cdot 6 \times 100$, $5 \cdot 6 \div 1000$. **16.** $\cdot 0834 \times 100$, $\cdot 0834 \div 10000$.

17. $\cdot 0056 \div 10000$, $\cdot 00037 \times 100$. **18.** $\cdot 00037 \div 1000$, $\cdot 0134 \div 100$.

ADDITION OF DECIMAL FRACTIONS.

59. Set down the quantities to be added under one another, the decimal points of the different quantities being made to come in the same vertical line. Add up exactly as in simple addition, and place the decimal point in the final result immediately under all the other decimal points.

Ex. *Add* 32·2879, 15·5, ·0347, 279·64, *and* 1·0035.

The addition is made just as if there were no decimal points, and the point introduced so as to be exactly under the others.

```
32·2879
15·5
 ·0347
279·64
 1·0035
328·4661
```

EXAMPLES XXXII.

Add together

1. ·13, ·48, ·75, ·89, and ·32.
2. ·05, ·32, ·71, 1·82, and 6·45.
3. ·843, ·057, ·937, ·285, and ·006.
4. ·3, ·03, ·003, ·033, ·0303, and ·030303.
5. 3·09, 4·65, 8·03, 14·25, and 7·89.
6. 5·008, 6·078, 9·109, 8·309, and 11·006.
7. 47·01913, 635·77, ·00187, and 352·9.
8. 63·5435, 21·207, 19·5, ·000835, 357, and 9·0625.
9. 25·87, 98·00834, ·083, ·0006352, and 8325·8.
10. 131·121, 2·987463, ·00101, and 61·305699.
11. ·0013, 1·010305, 777·8989, and ·235601.
12. 6·05, 8·008, 7·0007, and 9·090090009.
13. ·000367, ·0000821, ·0035, and ·07.

SUBTRACTION OF DECIMAL FRACTIONS.

60. Place the number to be subtracted under the other number, so that the decimal point comes under the decimal point, the units under the units, the tenths under the tenths, and so on. Proceed as in simple subtraction, and in the result put the point under the points in the other two lines.

Ex. 1. *Subtract* 23·7695 *from* 347·9832.

```
347·9832
 23·7695
324·2137
```

Ex. 2. *Subtract* 2·3478 *from* 23·5.

On setting down the question as on the left hand there are no figures above the 478, since the number of decimal places in the second number is greater than the number in the first. But by Art. 57 a decimal is unaltered by affixing cyphers to its right

```
23·5
 2·3478
21·1522
```

```
23·5000
 2·3478
21·1522
```

hand. Hence the question may be written as on the right. The subtraction can now be made as before, and the answer is 21·1522.

In actual practice there is no necessity to affix the cyphers. The student may assume their presence.

EXAMPLES XXXIII.

Subtract

1. ·3 from ·9. **2.** ·345 from ·632. **3.** 8·07 from 11·3.

4. ·007 from ·07. **5.** ·007 from ·7. **6.** ·00035 from ·006.

7. 7·08095 from 70·8095. **8.** ·003045 from ·01.

9. 37·589 from 86·1. **10.** 7·08036 from 9·007.

Find the difference between

11. 3·8721 and 1·6435. **12.** 8·972 and 3·0067. **13.** 1·834 and 183·4.

14. Subtract from 50 the sum of 1·23, 3·874, 15·9327, and 6·0003.

15. What number subtracted from 103·8745 leaves 47·3825?

16. What number added to 174·387 gives 257·32?

17. By how much does the sum of 34·2375 and 16·0007 exceed the sum of 13·9324 and 15·9874?

MULTIPLICATION OF DECIMALS.

61. **I. Multiplication by a whole number.**

Consider the following examples:

(1) ·4 $\times 6 = \frac{4}{10} \times 6 = \frac{24}{10} = 2\frac{4}{10} = 2{\cdot}4$ (Art. 55).

(2) ·07 $\times 39 = \frac{7}{100} \times 39 = \frac{273}{100} = 2\frac{73}{100} = 2{\cdot}73$.

(3) ·009 $\times 13 = \frac{9}{1000} \times 13 = \frac{117}{1000} = {\cdot}117$.

(4) ·000082 $\times 19 = \frac{82}{1000000} \times 19 = \frac{1558}{1000000} = {\cdot}001558$.

In each of these examples we see that the multiplication might be effected by multiplying the two numbers together as if they had been integers, and in the result placing the decimal point so that there are as many figures to the right of it as there were in the multiplicand.

EXAMPLES XXXIV.

Multiply

1. ·7 by 3, 8, and 9. **2.** ·9 by 5, 6, and 11.

3. ·13 by 6, 9, 14, and 23. **4.** ·25 by 9, 15, and 34.

5. ·08 by 13, 25, and 49. **6.** ·007 by 3, 8, 11, 28, and 65.

7. 8·5 by 7, 11, and 27. **8.** 7·6 by 22, 42, and 75.

9. 16·05 by 14, 32, and 83. **10.** 21·02 by 11, 17, and 35.

11. ·00125 by 17, 77, and 176. **12.** ·0000438 by 3, 11, and 63.

13. ·000006375 by 9, 27, and 342.

62. **II. Multiplication by a decimal fraction.**

Ex. 1. $\cdot 98 \times \cdot 4 = \frac{98}{100} \times \frac{4}{10} = \frac{392}{1000} = \cdot 392$ (Art. 55).

Ex. 2. $\cdot 0198 \times 4 \cdot 3 = \frac{198}{10000} \times \frac{43}{10} = \frac{198 \times 43}{100000} = \frac{8514}{100000} = \cdot 08514$.

We see that in each of these examples the multiplication might have been effected by the following rule:

Multiply the numbers as if they were whole numbers, and in the product place the decimal point so that the number of decimal places in it is equal to the sum of the number of decimal places in the multiplier and multiplicand. Cyphers are to be prefixed if necessary.

Thus, in Ex. 1, $4 \times 98 = 392$. Now in ·98 there are 2 digits following the point, and in ·4 there is 1; also $2 + 1 = 3$. Hence in 392 we count three digits from the right, and then place the decimal point so that we have the result ·392.

Ex. 2 is like Ex. 1, except that a nought has to be prefixed to make up 5 (*i.e.* $4 + 1$) places in the result.

On trial the rule will be found to be true for all cases.

Ex. 3. *Multiply* 43·62 *by* 2·153, *and* 1·00034 *by* ·0123.

43·62	1·00034
2·153	·0123
13086	300102
21810	200068
4362	100034
8724	12304182
9391386	
Now $3+2=5$.	Now $5+4=9$.
$\therefore$ *Ans.* = 93·91386.	$\therefore$ *Ans.* = ·012304182.

63. From the rule in the last article it is readily seen that if a number with 3 decimal places be multiplied by one with 2 decimal places, the extreme right-hand digit of the product will be in the fifth (that is, $3 + 2$) decimal place.

Similarly, if a number with 4 decimal places be multiplied by one with 5, the extreme right digit of the product will be in the ninth (that is, $4 + 5$) decimal place.

We thus have a second working rule for the multiplication of two decimals:

Set down the multiplier under the multiplicand so that the decimal point of the one is under that of the other; multiply as if both numbers were whole numbers, but place the first figure of the third line of the work as many places to the right of the first figure of the multiplier as there are decimal places in the multiplicand. The

decimal point of the final product will then come exactly under the decimal points of the multiplier and multiplicand.

The two last multiplications will then appear as follows:

Ex. 1. *Multiply* 43·62 *by* 2·153.

In the multiplicand there are two decimal places, and in the multiplier there are three. The right-hand figure of the product is therefore in the fifth (3+2) place of decimals.

Place the first figure of the product, 6, in the fifth place of decimals, that is, two places to the right of the 3 of the multiplier. The rest of the multiplication then proceeds as with whole numbers.

43·62
2·153
·13086
2·1810
4·362
87·24
93·91386

Ex. 2. *Multiply* 1·00034 *by* ·0123.

Here the multiplicand has five decimal places, and the multiplier has four.

The first figure of the product must therefore be in the ninth (4+5=9) place of decimals, that is, five places to the right of the first figure of the multiplier.

1·00034
·0123
·000300102
·00200068
·0100034
·012304182

64. A third method differs from the second only in the order of multiplication. Here we begin with the highest digit of the multiplier.

The two preceding examples worked in this way appear then as follows:

43·62	1·00034
2·153	·0123
87·24	·0100034
4·362	·00200068
2·1810	·000300102
·13086	·012304182
93·91386	

We subjoin an example worked by all three methods.

Ex. *Multiply* ·000325 *by* ·0827.

First Method.	*Second Method.*	*Third Method.*
·000325	·000325	·000325
·0827	·0827	·0827
2275	2275	2600
650	650	650
2600	2600	2275
268775	·0000268775	·0000268775

There are to be ten places following the point.

∴ *Ans.* = ·0000268775.

If the second or third methods be used, it is well to verify the result by seeing that the number of decimal places in it is

equal to the sum of the number of places in the multiplier and multiplicand.

EXAMPLES XXXV.

Multiply

1. 3·6 by 4·2. **2.** 5·7 by 9·6. **3.** ·59 by 3·4. **4.** 6·7 by 30·5.

5. 5·3 by ·67. **6.** ·63 by ·97. **7.** ·085 by ·037. **8.** ·935 by ·081.

9. ·079 by ·85. **10.** 11·687 by ·45. **11.** 8·345 by 7·36.

12. 9·325 by 8·41. **13.** 1·005 by ·097. **14.** ·00053 by ·037.

15. 3·753 by 9·08. **16.** 270·26 by ·00025. **17.** ·00034 by ·0057.

18. 2·083 by ·059 and 72300 successively. **19.** 41·372 by ·00083.

Find the continued product of

20. ·2, ·02, and ·002. **21.** 3·4, ·34, and ·034. **22.** 5·6, 8·4, and 10·8.

23. Simplify $(6\cdot34+3\cdot07-4\cdot892)\times(8\cdot97+5\cdot34-6\cdot71)$.

Find the values of

24. $(\cdot02)^3$, $(\cdot03)^4$, and $(\cdot005)^3$.

25. In a town with 15,000 electors, ·72 of the whole number voted for one candidate, and ·125 for the other; how many electors did not vote?

26. Find the value of 1·683 lbs. of gold, if 1 oz. is worth £4·0099.

27. The true length of a year being 365·24224 days, what does the error amount to in the five centuries commencing with A.D. 1801?

DIVISION OF DECIMALS.

65. I. When the divisor is a whole number.

Ex. 1. *Divide* 36·638 *by* 7.

In this example the division proceeds just as in the ordinary division of integers, but as soon as we use the first of the digits following the decimal point in the dividend, we set down the decimal point in the quotient.

$$7\overline{)36\cdot638}\quad 5\cdot234$$

Ex. 2. *Divide* ·0070065 *by* 9.

Here 9 into the first two zeros goes zero twice in succession; 9 into 7 will not divide, and we thus have a third nought; 9 into 70 goes **7** and 7 over, and the rest follows as in ordinary division. We write down the decimal point at once as there is no whole number in the dividend.

$$9\overline{)\cdot0070065}\quad \cdot0007785$$

Ex. 3. *Divide* ·000391 *by* 23.

As in Ex. 2 we write down the decimal point in the quotient at once.

Now 23 into 0 tenths, 0 hundredths, and 0 thousandths goes **0** tenths, **0** hundredths, and **0** thousandths. We therefore set down three 0's following the decimal point. 23 into 3 will not divide; so we set down a fourth 0. 23 into 391 goes **17**. The answer therefore = ·000017.

$$\begin{array}{r|l} 23 & \cdot000391\,(\,\cdot000017 \\ & \underline{23} \\ & 161 \\ & \underline{161} \end{array}$$

Ex. 4. *Divide* 100·62 *by* 42 *to four places of decimals.*

```
42 ) 100·62 ( 2·3957
       84
      ----
      166
      126
      ----
       402
       378
       ---
        240..... (1)
        210
        ---
         300
         294
         ---
           6
```

Notice that at the line marked (1) we begin to put down **0**'s. The reason we can do this is that 100·62=100·62000000....

EXAMPLES XXXVI.

Divide

1. 15·375 by 3, 5, and 8. **2.** 913·8768 by 3, 4, and 5.
3. ·01344 by 7 and 5. **4.** ·002871 by 11 and 8.
5. 2·21 by 13 and 13000. **6.** ·053 by 25. **7.** ·0918 by 18.
8. ·14616 by 72. **9.** ·27 by 144. **10.** 5752·8 by 376.
11. 125·837 by 3401. **12.** 6600·5 by 307. **13.** 3·6387 by 117.
14. 12191·92 by 304. **15.** 32585·85 by 681. **16.** 99·994 by 2890.
17. 19·69714 by 1301. **18.** 18596·508 by 98760.

Perform the following divisions to four places of decimals:

19. ·43÷7. **20.** ·00389÷17. **21.** 16·03857÷19.
22. ·05005÷63. **23.** ·0976÷83. **24.** Simplify 839·7527÷5×13.

66. II. When the divisor is itself a decimal fraction.

Consider the following:

Ex. 1. $\cdot48 \div \cdot6 = \frac{\cdot48}{\cdot6} = \frac{\cdot48\times10}{\cdot6\times10} = \frac{4\cdot8}{6} = \cdot8.$

Ex. 2. $4\cdot2 \div \cdot0006 = \frac{4\cdot2}{\cdot0006} = \frac{4\cdot2\times10000}{\cdot0006\times10000} = \frac{42000}{6} = 7000.$

Ex. 3. $\cdot0221 \div \cdot00013 = \frac{\cdot0221}{\cdot00013} = \frac{\cdot0221\times100000}{\cdot00013\times100000} = \frac{2210}{13} = 170.$

In each case we have multiplied the divisor and dividend by a sufficient number of tens to make the divisor a whole number.

The working rule is therefore as follows: *Move the decimal point as many places to the right in both divisor and dividend as will make the divisor a whole number, annexing cyphers to the dividend, if necessary; divide in the usual way as in Simple Division; but as soon as in the course of the work any figure is used from the decimal part of the dividend so altered, the decimal point must be inserted in the quotient.*

Ex. 4. *Divide* ·01324554 *by* ·00987.

Moving the decimal point five places to the right in each, we have to divide 1324·554 by 987.

In the third line of the work, where the remainder is 337, we take down the first digit of the decimal part of the dividend, as altered, and therefore we then insert the decimal point in the quotient.

```
987 ) 1324·554 ( 1·342
      987
      ----
      3375
      2961
      ----
       4145
       3948
       ----
        1974
        1974
        ----
```

Ex. 5. *Divide* ·00031 *by* ·0032.

Moving the decimal point four places to the right in both divisor and dividend, we have to divide 3·1 by 32.

EXAMPLES XXXVII.

Divide

1. ·2 by ·02 and ·1. **2.** ·0505 by ·05 and ·001.

3. ·0666 by ·0001 and ·003. **4.** 54 by ·06, ·009, and ·027.

5. 4·73 by 1·1 and ·011. **6.** 2·21 by 1·3, ·13, ·013, and ·000013.

7. ·00108 by ·012, ·08, and ·0003. **8.** 2·86 by ·013.

9. 18·2 by ·91. **10.** 2·5 by ·0625. **11.** ·504 by ·00042.

12. 3·1 by ·0025. **13.** 6·82 by ·0125. **14.** 42·547 by ·00542.

15. 53 by ·0025. **16.** 1346·8 by ·00148. **17.** 177·498 by ·0519.

18. ·0036 by ·0144. **19.** 30·5118 by 50·6. **20.** ·003634 by 6·32.

21. ·00427 by 3·05. **22.** 1·37852 by 28·6. **23.** ·060248 by 27·2.

24. 8404·99599 by 834·657. **25.** ·00012 by ·0192.

26. ·39908027 by 248·03. **27.** ·0832 by 2·56.

28. 53·9295 by ·00687.

Find, to four places of decimals, the value of the following :

29. 68·8085 ÷ 1·5. **30.** 7 ÷ ·091. **31.** ·3728 ÷ ·0283.

32. ·01234 ÷ 23·47. **33.** ·0064 ÷ ·51863.

34. ·015 × 2·2 ÷ ·035. **35.** 15·341 × 7·008 ÷ 3·29.

Divide

36. ·3833336 by the product of ·031 and 2·05.

37. the product of ·004 and 32·4 by 6·4.

38. the product of ·0374 and ·0075 by the difference between ·675 and ·6375.

Find the value of

39. (·004 ÷ ·025) ÷ (124 ÷ ·0062).

40. (·002 × 36·25 ÷ 0·029) − (102·85 × 0·04 ÷ 1·7).

41. (1·61 × 0·0209 ÷ 0·00253) − (2·03 × 0·336 ÷ 32·48).

42. $\dfrac{\cdot 23 \div \cdot 023}{\cdot 0023 \div 23000}$.

43. The lengths of two lines are respectively 25·3847 and ·0385 feet; how many lines as long as the latter can be cut off from the former, and what length will then be left?

44. A wire ·2346 yards long is cut up into pieces, each ·007 of a yard long; how many such pieces will there be, and what length will be left over?

45. A gallon contains 277·274 cubic inches; how many gallons (to the fourth place of decimals) are there in a cubic foot?

46. On dividing a certain number by ·059, the quotient is 538 and the remainder ·027; what is the number?

47. If 1869 sovereigns are coined out of 40 Troy pounds weight of standard gold, what is the weight of a sovereign in grains? Also what is the value of an ounce of standard gold?

CONVERSION OF A VULGAR TO A DECIMAL FRACTION.

67. We have seen (Art. 49) that a vulgar fraction may be looked upon as the quotient of the numerator divided by the denominator.

To convert a vulgar fraction into a decimal fraction we therefore reduce it to its lowest terms, and then divide the numerator by the denominator.

Ex. $\frac{7}{8} = 7 \div 8$;

therefore $\frac{7}{8} = \cdot 875$.

$$8 \overline{)\, 7 \cdot 000} \quad \cdot 875$$

Ex. $\frac{11}{25} = 11 \div 25$;

therefore $\frac{11}{25} = \cdot 44$.

$$25 \overline{)\, 11 \cdot 0} \quad \cdot 44$$
$$10\,0$$
$$100$$
$$100$$

Ex. $\frac{63}{64} = 63 \div 64$;

therefore $\frac{63}{64} = \cdot 984375$.

$$64 \begin{cases} 8 \overline{)\, 63} \\ 8 \overline{)\, 7 \cdot 875} \end{cases} \quad \cdot 984375$$

68. In examples such as those given in the last article, where the factors of the denominator are either 2's or 5's, the conversion can often be more expeditiously made without division.

For $$\frac{7}{8} = \frac{7}{2 \times 2 \times 2} = \frac{7 \times 5 \times 5 \times 5}{2 \times 2 \times 2 \times 5 \times 5 \times 5} = \frac{875}{1000} = \cdot 875.$$

Here we resolved the denominator into products of 2, and then multiplied both numerator and denominator by as many 5's as there were 2's in the denominator. The denominator then consists of powers of 10, and the result is convertible at once into a decimal as in Art. 55.

Again, $\frac{11}{25}=\frac{11}{5\times5}=\frac{11\times2\times2}{5\times5\times2\times2}=\frac{44}{100}=\cdot44$. Here we multiply by as many 2's as there are 5's in the denominator.

EXAMPLES XXXVIII.

Express as decimal fractions:

1. $\frac{1}{4}$, $\frac{7}{8}$, $\frac{3}{5}$, $\frac{11}{16}$. **2.** $\frac{3}{4}$, $\frac{7}{5}$, $\frac{13}{25}$, $\frac{17}{8}$. **3.** $\frac{15}{16}$, $\frac{3}{32}$, $\frac{14}{25}$, $\frac{8}{125}$.

4. $3\frac{11}{32}$, $5\frac{69}{125}$. **5.** $\frac{1001}{1000}$. **6.** $\frac{189}{675}$. **7.** $\frac{711}{225}$. **8.** $10\frac{3}{8}$, $14\frac{25}{256}$.

9. $\frac{53}{400}$, $\frac{69}{800}$. **10.** $\frac{1}{2}+\frac{3}{4}+\frac{5}{8}+\frac{11}{16}$. **11.** $3\frac{1}{2}+2\frac{4}{5}-1\frac{7}{8}$.

12. $3\frac{1}{2}+4\frac{2}{5}+3\frac{5}{16}$. **13.** $3\frac{11}{16}+8\frac{3}{32}+6\frac{4}{5}$. **14.** $4\frac{3}{4}+3\frac{24}{25}+5\frac{7}{8}+\frac{2}{5}$.

69. As other examples of Art. 68 we may consider the following:

Ex. 1. $\frac{2}{3}=2\div3$.

$$3\overline{)2\cdot000}$$
$$\cdot6666\ldots$$

On performing the division the remainder is always 2, and the quotient consists of an unending row of 6's.

Thus $\frac{2}{3}=\cdot6666\ldots$, the dots denoting that the figures never stop. The notation $\cdot\dot{6}$ is used to denote this unending row of 6's.

Ex. 2. $\frac{1}{7}=1\div7$.

$$7\overline{)1\cdot000}$$
$$\cdot142857142857142857\ldots$$

On performing the division we obtain as the first six places the digits 142857, and at this stage the remainder is 1, that is, *the remainder is the same number as that with which we started.* The digits 142857 therefore repeat again, and once more the remainder is 1, and the repetition of the same figures takes place again, and so on. The process therefore never comes to an end.

Hence $\frac{1}{7}=\cdot142857\ 142857\ 142857\ldots$, the dots denoting that the figures never stop. The notation $\cdot\dot{1}42857\dot{}$ is used to denote this unending row of figures, so that $\frac{1}{7}=\cdot\dot{1}42857\dot{}$.

Ex. 3. $\frac{17}{132}=17\div132=12878787\ldots=12\dot{8}\dot{7}$.

70. Decimal fractions, such as those of the last article, where some or all of the figures are repeated continually without any end, are called **Recurring Decimals,** or Recurrers, since the figures continually recur.

The part that is repeated is called the **Period.**

A Pure Recurring Decimal is one in which the period begins immediately after the decimal point. Examples are

$$\cdot4444\ldots,\ \cdot123123123\ldots,\ \cdot706706706\ldots.$$

A Mixed Recurring Decimal is one in which the period does not begin immediately after the decimal point, but in which the decimal point is followed by some figures which do not recur. Examples are

·11363636... (in which the 36 recurs but not the 11),

·0134134134... (in which only the 134 recurs but not the 0).

These are denoted by $\cdot 11\dot{3}\dot{6}$ and $\cdot 0\dot{1}3\dot{4}$.

71. It is easy to predict beforehand whether a vulgar fraction, expressed in its lowest terms, can be converted into a terminating decimal or not. For a terminating decimal is really a vulgar fraction with 10, or some power of 10, such as 100, 1000, ... as its denominator.

A vulgar fraction may therefore, as in Art. 68, be converted into a decimal by multiplying both its numerator and its denominator by such a number that the latter becomes 10, or a power of 10. Now the only numbers that can be multiplied so as to become powers of 10 are 2 and 5 and numbers whose only factors are 2 and 5.

Hence if the denominator of a vulgar fraction, which is expressed in its lowest terms, contains any other factors besides 2 or 5, it cannot be converted into a terminating decimal.

EXAMPLES XXXIX.

Convert the following vulgar fractions into decimals:

1. $\frac{2}{3}, \frac{4}{9}, \frac{5}{9}$. **2.** $\frac{23}{75}$. **3.** $\frac{29}{45}$. **4.** $\frac{3}{11}, \frac{5}{11}$. **5.** $\frac{4}{33}, \frac{8}{55}$. **6.** $\frac{19}{33}$.

7. $\frac{1}{99}$. **8.** $\frac{17}{27}$. **9.** $\frac{134}{375}$. **10.** $\frac{3}{7}, \frac{11}{7}$. **11.** $\frac{5}{13}, \frac{19}{13}$. **12.** $\frac{3}{130}$.

13. $\frac{1}{7}, \frac{6}{35}$. **14.** $\frac{29}{84}$. **15.** $\frac{43}{105}$. **16.** $\frac{1}{81}$. **17.** $\frac{23}{275}$.

PURE RECURRING DECIMALS.

72. *To convert a recurring decimal fraction into a vulgar fraction.*

Consider the following:

Ex. 1. *Express as a vulgar fraction the recurring decimal* $\cdot\dot{7}$.

We have $\cdot\dot{7} = \cdot 7777\ldots$.

Multiplying both sides by 10, we have

$$10 \text{ times } \cdot\dot{7} = 7\cdot 7777. \quad \ldots\ldots (1)$$

But $$\cdot\dot{7} = \cdot 7777. \quad \ldots\ldots (2)$$

Now the decimal part on the right hand side of (1) and (2) is the same. Subtracting the line (2) from the line (1), we therefore have

$$9 \text{ times } \cdot\dot{7} = 7,$$

that is, $$9 \times \cdot\dot{7} = 7, \text{ that is, } \cdot\dot{7} = \tfrac{7}{9}.$$

Ex. 2. *Express as a vulgar fraction the recurring decimal* $\cdot\dot{5}8\dot{9}$.

We have $\cdot\dot{5}8\dot{9} = 589589589\ldots.$

Multiply both sides by 1000, and we have

1000 times $\cdot\dot{5}8\dot{9} = 589\cdot589589\ldots\ldots$ (1)

But $\cdot\dot{5}8\dot{9} = \quad\cdot589589\ldots\ldots$ (2)

Subtracting (2) from (1) we have, since the decimal part on the right hand side is the same in each line,

999 times $\cdot\dot{5}8\dot{9} = 589$, that is, $999 \times \cdot\dot{5}8\dot{9} = 589$.

Hence $\cdot\dot{5}8\dot{9} = \frac{589}{999}$.

We thus see that in the above two, and all such, cases the following rule holds: *A pure recurring decimal is equal to a vulgar fraction, whose numerator is a whole number consisting of the figures that recur, and whose denominator is a whole number consisting of as many nines as there are digits in the recurring decimal.*

Thus we have $\cdot\dot{0}3\dot{7} = \frac{37}{999} = \frac{1}{27}$, $\cdot\dot{0}09\dot{9} = \frac{99}{9999} = \frac{1}{101}$.

EXAMPLES XL.

Convert into vulgar fractions, expressed in their lowest terms, the following pure recurring decimals:

1. $\cdot\dot{3}$, $\cdot\dot{5}$, $\cdot\dot{7}$, $\cdot\dot{8}$. **2.** $\cdot\dot{1}$, $\cdot\dot{0}\dot{1}$, $\cdot\dot{1}\dot{8}$, $\cdot\dot{7}\dot{2}$. **3.** $\cdot\dot{5}\dot{5}$, $\cdot\dot{8}\dot{4}$, $\cdot\dot{1}0\dot{1}$.

4. $\cdot\dot{7}\dot{5}$, $\cdot\dot{0}3\dot{7}$, $\cdot\dot{3}1\dot{5}$. **5.** $\cdot\dot{2}9\dot{7}$, $\cdot\dot{0}7\dot{4}$, $\cdot\dot{6}1\dot{5}$. **6.** $\cdot\dot{5}\dot{4}$, $\cdot\dot{1}3\dot{5}$, $\cdot\dot{1}4\dot{8}$.

7. $\cdot\dot{0}9$, $\cdot\dot{0}09\dot{9}$, $\cdot9\dot{9}\dot{9}$. **8.** $\cdot\dot{0}7\dot{4}$, $\cdot\dot{0}00037\dot{}$. **9.** $\cdot\dot{3}07692\dot{}$.

10. $\cdot\dot{4}28571\dot{}$. **11.** $\cdot\dot{1}42857\dot{}$. **12.** $\cdot\dot{2}85714\dot{}$.

13. $\cdot\dot{8}57142\dot{}$. **14.** $\cdot\dot{5}71428\dot{}$. **15.** $\cdot\dot{7}14285\dot{}$. **16.** $\cdot\dot{1}53846\dot{}$.

MIXED RECURRING DECIMALS.

73. Consider the following:

Express as a vulgar fraction the recurring decimal $\cdot5\dot{7}\dot{8}$.

We have $\cdot5\dot{7}\dot{8} = \cdot5787878\ldots\ldots$ (1)

Multiplying both sides of this line by 1000, we have

1000 times $\cdot5\dot{7}\dot{8} = 578\cdot7878\ldots\ldots$ (2)

Again, multiplying both sides of (1) by 10, we have

10 times $\cdot5\dot{7}\dot{8} = \quad5\cdot7878\ldots\ldots$ (3)

Subtracting line (3) from line (2), we have

990 times $\cdot5\dot{7}\dot{8} = 578\cdot7878\ldots - 5\cdot7878\ldots$,

that is, $990 \times \cdot5\dot{7}\dot{8} = 578 - 5$.

Hence, $\cdot5\dot{7}\dot{8} = \frac{578-5}{990}$.

The value of this recurring decimal might therefore have been written down by the following rule:

The numerator is the difference between a whole number, made up of the non-recurring and the recurring figures to the end of the first

period, and a whole number consisting of the non-recurring figures only. The denominator is a whole number consisting of as many nines as there are recurring figures, followed by as many cyphers as there are non-recurring figures.

Ex. 1. $\cdot 285\dot{7}\dot{4}\dot{2} = \frac{285742-28}{999900} = \frac{285714}{999900} = \frac{25974}{90900} = \frac{2886}{10100} = \frac{1443}{5050}$.

Here there are four figures that recur and two that do not; the denominator therefore consists of four 9's followed by two 0's.

Ex. 2. $\cdot 007\dot{6}4\dot{8} = \frac{7648-7}{999000} = \frac{7641}{999000} = \frac{849}{111000} = \frac{283}{37000}$.

Ex. 3. $\cdot 18\dot{9}1\dot{8} = \frac{18918-18}{99900} = \frac{18900}{99900} = \frac{189}{999} = \frac{21}{111} = \frac{7}{37}$.

EXAMPLES XLI.

Find the value of the following expressed as vulgar fractions in their lowest terms:

1. $\cdot 0\dot{9}$, $\cdot 6\dot{3}$. **2.** $\cdot 0\dot{3}\dot{6}$, $\cdot 01\dot{3}\dot{5}$. **3.** $\cdot 07\dot{2}$, $\cdot 12\dot{3}$. **4.** $\cdot 24\dot{9}$, $\cdot 0099\dot{9}$.
5. $\cdot 50\dot{7}\dot{0}$, $\cdot 1249\dot{9}$. **6.** $1\cdot 23\dot{4}\dot{5}$, $\cdot 123\dot{4}\dot{1}$. **7.** $\cdot 708\dot{3}$. **8.** $1\cdot 48\dot{3}$. **9.** $\cdot 236\dot{1}$.
10. $\cdot 6022\dot{7}$. **11.** $\cdot 2\dot{1}42857\dot{1}$. **12.** $\cdot 9\dot{2}8571\dot{4}$.

74. Addition and Subtraction of Recurring Fractions may be performed as in Art. 59. The result is, in general, a never-ending decimal.

Ex. 1. *Add* $3\cdot 9\dot{4}87\dot{1}$, $9\cdot 6539\dot{1}$, *and* $11\cdot 4896\dot{1}$.

The answer is clearly $25\cdot 0924\dot{1}$. For the last figure is too little, since if we had continued the decimals to the next few decimal places we should have had a 2 to carry to the twelfth place.

$$\begin{array}{r} 3\cdot 948794879487\ldots \\ 9\cdot 653965396539\ldots \\ 11\cdot 489648964896\ldots \\ \hline 25\cdot 092409240922\ldots \end{array}$$

This result is also easily obtained by converting the decimals to vulgar fractions.

The sum $= 3\frac{9487}{9999} + 9\frac{6539}{9999} + 11\frac{4896}{9999} = 23\frac{20922}{9999} = 25\frac{924}{9999} = 25\cdot 0\dot{9}2\dot{4}$.

Ex. 2. *Subtract* $6\cdot 384\dot{3}\dot{7}$ *from* $8\cdot 74\dot{8}9\dot{2}$.

Here both fractions are repeating after the third place of decimals, and as the L.C.M. of 2 and 3 is 6, the figures to the right of the second dotted line are the same as those to the right of the first dotted line. It is thus only necessary to continue the subtraction for two or three places beyond this second dotted line.

$$\begin{array}{r} 8\cdot 748|928928|92\ldots \\ 6\cdot 384|373737|37\ldots \\ \hline 2\cdot 364|555191|55\ldots \end{array}$$

$Ans. = 2\cdot 364555191\dot{1}$.

EXAMPLES XLII.

Add together

1. $\cdot 5\dot{5}$, $\cdot 7\dot{7}$, $\cdot 9\dot{9}$. **2.** $\cdot 8\dot{8}$, $\cdot 4\dot{4}$, $\cdot 3\dot{3}$. **3.** $\cdot 7\dot{7}$, $\cdot 3\dot{3}$, $\cdot 2\dot{3}\dot{3}$.
4. $\cdot 5\dot{5}$, $\cdot 9\dot{9}$, $\cdot 3\dot{7}\dot{7}$. **5.** $\cdot 3\dot{1}\dot{1}$, $\cdot 4\dot{4}\dot{5}$, $\cdot 6\dot{7}\dot{7}$, $\cdot 8\dot{3}\dot{3}$. **6.** $\cdot 4\dot{4}\dot{5}$, $\cdot 8\dot{3}0\dot{7}\dot{7}$, $\cdot 7\dot{9}\dot{9}$.
7. $3\cdot 3\dot{3}$, $4\cdot 42\dot{9}\dot{9}$. **8.** $\cdot 32\dot{6}\dot{1}$, $\cdot 3\dot{9}9\dot{0}$.

Subtract

9. $\cdot 45$ from $\cdot 4\dot{5}$. **10.** $\cdot 3\dot{4}\dot{5}$ from $\cdot 5\dot{4}\dot{3}$. **11.** $2\cdot 29\dot{7}$ from $2\cdot 30\dot{2}$.
12. $\cdot 00\dot{8}5\dot{4}$ from 1. **13.** $286\cdot 3\dot{8}\dot{7}$ from 543. **14.** $4\cdot 0\dot{1}\dot{5}$ from $6\cdot 3\dot{6}$.
15. $\cdot 6$ from $3\cdot 37$.

MULTIPLICATION AND DIVISION OF RECURRING DECIMALS.

75. In many cases it is better to convert recurring decimals into vulgar fractions than to multiply them together, even if the contracted methods of a later chapter are used.

Ex. 1. $\dot{2}\dot{7} \times 9\cdot1\dot{6} = \frac{27}{99} \times \frac{916-91}{900} = \frac{\overset{3}{27}}{\underset{11}{99}} \times \frac{\overset{11}{825}}{\underset{12}{900}} = \frac{3}{12} = \frac{1}{4} = \cdot25.$

Ex. 2. $85\cdot\dot{8}\dot{5} \div 6\cdot\dot{8}0\dot{6} = 85\frac{85}{99} \div 6\frac{806}{999} = \frac{8500}{99} \div \frac{6800}{999}$

$$= \frac{\overset{5}{\cancel{8}\cancel{5}\cancel{0}\cancel{0}}}{\underset{11}{\cancel{9}\cancel{9}}} \times \frac{\overset{111}{\cancel{8}\cancel{5}\cancel{0}\cancel{0}}}{\underset{4}{\cancel{6}\cancel{8}\cancel{0}\cancel{0}}} = \frac{555}{44} = 12\cdot61\dot{3}\dot{6} \text{ (Art. 69).}$$

EXAMPLES XLIII.

Perform the following multiplications and divisions:

1. $\cdot6\dot{3} \times 3.$ **2.** $\cdot\dot{7}\dot{8} \times 9.$ **3.** $\cdot5\dot{3} \times 11.$

4. $\cdot1\dot{3}2\dot{4} \times 111.$ **5.** $\cdot03\dot{8} \times 13\cdot68 \div 1\cdot33.$ **6.** $\cdot5\dot{4} \times 0\dot{3}\dot{7}.$

7. $\cdot\dot{7}\dot{2} \times \cdot0\dot{9}.$ **8.** $\cdot4\dot{5} \times \cdot6\dot{6}.$ **9.** $1\cdot1\dot{6} \times \cdot4285\dot{7}\dot{1}.$

10. $\cdot\dot{7}\dot{2} \times \cdot458\dot{3}.$ **11.** $\cdot2\dot{3} \div \cdot287\dot{5}.$ **12.** $31\cdot\dot{6} \div \cdot04\dot{2}.$

13. $\cdot2\dot{1} \div 21\dot{1}.$ **14.** $\cdot296 \div \cdot513\dot{8}.$ **15.** $4\cdot76\dot{1} \times \cdot01 \div 1\cdot7\dot{3}\dot{1}.$

Find the value expressed as vulgar fractions in their lowest terms of the following:

16. $\dfrac{2\cdot8 \times 11\cdot3\dot{6}}{5\cdot68\dot{1}}$ **17.** $\dfrac{53\cdot8\dot{1}\dot{8}}{32\cdot5\dot{9}} \div 4\frac{14}{163}.$ **18.** $\dfrac{\cdot392}{\cdot1225} - \dfrac{\cdot3\dot{0}\dot{9} - \cdot0\dot{1}\dot{8}}{\cdot\dot{0}\dot{9}}.$

19. $\dfrac{\cdot\dot{9} + \cdot9}{1\cdot9}.$ **20.** $\dfrac{\cdot\dot{7}14285\dot{7} - \cdot62\dot{1}}{\cdot\dot{7}14285\dot{7} + \cdot62\dot{1}}.$

VALUE OF A GIVEN DECIMAL OF A CONCRETE QUANTITY.

76. In Art. 50 we have shown how to find the value of a given vulgar fraction of a concrete quantity. We could convert the given decimal into a vulgar fraction and proceed as in that article. We may, however, proceed as in the following examples, and avoid the reduction to a vulgar fraction.

Ex. 1. *Find the value of* 3·45625 *of* £1.

Multiply the decimal part of the pounds by 20 to reduce it to shillings. We thus have 9·125 shillings.

Multiply the decimal part of these shillings by 12 to bring it to pence. We thus have 1·5 pence.

Multiply the decimal part of these pence by 4 to bring it to farthings. We thus have 2 farthings.

$3\cdot45625 \text{ of } £1 = £3\cdot45625$

$\times 20$

$9\cdot12500$ shillings.

$\times 12$

$1\cdot500$ pence

$\times 4$

$2\cdot0$ farthings.

Hence $£3\cdot45625 = £3.\ 9s.\ 1\frac{1}{2}d.$

Ex. 2. *Reduce* ·0625 *of* £1. 13*s.* 6*d.* to £. *s. d.*

£1. 13*s.* 6*d.* = 33*s.* 6*d.* = 402 pence.

Also ·0625 of 402 pence

= ·0625 × 402 pence

= 25·125 pence

$= 25\frac{1}{10}\frac{2}{00}\frac{5}{0}d.$

$= 25\frac{1}{8}d. = 2s.\ 1\frac{1}{8}d.$

·0625
402
1250
25·00
25·1250

Ex. 3. *Find the value of* ·364 *of* 1 *cwt.* 16 *lbs.* + 1·28 *of* 11 *lbs.* 5 *oz.* − 3·76 *oz. in ounces and decimals of an ounce.*

Now ·364 of 1 cwt. 16 lbs. = ·364 × 128 lbs. = 46·592 lbs.

= 46·592 × 16 oz. = 745·472 oz.

Also 1·28 of 11 lbs. 5 oz. = 1·28 × 181 oz. = 231·68 oz.

∴ *Ans.* = (745·472 + 231·68 − 3·76) oz. = 973·392 oz.

EXAMPLES XLIV.

Find the value in pounds, shillings, and pence, and decimals of a penny of

1. ·375 of £1. **2.** ·525 of £1. **3.** ·325 of £10.

4. ·2387 of £1. **5.** ·4639 of £1. **6.** ·3842 of £3.

7. ·159375 of £1. **8.** ·775625 of £5. **9.** ·5376 of 5*s.*

10. 1·325 of £2. 10*s.* **11.** 3·827 of £3. 15*s.* **12.** 5·362 of 7*s.* 6*d.*

13. 2·613 of £2. 8*s.* 9*d.* **14.** 1·0375 of £18. 17*s.* 10*d.*

15. ·01625 of £204. 3*s.* 4*d.* **16.** ·3 of £1 + ·9 of 10*s.* + ·675 of 1*s.*

17. ·8752 of a guinea − ·52521 of £1.

18. $\frac{5}{8}$ of £1 + $\frac{3}{7}$ of a guinea − $2\frac{3}{16}$*s.*

19. ·45 of £1 − (·375 of 13*s.* 4*d.* + ·07 of £2. 10*s.*).

20. ·675 of a guinea + $1\frac{1}{16}$ of £1 + $\frac{3}{4}$ of 2*s.* 6*d.*

21. 27·135 of 1*s.* $10\frac{1}{2}d.$ + ·7 of 2*s.* 6*d.* + ·125 of a guinea.

22. 11·75 of 8*s.* $0\frac{3}{4}d.$ + 7·125 of £3·675 + $1\frac{1}{16}$ of 3*d.*

23. ·625 of a guinea + ·5625 of a crown − ·75 of 7*s.* 6*d.* + ·625 of 5*d.*

24. 5·35 of 15*s.* + 3·45 of 17*s.* 6*d.* − 2·55 of 12*s.* 6*d.*

25. $\cdot\dot{3}$ of 6*s.* 6*d.* **26.** $\cdot\dot{5}$ of 14*s.* 3*d.* **27.** $1\cdot1\dot{6}$ of 2*s.* 6*d.*

28. $11\cdot8\dot{3}$ of 8*s.* **29.** $2\cdot44\dot{9}$ of a crown. **30.** $3\cdot5\dot{4}\dot{9}$ of 9*s.* 3*d.*

31. ·3375 of £1 + $\cdot21\dot{6}$ of £15 + 1·025 of 6*s.* 8*d.*

32. $\cdot0\dot{3}$ of £2 + 2·375 of a florin + ·25 of 4*s.* 4*d.*

33. $\cdot3\dot{7}\dot{8}$ of 2*s.* 9*d.* − $\cdot\dot{3}7\dot{8}$ of 1*s.* $6\frac{1}{2}d.$

34. Which is the greater, ·0231 of a guinea or ·19 of half-a-crown, and by how much?

Find the value of

35. ·34 ton in lbs. **36.** ·67 mile in feet. **37.** ·95 day in minutes.

38. ·37 ac. in sq. yards. **39.** ·345 lb. Troy in grains.

40. 6·938 cwts. in ounces. **41.** ·8347 mile in yards.

42. ·4875 ac. in sq. yards. **43.** 29·5305 days in seconds.

44. ·109375 ton in lbs. **45.** ·9043 of 3 cwt. 3 qrs. 9 lbs. in ounces.

46. ·390625 of 6 ac. 1 r. 24 sq. po. in sq. yards.

47. 3·0175 of 9 bush. 3 pecks in pints.

48. ·326 of a day + ·018 of an hour in minutes and decimals of a minute.

49. ·0185 of a week + ·634 of an hour in minutes.

50. 18·42 quarts − ·706 of a gallon in pints and decimals of a pint.

51. ·4 of a mile + ·424 of a pole + ·4246 of a yard in feet.

52. 91·6634375 sq. yds. − (·00125 ac. + ·0625 r. + ·00375 sq. po.) in sq. yds.

53. $\cdot 11\dot{3}\dot{6}$ of a mile in yards. **54.** $\cdot 00\dot{3}\dot{7}\dot{8}$ of a mile in yards.

55. $\cdot 142857\dot{}$ of 1 fur. 18 po. 3 yds. in yards.

77. *To find what decimal fraction one given quantity is of a second given quantity of the same kind.*

As in Art. 51 we may reduce both quantities to the same denomination, and then divide one by the other.

Ex. 1. *What decimal fraction is* £3. 14*s.* 8¼*d.* *of* £5?

£3. 14*s.* 8¼*d.* = 74*s.* 8¼*d.* = 896¼*d.* = 3585 farthings.

Also £5 = 100*s.* = 1200*d.* = 4800 farthings.

Therefore required ans. $= \frac{3585}{4800} = \frac{717}{960} = \frac{89{\cdot}625}{120} = \frac{7{\cdot}46875}{10} = {\cdot}746875.$

The work may be also shown as follows:

We set down the pence and farthings expressed as a decimal of a penny and divide by 12 to reduce them to a decimal of a shilling. We then add to the result the 14*s.*, and divide by 20 to give the decimal of £1. To this result we add the £3, and divide by 5 to give the required answer.

12) 8·25*d.* = 8¼*d.*
20) **14**·6875*s.*
5) **3**·734375£
·746875

Ex. 2. *What decimal is* 3 *cwt.* 2 *qrs.* 3 *lbs.* 8 *oz. of* 1 *ton?*

28) 3·5 lbs. = 3 lbs. 8 ozs.
4) **2**·125 qrs.
20) **3**·53125 cwt.
·1765625 ton.

EXAMPLES XLV.

In the following questions express the first quantity as a decimal fraction of the second:

1. 3*d.*; 1*s.* **2.** 4½*d.*; 1*s.* **3.** 7½*d.*; 1*s.* **4.** 7*s.* 6*d.*; £1.

5. 5*s.* 9*d.*; £1. **6.** 11*s.* 6*d.*; £1. **7.** ¾*d.*; £1. **8.** 2¼*d.*; 7*s.* 6*d.*

9. 5*s.* 6*d.*; £4. **10.** 7½*d.*; £1. **11.** £1. 0*s.* 2¼*d.*; £1.

12. 3*s.* 9*d.*; one crown. **13.** 2½ guineas; £5. **14.** £5. 17*s.* 9½*d.*; 4*s.* 7*d.*

15. £5. 7*s.* 6¾*d.*; 1*s.* **16.** £3. 12*s.* 2¼*d.*; eleven guineas.

In the following questions express the first quantity as a decimal fraction of the second :

17. $\frac{3}{20}$ of 4 guins. ; £7. **18.** $(\frac{1}{1\frac{1}{2}}+\frac{1}{1\frac{1}{5}}-\frac{1}{2\frac{1}{7}}$ of $2\frac{1}{40}+\frac{1}{2\frac{1}{4}})$ of a guinea; £2. 9*s.*

19. £15·125 + 17·3025*s.* + 9·75*d.* ; £25.

20. $6\frac{1}{2}$ guineas – £3·525 ; a crown. **21.** $3\frac{1}{4}$ guineas – £1·7625 ; 5*s.*

22. 4*d.* ; 1*s.* **23.** 3*s.* 4*d.* ; £1. **24.** $2\frac{1}{3}$*s.* ; 10*s.* 6*d.*

25. 4*s.* $3\frac{1}{2}$*d.* ; £1. 5*s.* 9*d.* **26.** $3\frac{1}{2}$ shillings ; $\frac{5}{11}$ of a guinea.

27. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{7}{11}$ of £16. 10*s.* ; £36.

28. $\frac{3}{5}$ of 17*s.* 6*d.* + $\frac{5}{2\frac{1}{7}}$ of 21*s.* + $\frac{7}{3\frac{1}{2}}$ of 10*s.* ; £4. 19*s.* $5\frac{1}{2}$*d.*

29. $\cdot 8\dot{3}$ of 8*s.* + $\cdot 0\dot{5}$ of 2 guineas + 5*s.* ; half a guinea.

30. $\frac{1\frac{7}{20}}{2}$ of 2*s.* 6*d.* + $\frac{5}{7}$ of a guinea + $1\frac{\frac{7}{36}}{6}$*s.* + $\frac{1}{6}$*d.* ; £1.

31. Express the difference between £15·125 and 30·25*s.* as a decimal of the former sum.

32. Find the value of $\cdot 3\dot{8}\dot{4}$ of £1. 7*s.* 9*d.* + $\cdot 3\dot{8}\dot{4}$ of £1. 7*s.* 6*d.* + $\cdot 384$ of £1. 17*s.* 6*d.*, and express the result as the decimal of £10.

In the following questions express the first quantity as a decimal fraction of the second :

33. $\frac{1}{4}$ fur. ; 1 mile. **34.** $3\frac{1}{2}$ qrs. ; 1 ton.

35. $5\frac{1}{2}$ yds. ; $2\frac{1}{2}$ miles. **36.** 15 lbs. 12 oz. ; 1 ton.

37. 2 qrs. 21 lbs. ; 1 ton. **38.** 2 cwt. 3 qrs. 7 lbs. ; $1\frac{1}{2}$ tons.

39. 1 r. 14 sq. po. ; 1 acre. **40.** 4 ac. 2 r. 16 sq. po. ; 1 sq. mile.

41. 2 cwts. 1 qr. 7 lbs. ; 1 ton. **42.** 16 cwts. 3 qrs. 10 lbs. 8 oz. ; 1 ton.

43. 3 pks. 2 qts. $\frac{4}{5}$ pt. ; 11 bush. **44.** 2 dwt. 14 grs. ; 1 lb. Troy.

45. 11 miles 3 fur. 18 po. 2 yds. 2 ft. 3 in. ; 1 mile.

46. 1 day 4 hrs. 31 mins. $52\frac{1}{2}$ secs. ; 3 days 4 hrs. 5 mins.

47. 3 tons 4 cwt. 1 qr. $2\frac{4}{5}$ lbs. ; 5 tons 7 cwts. 14 lbs.

48. 1·3 of 9 lbs. – ·046 of 2 cwt. 1 qr. ; 5 oz.

49. 5 hrs. 20 mins. ; 1 day. **50.** $\frac{1\frac{7}{19}}{1}$ of 1 hour 54 mins. ; $8\frac{1}{2}$ days.

51. An oz. Av. ; an oz. Troy. **52.** 3 qrs. 18 lbs. $10\frac{2}{3}$ oz. ; 1 cwt.

53. Which is the greater, $\cdot 5\dot{7}$ of a league or $\cdot 7\dot{2}$ of a mile? Express the difference as the decimal of a furlong.

54. Add together ·224 of 1 lb., ·47 of 1 cwt., and ·39 of 1 ton, and express the result as the decimal of a ton.

MISCELLANEOUS EXAMPLES XLVI.

Decimal Fractions.

1. ·378125 of an acre is let in allotments so as to produce a rental of £·75625 ; how much is the rent per sq. pole?

2. Find the sum of money such that $5\frac{2}{13}$ of it shall be equal to 7·2 of £5. 3*s.* $9\frac{1}{2}$*d.*

3. If ·511 of $\frac{1\frac{6}{7}}{3}$ of 1 qr. 3 lbs. 4 oz. of tin is worth ·65625 of a shilling, what is the value of a ton?

4. A gentleman left £4000 to be divided between his wife, son, and daughter. The wife received ·325 of it, and the son ·125 of what was then left. The daughter received the remainder. What was the daughter's share worth?

5. A person sold ·15 of an estate to one person and then $\frac{5}{17}$ths of what is left to another; what part of his estate did he still retain?

6. How many ounces of silver are there in a piece of silver plate costing £59. 10*s.*, the price of silver being ·255 of 10*s.* per ounce, and the cost of manufacture one-third of the value of the silver?

7. A hall is 80·73 ft. long and 45·63 ft. wide; find the length of the longest stick which will measure both its length and its breadth.

8. If a metre be 3·2809 feet and the length of a line drawn on the earth from the North Pole to the Equator be 10,000,000 metres, find the length of the circumference of the earth to the nearest mile.

9. *A* owes *B* ·6 of what *B* owes *C*, and *B* gives *A* five shillings to balance the amount between them all. How much does *B* owe *C*?

10. An army lost ·13425 of its number in a battle and, in addition, 2778 men were taken prisoners; if the number left be $\frac{3}{4}$ of the original strength, find the original strength.

11. Find the value of ·$\dot{1}42857\dot{1}$ of £5 + ·$\dot{4}2857\dot{1}$ of £10 + ·$\dot{2}85714\dot{2}$ of £15 + ·$\dot{5}7142\dot{8}$ of £20 + ·$\dot{5}71428\dot{5}$ of £25 + $714285\frac{5}{7}$ of £30.

12. If an ounce of standard gold be worth £3·94375, find the value of 3·1$\dot{6}$ ounces.

13. Find the value of ·01$\dot{4}8\dot{1}$ of a ton at 4*s.* 6*d.* per lb.

14. Find the value of 1·68$\dot{3}$ of a pound of gold at £4·$\dot{0}09\dot{9}$ per oz.

15. *A* is 2·$\dot{6}$ times as old as *B*, *B* is ·9 times as old as *C*, and *C* is 20 years of age; how old is *A*?

16. *A* has shares in an estate to the amount of ·25 of it and of ·$\dot{3}\dot{6}$ of it. *B* has shares in the same estate to the amount of ·25$\frac{7}{9}$ of it. Find the difference in value between the shares of *A* and *B* if ·$\dot{3}\dot{6}$ of the estate is worth £5000.

17. A person possesses ·$\dot{3}1\dot{5}$ of an estate; he sells ·63 of his share to one man and ·1$\dot{6}$ of what he had left to a second man: what share of the estate has he now left?

18. A man spent ·25 of his money, then ·4 of what was left, and afterwards ·$\dot{3}$ of the remainder, and found he had £752 left; how much had he at first?

19. The quotient being $2\frac{1}{12}$, and the divisor ·37$\dot{1}\dot{5}$, find the dividend and express it as a vulgar fraction in its lowest terms.

20. Three girls begin skipping at exactly the same moment. They jump at intervals of ·$\dot{6}$, ·75, and ·8 second respectively. How long will it be before they again jump simultaneously?

[If it is thought desirable, Chapter XX. may be read here.]

CHAPTER V.

THE METRIC SYSTEM. DECIMAL COINAGE.

78. In the present chapter we shall consider a system of Weights and Measures called the Metric System.

The fundamental principle is that in it each denomination is ten times greater than the next lower denomination, except in the case of areas. The arithmetic of questions given in the Metric System is therefore much simpler than in the British System of Weights and Measures.

For example, in our measurement of length 12 inches make 1 foot, 3 feet make 1 yard, $5\frac{1}{2}$ yards 1 pole, 40 poles 1 furlong, and 8 furlongs make 1 mile. Here in reducing inches to miles the numbers by which we successively divide are 12, 3, $5\frac{1}{2}$, 40, and 8.

In the Metric System the corresponding divisor is always 10, and the process of such reduction from a lower denomination to a higher consists simply in altering the position of the decimal point.

The adoption of the Metric System and the corresponding Decimal Coinage of money would, without doubt, cause a considerable temporary inconvenience to business men, but the ultimate gain would be immense.

79. Length. The fundamental unit of length in the Metric System is the **Metre.** [The word metre is derived from the Greek word *Metron*, which means Measure.]

The metre is defined to be the length of a certain platinum bar kept in the Archives at Paris. Its length is 39·37079... inches, that is, 3·28089... feet.

The metre (and also the corresponding units of area, volume, weight, etc.) is sub-divided into tenths, hundredths, thousandths, etc.

To signify the sub-divisions, the Latin words **Deci, Centi, Milli** are used as prefixes. Thus

1 decimetre $= \frac{1}{10}$th of a metre,
1 centimetre $= \frac{1}{100}$th ,,
1 millimetre $= \frac{1}{1000}$th ,,

To denote multiples of a metre, the Greek words **Deca, Hecto, Kilo, Myria,** etc., are used as prefixes. Thus

1 decametre $= 10$ metres,
1 hectometre $= 100$,,
1 kilometre $= 1000$,,

We thus have the following table: Rough Estimate.

1 millimetre	$= \frac{1}{1000}$th of a metre	= twice the thickness of a post-card.
1 centimetre	$= \frac{1}{100}$th "	= thickness of this book.
1 decimetre	$= \frac{1}{10}$th "	= breadth of a man's hand.
1 metre		= length of a walking-stick.
1 decametre	= 10 metres	= half length of cricket pitch.
1 hectometre	= 100 "	= length of football field.
1 kilometre	= 1000 "	= 10 min. sharp walking.

It may be noted that 2·5 cms. = 1 in. (nearly), and 1 kilometre $= \frac{5}{8}$ mile (nearly).

80. Surface. The unit of area in the Metric System is called the **Are.** It is equal to a square, each of whose sides is 10 metres. The Are therefore = 100 square metres.

$\therefore$ The Centiare $= \frac{1}{100}$ of an Are = 1 sq. metre.
The Hectare = 100 Ares = 10000 sq. metres.

The area of farms, etc., is in France usually expressed in hectares. One hectare = 2·47... acres.

81. Volume, or Cubic, Measure. The unit of volume in the Metric System is the cubic metre and = 1·308... cub. yds. This, especially in measurements of wood, is often called a Stere.

82. Capacity. The unit of capacity in the Metric System is called a **Litre** and = 1·76... pints. The litre is equal in volume to a cube, each of whose sides is equal to a length of 10 centimetres. It therefore contains 1000 cub. cms.

As before, we have 1 millilitre $= \frac{1}{1000}$ of a litre,
1 centilitre $= \frac{1}{100}$ of a litre, etc.

83. Weight. The unit of weight in the Metric System is called a **Gramme.** A gramme is the weight *in vacuo* of a cubic centimetre of pure water at a temperature of 4° C. [This temperature, which is equal to 39·2 F., is chosen because at this temperature water has its greatest density.]

The kilogramme (1000 grammes) is defined, in practice, as the weight of a standard platinum cylinder, which is kept in the Archives at Paris, and the litre as that quantity of water whose weight, at 4° C., is a kilogramme.

A gramme = 15·4323487... grains.
A kilogramme = 15432·3487... " = 2·20462... lbs. Av.

84. **Decimal Coinage.** In America the Standard coin is the **Dollar,** which is worth about 4*s.* 2*d.* Also 1 Dollar = 100 Cents.

In France the Standard coin is the **Franc,** which is worth about 10*d.* Also 1 Franc = 100 Centimes.

EXAMPLES XLVII.

Change the following (1–6) where necessary into metres and find their sum :

1. 2475 cms., 834 mm., 234 metres, 945 decimetres, 8576 mms., 3984 decimetres.

2. 8976 mm., 4521 cms., 176 decimetres, 32794 mm., 80·93 decimetres, 924·5 metres.

3. 456 kilometres, 927 metres, 846·2 hectometres, 3786 decametres, 9534 hectometres.

4. 456·892 decametres, 8·952 metres, 976·783 hectometres, 8·75 kilometres.

5. 3472 mm., 946·2 cms., 27·4 kilometres, 8234 hectometres.

6. 27·69 metres, 2·1 kilometres, 45 mm., 856 cms., 92,465 mm., 3·46 kilometres, 9287 decametres.

7. From the sum of 34,786 mm. and 859 metres take the sum of 4·2 hectometres and ·12 decametres.

8. Find the value of (1 kilometre + 1 decametre + 1 mm.)
− (9 metres + 8 decimetres + 7 cms. + 6 mms.).

9. A man walks at the rate of 4·756 kilometres per hour for 6·347 hours. Find, to the nearest metre, how far he walks.

10. A stick is 15·97 decimetres in length : how many kilometres are there in 3569 of such sticks ?

11. In the preceding question how many such sticks would be required to cover a distance of 9·751075 hectometres ?

12. A train goes 52·347 kilometres an hour. How long would it take to travel 33·763815 kilometres ?

13. A railway ticket costs 254 cents : how many dollars will 358 of them cost ?

14. Multiply 182 dollars 35 cents by 8 and then divide this result by 7. Assuming a dollar is worth 4*s.* 2*d.* find the value of your answer in English money.

15. Find the total cost of 284·5 metres at 25·6 centimes per metre and 479 cms. at 2·4 francs per cm.

16. If the frontage on a road cost 75·42 francs per annum, find how many centimes 59 such frontages would cost.

17. If a road cost 254·2 francs per metre to make, find the cost of making 395·7 metres of it in English money, assuming £1 = 25 francs.

18. A cistern is filled by a pipe which lets in 27 litres per minute : how many kilolitres come in during 3 hrs. 41 mins. ?

19. How many kilogrammes do 378·46 kilolitres of water weigh ?

[In the following examples it may be assumed as a sufficient approximation, unless it is otherwise stated, that 1 metre = 39·371 inches, and that a kilogramme = 2·2046... lbs. Av.]

Find the approximate number of metres, etc., in

20. 1 yard. **21.** 1 chain. **22.** 1 mile. **23.** 7¼ inches.

Express in appropriate British measurement:

24. 10 metres 4 decimetres 9 centimetres.

25. 77·452 metres. **26.** 1 kilometre.

27. the earth's polar circumference which is forty million metres.

28. Prove that 8 kilometres are very nearly 5 miles.

29. The seconds' pendulum being 1·0872 yards long, find its length in metres to four places of decimals.

Find the area of a rectangular surface whose sides are

30. each 485 decimetres.

31. each 523 metres 7 decimetres 5 centimetres.

32. 47 metres 8 decimetres, and 79 metres 3 decimetres.

Express in square metres, etc.

33. 1 sq. ft. **34.** 1000 sq. yds. **35.** ¼ acre. **36.** 1 sq. chain.

37. An estate of 185 hectares 65 ares is divided into 79 fields of equal area; what is the size of each?

38. Find the number of cub. inches in 25 litres.

39. Find in £. s. d. the cost of 47 kilog. 84 grammes of an article at £23·57 per kilo.

Express in kilogrammes, grammes, etc.,

40. 1 English ton. **41.** 2 cwt. 3 qrs. 11 lbs. **42.** 17¼ lbs. Troy.

43. Express the atmospheric pressure of 14¾ lbs. per sq. inch in grammes per sq. centimetre.

44. A farmer lays 2 tons of lime on an acre of ground; how many grammes is this per square metre?

45. A cubic foot of water weighs 62·4 lbs., express the weight of a cubic metre of water as a decimal of a ton, to three places of decimals.

Also how many cubic centimetres will weigh 1 lb.?

Given that £1 = 25·22 francs = 4·87 dollars, express

46. £3. 7*s.* 5¼*d.* in francs. **47.** £240. 6*s.* 9*d.* in francs.

48. 10 million francs in £. s. d. **49.** £416. 13*s.* 4*d.* in dollars.

50. £32. 7*s.* 5½*d.* in dollars. **51.** 575 dollars 47 cents in £. s. d.

52. 2760 dollars 25 cents in £. s. d.

53. Find to the nearest farthing the price per lb. in English money of tea which costs in France 4·50 francs per kilogramme.

54. If a litre be ·52 gallon, find to the nearest penny in English money the value of a pint of liquid worth 20 francs the litre.

55. What is the value in English money of a yard of silk worth 7½ francs per metre? [1 franc = 9·38 pence.]

56. What charge in centimes per kilometre would be equivalent to our third-class fare of a penny per mile?

CHAPTER VI.

PRACTICE. INVOICES.

85. Practice is a convenient way of performing Compound Multiplication.

Suppose that we have to find the value of 347 articles at £1. 11*s.* 8*d.* each. We may multiply £1. 11*s.* 8*d.* by 347, or we may proceed thus:

	£.	*s.*	*d.*			
	347.	0.	0 = value of 347 arts. at £1			
10*s.* = $\frac{1}{2}$ of £1, ∴	173.	10.	0 =	,,	,,	10*s.*
1*s.* 8*d.* = $\frac{1}{6}$ of 10*s.*, ∴	28.	18.	4 =	,,	,,	1*s.* 8*d.*
	£549.	8.	4 =	,,	,,	£1. 11*s.* 8*d.*

Fractions like $\frac{1}{2}$ and $\frac{1}{6}$, whose numerators are unity, are called **Aliquot Parts.** The student should always try to choose such fractions, as they make the work easier. It is a useful plan to make each aliquot part just one half the previous one as far as possible.

Thus 5 is an aliquot part of 15, being $\frac{1}{3}$ of 15.

2*s.* 6*d.* ,, ,, £1, ,, $\frac{1}{8}$ of £1.

3*s.* 4*d.* ,, ,, £1, ,, $\frac{1}{6}$ of £1.

6*s.* 8*d.* ,, ,, £1, ,, $\frac{1}{3}$ of £1.

86. We subjoin some further examples.

Ex. 1. *Find the value of* 7643 *articles at* 3*s.* 7$\frac{1}{4}$*d. each.*

	£.	*s.*	*d.*			
	7643.	0.	0 = cost of 7643 arts. at £1 each.			
2*s.* 6*d.* = $\frac{1}{8}$ of £1, ∴	955.	7.	6 =	,,	,,	2*s.* 6*d.* each.
1*s.* = $\frac{1}{20}$ of £1, ∴	382.	3.	0 =	,,	,,	1*s.* ,,
1*d.* = $\frac{1}{12}$ of 1*s.*, ∴	31.	16.	11 =	,,	,,	1*d.* ,,
$\frac{1}{4}$*d.* = $\frac{1}{4}$ of 1*d.*, ∴	7.	19.	2$\frac{3}{4}$ =	,,	,,	$\frac{1}{4}$*d.* ,,
	£1377.	0.	7$\frac{3}{4}$ =	,,	,,	3*s.* 7$\frac{1}{4}$*d.* ,,

In this question the cost of each article is not so much as £1. We therefore, after the first line, draw a thick line to denote that it is not to be added up.

It will be noted that, in the third line, 1*s.* is not an aliquot part of 2*s.* 6*d.* The third line is therefore not obtained from the second line. It is derived by dividing the first line by 20.

This question may also be worked by dividing the 3*s.* 7¼*d.* into the parts 2*s.*, 1*s.*, 6*d.*, 1*d.*, and ¼*d.* The work is a little longer, but a little easier.

Ex. 2. *Find the value of* 3487 *cwt. at* £3. 7*s.* 3*d. per cwt.*

	£.	*s.*	*d.*				
	3487	0	0	= value of 3487 cwt. at £1.			
	3						
	10461	0	0	=	,,	,,	£3
5*s.* = ¼ of £1 ∴	871	15	0	=	,,	,,	5*s.*
2*s.* = 1/10 of £1 ∴	348	14	0	=	,,	,,	2*s.*
3*d.* = ⅛ of 2*s.*, ∴	43	11	9	=	,,	,,	3*d.*
	£11725	0	9	=	,,	,,	£3. 7*s.* 3*d.*

Here it will be noticed that 2*s.* is not an aliquot part of the 5*s.* of the previous line; it is 1/10th of £1. Thus the fifth line is not derived from the line before it, but it is found by dividing the first line by 10.

Ex. 3. *Find the value of* 256¼ *cwt. of metal at* £5. 16*s.* 2¼*d. per cwt.*

	£	*s.*	*d.*				
	256	5	0	= cost of 256¼ cwt. at £1.			
			5				
	1281	5	0	=	,,	,,	£5
10*s.* = ½ of £1, ∴	128	2	6	=	,,	,,	10*s.*
5*s.* = ½ of 10*s.*, ∴	64	1	3	=	,,	,,	5*s.*
1*s.* = ⅕ of 5*s.*, ∴	12	16	3	=	,,	,,	1*s.*
2*d.* = ⅙ of 1*s.*, ∴	2	2	8·5	=	,,	,,	2*d.*
¼*d.* = ⅛ of 2*d.*, ∴		5	4·0625	=	,,	,,	¼*d.*
	£1488	13	0·5625	=	,,	,,	£5. 16*s.* 2¼*d.*

∴ *Ans.* = £1488. 13*s.* 1*d.* nearly.

EXAMPLES XLVIII.

Find, by Practice, the cost of the following numbers of articles at the price given for each:

1. 65 at 10*s.*, and also at 15*s.*

2. 231 at 5*s.*, and also at 7*s.* 6*d.*

3. 294 at 5*s.*, and at 1*s.* 6*d.*

4. 57 at 2*s.* 6*d.*, 3*s.* 4*d.*, and 5*s.* 6*d.*

5. 337 at 6*d.*, and also at 2*s.* 6*d.*

6. 533 at 7*s.* 6*d.*, 11*s.* 6*d.*, and 13*s.* 9*d.*

7. 637 at 12*s.* 6*d.*, 18*s.* 9*d.*, and £1. 2*s.* 6*d.*

8. 700 at 5*s.* 7¾*d.* and 2*s.* 3½*d.*

9. 895 at 14*s.* 8*d.*, 17*s.* 10*d.*, and £3. 4*s.* 5*d.*

10. 94 at £2. 10*s.*, £8. 5*s.*, and £1. 2*s.* 6*d.*

11. 67 at £1. 6*s.* 8*d.* and £5. 12*s.* 6*d.*

12. 487 at £5. 5*s.* and £2. 5*s.* 6*d.*

13. 2375 at £4. 17*s.* 6*d.* and £3. 8*s.* 9*d.*

14. 135 at £5. 13*s.* 4*d.* and £8. 5*s.*

15. 293 at £1. 6*s.* 8*d.* and £2. 1*s.* 6*d.*

16. 1020 at £3. 15*s.* 9*d.* and £5. 3*s.* 7½*d.*

17. 439 at £5. 19*s.* 6*d.* and £3. 9*s.* 10*d.*

18. 4321 at £7. 11*s.* 8*d.*

19. 2347 at 13*s.* 5¾*d.* and 15*s.* 11¾*d.*

20. 3846 at £1. 17*s.* 9*d.* and at £2. 6*s.* 3½*d.*

Find, by Practice, the cost of the following numbers of articles at the price given for each :

21. 4834 at £2. 5*s.* 6½*d.* and at £7. 13*s.* 7¾*d.*

22. 1230 oz. of gold at £3. 17*s.* 10½*d.* per oz.

23. 1379 dozens at £10. 13*s.* 7¾*d.* per dozen.

24. 1236 sheep at £2. 11*s.* 10½*d.* each.

25. 569 cwt. at £2. 16*s.* 9¾*d.* per cwt.

26. 4379 scores of sheep at £33. 17*s.* 10*d.* per score.

27. 2037½ cwt. of soap at £1. 19*s.* 4½*d.* per cwt.

28. 348½ articles at 17*s.* 7½*d.* each.

29. 2843¼ tons of steel at £6. 14*s.* 9*d.* per ton.

30. 2391⅜ articles at 18*s.* 10½*d.* each.

31. 416,472 young trees at £1. 2*s.* 9*d.* per dozen.

32. Find the weight of 269 casks each weighing 6 cwt. 1 qr. 14 lbs.

33. Find the cost of 563,850 cub. ft. of gas at 2*s.* 11*d.* per 1000 cub. ft.

34. Find the total length of wool in 379 skeins, each being of length 63 yds. 2 ft. 3 ins.

35. What is the pay of 843 men for one week at the rate of 18*s.* 9*d.* per week ?

36. In the Royal Engineers the daily pay of a captain, lieutenant, and second lieutenant is respectively 11*s.* 7*d.*, 6*s.* 10*d.*, and 5*s.* 7*d.*; what is the yearly pay of each such officer ?

37. The pay of a colonel in the Foot Guards is £1. 6*s.* 9*d.* per day; how much is this per year ?

87. Examples like the preceding are often said to belong to **Simple Practice.**

When the quantity, whose value is to be found, is not necessarily expressed in terms of the unit whose value is given, the example is one of **Compound Practice.**

Ex. 1. *Find the value of* 16 *cwt.* 2 *qrs.* 14 *lbs. of copper at* £43. 10*s. per ton.*

	£.	s.	d.			
	43	10	0	= value of 1 ton.		
10 cwt. = ½ of 1 ton, ∴	21	15	0	=	,,	10 cwt.
5 cwt. = ½ of 10 cwt., ∴	10	17	6	=	,,	5 cwt.
1 cwt. = ⅕ of 5 cwt., ∴	2	3	6	=	,,	1 cwt.
2 qrs. = ½ of 1 cwt., ∴	1	1	9	=	,,	2 qrs.
14 lbs. = ¼ of 2 qrs., ∴		5	5¼	=	,,	14 lbs.
	£36	3	2¼	=	,,	16 cwt. 2 qrs. 14 lbs.

In this case instead of, as before, writing down the cost of the whole amount at £1 per ton, we set down the cost of 1 ton at the given price and draw a line, since the given weight is less than a ton ; we then take aliquot parts of the ton, etc. Thus, since 10 cwt. is half a ton, the value of 10 cwt. is one half that of 1 ton. The second line of the work is therefore obtained by dividing the first line by 2.

Ex. 2. *Taking a month as 4 weeks of 7 days, find the amount of a man's wages for 4 months 1 week and 6 days at £4. 13s. 2d. per month.*

£. s. d.
4 . 13 . 2 = wages for 1 month
4

	18 . 12 . 8	= ,,	4 months
1 week = $\frac{1}{4}$ of 1 month	1 . 3 . 3·5	= ,,	1 week
4 days = $\frac{1}{7}$ of 1 month	13 . 3·71	= ,,	4 days
2 days = $\frac{1}{2}$ of 4 days	6 . 7·85	= ,,	2 days
	£20 . 15 . 11·06	= ,,	4 months 1 week 6 days.

Ans. = £20. 15*s.* 11*d.* nearly.

In this example it is not easy to split up 6 days into aliquot parts of one week. We therefore take 4 days as $\frac{1}{7}$ of one month. The fifth line of the work is therefore not derived from the fourth line, but is obtained by dividing the first line by 7.

EXAMPLES XLIX.

Neglecting fractions of a farthing, find, by Practice, the cost of

1. 5 cwt. 2 qrs. 21 lbs. at £12 per ton.
2. 13 cwt. 2 qrs. 21 lbs. at £16. 6*s.* 8*d.* per ton.
3. 69 yds. 2 ft. 7½ in. at £1. 3*s.* 6*d.* per yard.
4. 3 oz. 17 dwt. 12 grs. at 18*s.* 6*d.* per oz.
5. 2 tons 7 cwt. 22 lbs. 5 oz. at £32 per ton.
6. 4 tons 3 cwt. 2 qrs. 24 lbs. at £72. 18*s.* 4*d.* per ton.
7. 4 cwt. 3 qrs. 15 lbs. at £21. 11*s.* 8*d.* per ton.
8. 22 cwt. 3 qrs. 21 lbs. at £2. 9*s.* 6*d.* per cwt.
9. 15 gals. 3 qts. 1½ pt. at £1. 6*s.* 9*d.* per gallon.
10. 2 tons 3 qrs. 22 lbs. at £8. 19*s.* 2*d.* per ton.
11. 3 tons 11 cwt. 3 qrs. 10½ lbs. at £8. 6*s.* 8*d.* per ton.
12. 7 tons 17 cwt. 2 qrs. 8 lbs. at £32. 17*s.* 4*d.* per ton.
13. 10 oz. 18 dwt. 21 grs. of gold plate at £46. 14*s.* 6*d.* per Troy lb.
14. 1 lb. 13 dwt. 8 grs. at £3. 17*s.* 6*d.* per oz.
15. 3 lbs. 4 oz. 13 dwt. 19 grs. of gold at £3. 17*s.* 10½*d.* per oz.
16. 27 tons 13 cwt. 3 qrs. 24 lbs. 4 oz. at £9. 6*s.* 8*d.* per ton.
17. 9 tons 4 cwt. 3 qrs. 21 lbs. at £14. 15*s.* 9*d.* per cwt.
18. 5 fur. 27 po. 3 yds. of fencing at £24. 9*s.* 6*d.* per furlong.
19. A railway 173 mi. 3 fur. 15 po. long at £5638 per mile.
20. A road 6 fur. 12 po. 2 yds. long at £15. 17*s.* 6*d.* per furlong.
21. A road 29 mi. 7 fur. 200 yds. long at £34. 16*s.* 8*d.* per mile.
22. 127 yds. 1 ft. 3 in. of silk at 7*s.* 9*d.* per yard.
23. 47 ac. 2 r. 25 sq. po. at £258. 3*s.* 4*d.* per half acre.
24. 347 ac. 2 r. 605 sq. yds. at £45. 10*s.* per acre.
25. 214 ac. 3 r. 29 sq. po. at £125. 7*s.* 6*d.* per acre.

Neglecting fractions of a farthing, find, by Practice, the cost of

26. 237 gals. 1 qt. 1¾ pt. at £1. 3*s.* 9*d.* per gallon.

27. 10 tons 13 cwt. 2 qrs. 20 lbs. at £43·6785 per ton.

28. 18·675 miles of wire at £12. 3*s.* 6*d.* per mile.

29. Find the rent of 7 ac. 3 r. 15 po. at £1. 10*s.* 8*d.* per acre.

30. Find the taxes on 124 ac. 1 r. 20 sq. po. at 5*s.* 6*d.* per acre.

31. Find the cost of warming a building for 11 days 17 hrs. 28 mins. at £4. 10*s.* per day.

32. Find the cost of a fence enclosing a rectangular field 347 yards long and 152 yards wide at 2*s.* 10*d.* a yard.

33. What is the value of a piece of gold plate weighing 13 oz. 11 dwt. 18 grs. at the rate of £4. 4*s.* 9*d.* per oz.?

34. Taking a month as 28 days, what is a man's pay for 7 months 19 days at the rate of £4. 14*s.* 6*d.* per month?

35. Find a servant's wages for 5 months 3 weeks and 6 days at £1. 4*s.* 6*d.* per month, reckoning 7 days to a week and 4 weeks to a month.

36. What is the whole cost of 25 quarters 5¾ bushels of wheat at 22*s.* 6*d.* per quarter, and 17 quarters 6¼ bushels of barley at 25*s.* 7½*d.* per quarter?

37. An estate consists of 89 ac. 3 r. 37 sq. po. of pasture, 73 ac. 2 r. 17 sq. po. of arable land, and 10 ac. 1 r. 12 sq. po. of plantation; what is its annual value at a rent of 27*s.* 6*d.* an acre?

38. A gold wreath weighing 3 lbs. 7 oz. 12 dwt. costs £255; if it is made of gold worth £3. 17*s.* 6*d.* per oz., what is charged for manufacture?

39. A field of 10 ac. 3 r. 8 sq. po. when sown with wheat produced 36 bushels of wheat per acre, which was sold at 25*s.* per quarter. The next time the field was sown with wheat the product was 27 bushels an acre, and the selling price of wheat was 30*s.* a quarter; find the difference in the value of the two crops.

40. What is the value of a piece of gold plate weighing 10 oz. 8 dwt. 12 grs. at the rate of £4. 2*s.* 6*d.* per oz.?

41. What change should I receive out of 100 guineas after buying 3 gold cups, each weighing 4 oz. 2 dwt. 2 grs., if gold is valued at £3. 19*s.* 6*d.* per oz.?

42. What is the rent of a furnished house for 5 months 3 weeks at the rate of 10 guineas per month?

43. Find the value of seven table-spoons and a dozen tea-spoons at 3*s.* 6*d.* per oz., if a table-spoon and a tea-spoon weigh 3 oz. 10 dwt. and 15 dwt. respectively?

INVOICES.

88. When purchases are made at a shop the seller of the goods delivers to the buyer a list of the articles bought, with their prices set down. Such a list is called an Invoice or Bill.

89. Specimen of a Bill or Invoice.

Mrs. Smith, London, *Feb. 7th*, 1905.

Bought of Messrs. Robinson & Co., 237 Strand, W.C.

	£.	s.	d.
27 yds. of flannel at 3*s.* 4*d.* per yd.	4	10	0
32 yds. of calico at 5½*d.* per yd.		14	8
3½ doz. of socks at 8*s.* 4*d.* per doz.	1	9	2
6 pairs of gloves at 3*s.* 6*d.* per pair	1	1	0
4 collars at 1*s.* 5½*d.* each		5	10
	£8	0	8

EXAMPLES L.

Make out invoices for, and find the total cost of, the following purchases, and insert any names and dates :

1. 5 pairs of gloves at 2*s.* 6*d.* each, 24 yards of muslin at 1*s.* 9*d.* per yard, 17 yards of ribbon at 8½*d.* per yard, and 35 yards of flannel at 1*s.* 7¾*d.* per yard.

2. 10 lbs. of sugar at 2½*d.* per lb., 6 lbs. of tea at 1*s.* 8*d.* per lb., 8 of coffee at 1*s.* 4*d.*, 12 of currants at 4½*d.*, 10 of rice at 2½*d.*, and 9 of cheese at 11*d.*

3. 18½ yards of silk at 5*s.* 6*d.* per yard, 64 yards of sheeting at 1*s.* 4½*d.*, 4 pairs of blankets at 18*s.* 4*d.* per pair, and 3 pairs at 12*s.* 8*d.*, 26 yards of linen at 10¾*d.*, 4 rugs at 19*s.* 6*d.*, and 5 at £1. 6*s.* 6*d.* each.

4. 13¾ yards of velvet at 7*s.* 6*d.* per yard, 7½ yards of crape at 2*s.* 9*d.* per yard, 3½ yards of cloth at 10*s.* 8*d.* per yard, 26½ yards of linen at 1*s.* 10*d.* per yard, and 18 yards of flannel at 1*s.* 4*d.* per yard.

5. 2 doz. pairs of gloves at 2*s.* 11½*d.* per pair, 38 yards of flannel at 9¾*d.* per yard, a gross of buttons at 3¼*d.* per doz., 136 yards of trimming at 2*s.* 7½*d.* per doz. yards.

6. 7½ doz. of knives at 7¾*d.* each, 2 score of corkscrews at 5½*d.* each, 11 teapots at 2*s.* 3½*d.* each, and 40 files at 7*s.* 6*d.* per dozen.

7. 5 doz. slates at 3 for 10½*d.*, 6 score of pencils at 5¼*d.* per half doz., 18 boxes of chalk at 1*s.* 5¼*d.* per box, and ½ gallon of ink at 5*d.* per pint.

8. 3 pieces of cloth, 32 yards each, at 7*s.* 9*d.* a yard, 75 yards of calico at 3½*d.* a yard, 10 doz. reels of cotton at 1¾*d.* a reel, and 3 doz. skeins of silk at 7¾*d.* a skein.

9. 12 bars of soap, each weighing 3½ lbs., at 5½*d.* per lb., 164 lbs. of tea at 30*s.* per doz. lbs., and 12 packets of candles, each containing 6½ lbs., at 11½*d.* per lb.

10. 1 cwt. of indigo at 14*s.* 6*d.* per lb., 1 ton of cloves at 1*s.* 2*d.* per lb., 5 cwt. 3 qrs. 18 lbs. of zinc at 2½*d.* per lb., and 7 cwt. 1 qr. 14 lbs. of black tin at £64 per ton.

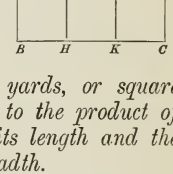
CHAPTER VII.

SQUARE AND CUBIC MEASURE.

90. The area of any surface is the number of square feet (or square yards, or square inches) contained in that surface.

Thus the area of a square, each of whose sides is 9 feet, is 9^2, that is 81, square feet.

91. A surface like the floor or the wall of an ordinary room is called a **rectangle.** Its opposite sides are equal and its corners form right angles.



Thus, if $ABCD$ be a rectangle, then

$$AB = CD, \quad AD = BC,$$

and the angles at A, B, C, and D are all right angles.

92. *The number of square feet (or square yards, or square inches, etc.) in the area of a rectangle is equal to the product of the number of feet (or yards, or inches, etc.) in its length and the number of feet (or yards, or inches, etc.) in its breadth.*

Consider a rectangle $ABCD$, whose length AD is 3 feet and whose breadth AB is 2 feet. We have to show that the number of square feet in its area $= 3 \times 2 = 6$.

Divide AD at E and F, so that $AE = EF = FD =$ one foot.

Divide AB at G, so that $AG = GB =$ one foot. Similarly, divide BC at H and K and also DC at L.

Join the points thus obtained as in the figure.

We then have six figures within the area $ABCD$, and each of these figures has each of its sides equal to one foot. Hence the area of each of these six figures is one square foot.

Hence the area of $ABCD$ is 6 square feet, and this is the result the rule above would give.

93. We must next consider the case where the sides are given as fractions of a foot.

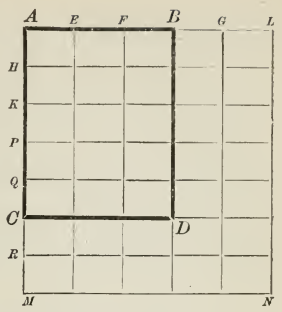
Let $ALNM$ be a square whose side is 1 ft. long and which therefore contains one square ft. of area.

If we divide AL into 5 equal parts and AM into 7 equal parts, and draw parallel lines as in the figure, we shall get 35 small rectangles making up the area of the square, and each of these will equal $\frac{1}{35}$ sq. ft. The rectangle $ABDC$ contained by the thick lines in the fig., where $AB = \frac{3}{5}$ ft. and $AC = \frac{5}{7}$ ft., will be found to contain 15 of the small rect-

angles, *i.e.* $\frac{1}{3}\frac{5}{5}$ sq. ft. But this is the result given by Art. 92, since

$$\frac{3}{5} \times \frac{5}{7} = \frac{1}{3}\frac{5}{5}.$$

In a similar manner the rule may be shown to be true for any other fractional length of the sides.



94. For a rectangle we therefore have the following important rule:

Length × Breadth = Area...(1)

The student must be careful about the units employed; *e.g.* he must not multiply feet by inches to find an area.

Ex. 1. *Find the area of a room whose length is* 18 *ft.* 5 *in. and whose width is* 14 *ft.* 9 *in.*

18 ft. 5 in. $= 18\frac{5}{12}$ ft., and 14 ft. 9 in. $= 14\frac{3}{4}$ ft.;

$\therefore$ area $= 18\frac{5}{12} \times 14\frac{3}{4}$ sq. ft. $= \frac{221}{12} \times \frac{59}{4}$ sq. ft.

$= \frac{13039}{48}$ sq. ft. $= 271\frac{31}{48}$ sq. ft. $= 271$ sq. ft. 93 sq. in.

Ex. 2. *A man desires a rectangular plot of ground of area* $\frac{2}{3}$ *of an acre on which to build a house. If the frontage to the road be* 40 *yards, what must be the depth of the plot?*

$\frac{2}{3}$ of an acre $= \frac{2}{3} \times 4840$ sq. yards;

$\therefore$ depth of the plot $= \frac{\text{area}}{\text{width}} = \frac{\frac{2}{3} \times 4840}{40}$ yds. $= \frac{2}{3} \times \frac{4840}{40}$ yds.

$= \frac{2}{3} \times 121$ yds. $= \frac{242}{3}$ yds. $= 80\frac{2}{3}$ yds. $= 80$ yds. 2 ft.

Ex. 3. *Find the cost of staining the floor of a room* 22 *ft.* 9 *in. by* 18 *ft.* 6 *in. at* 4*d. per square foot.*

Cost = area in sq. ft. × 4 pence $= 22\frac{3}{4} \times 18\frac{1}{2} \times 4$ pence

$= \frac{91}{4} \times \frac{37}{2} \times 4$ pence $= 1683\frac{1}{2}$ pence $=$ £7. 0*s.* $3\frac{1}{2}d.$

EXAMPLES LI.

Fill up the omissions in the following:

	Length.	Breadth.	Area.
1.	18 feet.	13 feet.	
2.	17 feet.	11 feet.	
3.	21 ft. 6 in.	19 feet.	
4.	3 yds. 2 ft.	5 yds. 1 ft. 6 in.	
5.	6 miles.	44 feet.	
6.	18 feet.		270 square feet.
7.	18 ft. 9. in.		273 sq. ft. 63 sq. in.

Find the acreage of the rectangular fields whose lengths and breadths are

8. 187 yards and 55 yards. **9.** 323 yards and 126 yards.

10. 56 chains 25 links and 25 chains 20 links.

11. 10 chains 25 links and 7 chains 35 links.

12. If land be worth £162. 15*s.* per acre, find the value of a field whose length and breadth are respectively 55 yds. and 44 yds.

13. Find the difference between one quarter of a square mile and one quarter of a mile square, and express the answer in square yards.

14. A carpet covers the floor of a room 22 ft. long, the cost at 5*s.* per square yard being £11 ; find the width of the room.

15. A piece of carpet 72 yds. long and 27 in. wide is used to cover a room. If the whole piece cost £12. 12*s.*, what is the cost per sq. yd.?

16. How many allotments of 4 ac. 2 r. 36 sq. po. each could possibly be taken out of a stretch of land 56 miles long by 20¼ miles?

17. A piece of paper 12 yards long by 21 inches wide is cut into strips 1 inch wide ; how many such sheets would be required to form a strip that would reach from London to Exeter, a distance of 171½ miles?

18. If 11,000 copies of the *Times* be issued daily, each copy consisting of 2 sheets, and each sheet being 4 feet by 3 feet, how many acres will one edition cover?

19. Each copy of a newspaper, whose circulation is 242,000 copies daily, has an area of 6 square feet ; how many square miles will 16 weeks' issue cover?

20. The length and breadth of a tennis court measured by a certain yard measure were meant to be 78 and 36 ft. respectively. If the measure be 1 inch too short, by what area was the court less than the supposed area?

21. The area of a rectangular enclosure is to be 1547 sq. yds., and each of the sides is to be an exact number of yards ; what are the possible dimensions of the enclosure?

22. An oblong field of which the area is 3 acres and one side 12 rods long is planted with trees at its corners, along its sides, and throughout in rows 6 ft. apart ; how many trees were there?

Find the cost of lining a cistern without a lid whose length, breadth, and depth are respectively

23. 7 ft. 10 in., 5 ft. 4 in., and 3 ft. 6 in., with lead at £1. 0*s.* 3*d.* per square yard.

24. 6 ft. 9 in., 5 ft. 3 in., and 2 ft. 6 in., with zinc at 6*s.* 9*d.* per square yard.

25. 5 ft. 6 in., 3 ft. 3 in., and 4 ft. 9 in., with zinc at 7½*d.* per square foot.

26. 12 ft. 9 in., 8 ft. 3 in., and 6 ft. 6 in., with lead which costs £1. 8*s.* per cwt. and weighs 8 lb. to the square foot.

27. Find the cost of papering the walls and ceiling of a room whose height is 12 feet, length 16 feet, and breadth 13 feet, the papering of the walls costing 9*d.* per sq. yd., and that of the ceiling $4\frac{1}{2}d.$ per sq. yd.

28. Find the cost at 3*d.* per sq. ft. of painting the walls of a room 18 feet long, 15 feet wide, and 10 feet high, allowing for a window 5 feet by 4 feet, 2 doors each 7 feet by 3 feet, and a fireplace 4 feet by 5 feet.

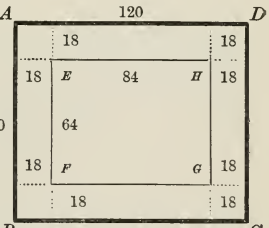
29. Find the expense of turfing a piece of ground which is 40 yards long and 100 feet wide, with turfs, each a yard long and a foot wide, their cost being 6*s.* 9*d.* a hundred.

30. How much turf will be required to cover a rectangular plot of ground which consists of a lawn-tennis court (26 yards by 12) and a margin of $3\frac{1}{2}$ yards at each end and 4 feet at each side? Find also the cost at 4*s.* 6*d.* per square pole.

31. The floor of a room 17 ft. 6 in. long by 16 ft. 3 in. wide is to be covered with tiles at £2. 5*s.* per thousand, the size of a tile being 3 inches by 5 inches; what is the cost of the tiles?

32. How many tiles $4\frac{1}{2}$ inches by 6 inches, will be required to cover a floor 18 feet long by 13 ft. 6 in. broad, and what is their cost at 3*s.* 4*d.* per dozen?

33. *A rectangular court-yard is* 120 *feet long, and* 100 *feet wide; around the court-yard is a gravel path,* 18 *feet wide and the rest of the court is a grass plot: find the area of the grass plot, and also that of the gravelled portion.*



Let *ABCD* be the court-yard, and *EFGH* the boundary of the grass plot.

Then *EF* is less than *AB* by twice the width of the gravel, that is, by twice 18 feet.

$\therefore EF = AB - 2 \times 18 = 100 - 36 = 64$ ft.

So $EH = 120 - 2 \times 18 = 120 - 36 = 84$ ft.

The area of the grass plot therefore $= 84 \times 64$ sq. ft. $= 5376$ sq. ft.

The area of the whole court-yard $= 120 \times 100$ sq. ft. $= 12{,}000$ sq. ft.

Hence the area of the gravelled portion $= 12{,}000 - 5376 = 6624$ sq. ft.

34. What will it cost to make a gravel walk 7 feet wide along the inner edge of each side of a square field, whose side is 110 yards long, at 1*s.* 6*d.* per sq. yard?

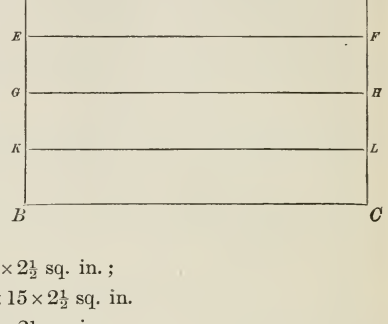
35. Find the expense of paving a pathway 6 feet wide round and immediately outside a lawn, 21 yards long and 10 yards broad, at $9\frac{1}{4}d.$ per sq. yard.

36. A rectangular court is 120 feet long and 90 broad, and a path of the uniform width of 10 feet runs round it and inside it; find the cost of covering the path with flagstones at 4*s.* 6*d.* per sq. yard, and the remainder with turf at 3*s.* 3*d.* per 100 sq. feet.

Carpeting of Rooms and Papering of Walls.

95. **Ex. 1.** *What length of carpet* $2\frac{1}{2}$ *ft. wide is required to entirely cover a room* 15 *ft. long by* 10 *ft. wide?*

Cut out of white paper a rectangle 15 in. long by 10 in. wide. Then cut strips of coloured paper each 15 in. long by $2\frac{1}{2}$ in. wide. Paste or pin these over the rectangle, parallel to its 15 in. side, so that they cover it without overlapping.



In the adjoining fig., $ABCD$ is the paper rectangle, while $AEFD$, $EGHF$, etc., are the spaces which the strips cover.

It will be found that 4 strips are required or *a total length of* 60 *in.* so that a length of 60 feet of carpet is wanted.

Now the area of 1 strip $= 15 \times 2\frac{1}{2}$ sq. in.;

$\therefore$ 4 strips $= 4 \times 15 \times 2\frac{1}{2}$ sq. in.

$= 60 \times 2\frac{1}{2}$ sq. in.

= total length × breadth of strip.

But this covers the white rectangle, and we get the rule:

Total length of carpet required × breadth of roll = area of room.

By this rule the result may be obtained more quickly.

For, total length $\times 2\frac{1}{2} = 15 \times 10$.

$$\therefore \text{ total length} = \frac{15 \times 10}{2\frac{1}{2}} = 60 \text{ ft.}$$

Ex. 2. *Find the cost of the paper for the walls of a room,* 24 *feet long,* 18 *feet wide, and* 10 *feet high, when the paper is* 21 *inches wide, and costs* 2*s.* 3*d. per piece of* 12 *yards.*

Since the area of each wall = its length × its height,

$\therefore$ the area of the 4 walls $= (24 \times 10 + 18 \times 10 + 24 \times 10 + 18 \times 10)$ sq. ft.

$= (24 + 18 + 24 + 18) \times 10$ sq. ft.

$= 840$ sq. ft.;

$\therefore$ length of paper $\times 1\frac{3}{4} = 840$;

$$\therefore \text{ length of paper} = \frac{840}{1\frac{3}{4}} = 480 \text{ ft.}$$

Now cost of the paper $= 27d.$ per 12 yds. $= \frac{3}{4}d.$ per ft.;

$\therefore$ total cost $= 480 \times \frac{3}{4}$ pence $= 360d. = £1.\ 10s.$

EXAMPLES LII.

[*In many of the following examples the student would do well if he drew a figure.*]

Find the cost of covering the floor of a room whose length and breadth are respectively

1. 19 ft. 6 in. and 13 ft. with carpet, 39 in. wide, at 4*s.* 9*d.* per yd.

2. 20 ft. 3 in. and 13 ft. 4 in. with carpet, 27 in. wide, at 5*s.* per yd.

3. each 17 ft. 6 in. with carpet, 2 ft. 4 in. wide, at 3*s.* 9*d.* per yard.

4. 19 ft. 6 in. and 16 ft. 6 in. with carpet, $\frac{3}{4}$ yd. wide, at 2*s.* 6*d.* per yard.

5. When 54 yds. $2\frac{1}{2}$ ft. of carpet exactly cover the floor of a room, $23\frac{1}{2}$ ft. long and 15 ft. 9 in. broad, what is the width of the carpet?

6. What length of carpet, 30 in. wide, will be wanted for a room 17 ft. 4 in. long and 13 ft. 9 in. wide? If the cost of the carpet be £5. 19*s.* 2*d.*, what is its price per yard?

7. A room is 20 ft. long and 16 ft. wide. The centre of the room is covered with carpet, 20 in. wide, and costing 3*s.* 6*d.* per yard, and there is a border all round the room, one yard wide, which is stained at a cost of 1*s.* per sq. yard; find the whole cost.

8. If it cost £14. 6*s.* $4\frac{1}{2}$*d.* to carpet a room 14 ft. 6 in. wide with carpet 27 in. wide at 6*s.* 9*d.* per yd., what is the length of the room?

What is the cost of papering a room whose length, breadth, and height are respectively

9. 24 ft., 18 ft., and 11 ft., with paper 21 in. wide, at 3*d.* per yard?

10. 27 ft., 18 ft., and 12 ft., with paper 21 in. wide, at $3\frac{1}{2}$*d.* per yard?

11. 17 ft., 13 ft., and 14 ft., with paper 21 in. wide, at $4\frac{1}{2}$*d.* per yard?

12. 16 ft. 9 in., 13 ft. 3 in., and 12 ft., with paper 21 in. wide, at $2\frac{1}{4}$*d.* per yard?

13. 14 ft., 13 ft. 6 in., and 7 ft. 8 in., with French paper 18 in. wide, at 13*s.* 6*d.* per dozen yards?

14. 28 ft. 6 in., 18 ft. 9 in., and 12 ft., with paper 1 ft. 9 in. wide, at 2*s.* 6*d.* per piece of 12 yards?

15. 20 ft., 17 ft., and 12 ft., with paper 21 in. wide, at 2*s.* 8*d.* per piece of 12 yards, allowing 42 sq. feet for doors and windows?

16. each 15 ft., with paper 1 yard wide, at 1*s.* 2*d.* a yard, there being a door 6 ft. by 4 ft., and a window 7 ft. by 3 ft.?

17. The cost of the papering of a room 17 ft. 6 in. long, 14 ft. wide, and 12 ft. high with paper at $4\frac{1}{2}$*d.* per yard is £2. 2*s.*; what is the width of the paper?

18. The roller used for a lawn is $6\frac{1}{2}$ ft. in circumference, and 2 ft. 3 in. wide. It turns 12 times in passing from one end of the lawn to the other. Find the area rolled when the roller has passed ten times the whole length.

19. The circumference of a roller is 3 ft. 4 in., and its width is 3 feet; how many revolutions must it make to roll a quarter of an acre of grass?

CUBIC MEASURE.

96. A solid is that which has length, breadth, and height.

A cube is a solid whose length, breadth, and height are equal, it being such that the six surfaces bounding it are all squares. It is easily seen that the number of cubic feet in a cube is obtained by cubing the number of feet in either of its edges.

A rectangular parallelopiped is a solid which is bounded by six rectangles.

97. *The number of cubic feet (or yards, or inches, etc.) in a rectangular parallelopiped is obtained by multiplying together the number of feet (or yards, or inches) in its length, the number of feet (or yards, or inches) in its breadth, and the number of feet (or yards, or inches) in its height, i.e.*

length × breadth × height = volume.

If the student draws a figure similar to that of an ordinary box with its length equal to 4 ft., its breadth 3 ft., and its height 2 ft., it will be found to contain 24 cub. ft., *i.e.* $2 \times 3 \times 4$ cub. ft. Any other such solid, whose edges are an exact number of feet in length, may be considered in the same way, when the rule will be found to hold good.

The same rule for the volume may by a method similar to that of Art. 93 be proved to be true when the edges are fractional lengths.

Ex. *Find the volume of a block of stone, whose length is* 2 *ft.* 8 *in., whose breadth is* 1 *ft.* 7 *in., and whose height is* 1 *ft.* 3 *in.*
Find also its value at 2*s.* 3*d. per cubic foot.*

The length $= 2$ ft. 8 in. $= 2\frac{2}{3}$ ft. $= \frac{8}{3}$ ft.

The breadth $= 1$ ft. 7 in. $= 1\frac{7}{12}$ ft. $= \frac{19}{12}$ ft.

The height $= 1$ ft. 3 in. $= 1\frac{1}{4}$ ft. $= \frac{5}{4}$ ft.

Hence the volume of the block $= \frac{\cancel{8}^{2}}{\cancel{3}} \times \frac{19}{\cancel{12}_{6}} \times \frac{5}{\cancel{4}}$ cub. ft. $= \frac{95}{18}$ cub. ft.

$= 5\frac{5}{18}$ cub. ft. $= 5$ cub. ft. 480 cub. in.

Its value $= \frac{95}{18} \times 2s.\ 3d. = \frac{95}{18} \times 2\frac{1}{4}s. = \frac{95}{\cancel{18}_{2}} \times \frac{\cancel{9}}{4}s. = \frac{95}{8}s. = 11\frac{7}{8}s. = 11s.\ 10\frac{1}{2}d.$

EXAMPLES LIII.

Find the volume of boxes, cisterns, etc., whose length, width, and depth are respectively

1. 5 ft., 4 ft., and 3 ft.

2. 8 ft., 7 ft., and 2 ft.

3. 10 ft., 5 ft., and 3 ft. 6 in.

4. 9 ft., 6 ft. 6 in., and 3 ft. 3 in.

5. 8 ft. 6 in., 5 ft. 6 in., and 3 ft. 4 in.

6. 7 ft. 9 in., 6 ft. 8 in., and 5 ft. 7 in.

A gallon of water being $277\frac{1}{4}$ cubic inches nearly, find the number of gallons (to the nearest unit) contained in cisterns whose length, breadth, and depth are

7. 4 ft., 3 ft., and 2 ft. **8.** 6 ft., $3\frac{1}{2}$ ft., and 2 ft. 4 in.

9. Find in tons, cwt., etc., the weight of water contained in a tank 6 yards long, 2 yards high, and 8 feet wide, the tank being full and the weight of a cubic foot of water being 1000 ounces.

10. From a piece of wood, whose volume is 3 cub. yds. 3 cub. ft. 3 cub. in., a cube is cut, each of whose edges measures 1 yd. 1 ft. 1 in.; how much is left?

11. Find the value of a balk of timber 39 ft. 6 in. long and 3 ft. 7 in. thick each way, at 2*s.* 6*d.* a cubic foot.

12. A block of marble is 29 ft. 6 in. long, 12 ft. 6 in. wide, and 10 ft. deep, and cost £407. 3*s.* $2\frac{3}{4}d.$; at what rate is this per cubic foot?

13. If 12 inches of rain fall in a year, find the total weight which will fall in that year upon a garden of one acre in extent, assuming that 36 cubic feet of water weigh one ton.

14. A pond whose area is half an acre is frozen over with ice two inches thick; find in tons to the nearest whole number the weight of the ice if a cubic foot of it weighs $57\frac{1}{4}$ lbs.

15. A cubic foot of Canadian elm weighs ·725 as much as a cubic foot of water, and a cubic foot of water weighs 1000 ounces; what is the weight of a beam of Canadian elm 12 ft. 6 in. long, 1 ft. 6 in. deep, and 1 ft. 3 in. thick?

16. Find the expense of digging out the gravel from a pit 22 yds. long, 14 ft. wide, and 4 ft. deep, at 1*s.* 6*d.* per cubic yard. If the gravel is sold at 2*s.* 6*d.* a cubic yard, what is the net sum received by the sale of the gravel after paying the cartage to its destination at 1*s.* per load of 3 cubic yards?

17. A stream, one inch square in section, pours into a cistern at the rate of 15 feet per minute; how many gallons enter the cistern in one hour? [A gallon = $277\frac{1}{4}$ cubic inches nearly.]

18. A rectangular grass plot is 200 ft. long and 120 ft. wide; a path surrounds it whose width is 10 ft.; find the cost of covering the path with gravel, 3 inches deep, if the gravel cost 4*s.* 6*d.* per cubic yard.

[*In the figure of page* 81 *the grass plot is EFGH and the boundary of the gravel is ABCD.*]

19. A quadrangle, 120 feet long by 100 feet wide, has a grass plot, 60 feet by 50 feet, in the centre; find the cost of gravelling the rest of it to a depth of 6 inches at 4*s.* per cubic yard.

20. *How many bricks, whose length, breadth, and thickness are respectively* 9 *inches,* $4\frac{1}{2}$ *inches, and* 3 *inches are required to build a wall* 20 *yards long,* 8 *feet high, and* $13\frac{1}{2}$ *inches thick, the space occupied by the mortar being neglected?*

The length, breadth, and thickness of a brick are respectively $\frac{3}{4}$, $\frac{3}{8}$, and $\frac{1}{4}$ ft. Hence the volume of a brick $= \frac{3}{4}$ ft. $\times \frac{3}{8}$ ft. $\times \frac{1}{4}$ ft. $= \frac{9}{128}$ cub. ft.

The length, breadth, and thickness of the wall are respectively 60, 8, and $\frac{9}{8}$ ft. Hence the volume of the wall $= 60$ ft. $\times 8$ ft. $\times \frac{9}{8}$ ft. $= 540$ cub. ft.

The number of bricks = volume of the wall $\div$ volume of one brick

$$= 540 \div \frac{9}{12 \cdot 8} = 540 \times \frac{12 \cdot 8}{9} = 60 \times 128 = 7680.$$

21. How many small cubes, each of edge 3 inches in length, can be cut out of a large cube whose edge is 18 inches?

22. A brick being 9 inches long, $4\frac{1}{2}$ inches wide, and 3 inches thick, find how many are contained in a stack 30 feet long, 16 ft. 6 in. wide, and 9 ft. 3 in. high.

23. A wall is 11 feet high, 20 feet long, and 1 foot thick; how many bricks will be wanted to build it, each brick being 11 inches long, 5 inches broad, and 6 inches thick?

24. How many tiles, each 6 in. square and 1 in. thick, can be put together in a stack 38 feet long, 10 ft. 6 in. wide, and 6 ft. 5 in. high?

25. A cistern contains $243\frac{3}{4}$ cub. feet of water; its length is 11 ft. 3 in. and its depth is 3 ft. 4 in.; what is its width?

26. If gold be beaten out so thin that a grain will form a leaf of area 56 sq. inches, how many of these leaves will make a pile one foot in thickness, the weight of a cubic inch of gold being taken to be 11 oz. Av.?

27. What must be the area to the nearest rood of the surface of a reservoir so that the withdrawal of one million gallons of water a day may lessen the depth of the water by 1 foot in a week? [A gallon contains $277\frac{1}{4}$ cub. in. nearly.]

28. By drawing off $2762\frac{1}{2}$ cub. feet I cause the surface of the water in a reservoir 19 ft. 6 in. wide to sink 30 inches; what is the length of the reservoir?

29. *A box with a cover is made of wood which is half an inch thick; if its external measurement be* 18 *inches by* 15 *inches by* 10 *inches, find the internal volume of the box. Find also the volume of the wood.*

Internal length = external length − twice the thickness of the wood
$= 18$ in. $- 1$ in. $= 17$ in.

So internal breadth $= 15$ in. − twice the thickness $= 15$ in. $- 1$ in. $= 14$ in.

Similarly, internal height $= 10$ in. $- 1$ in. $= 9$ in.

$\therefore$ internal volume $= 17 \times 14 \times 9$ cub. in. $= 2142$ cub. in.

Also volume occupied by the box
= its external length × its external breadth × its external height
$= 18 \times 15 \times 10$ cub. in. $= 2700$ cub. in.

$\therefore$ volume of the wood $= 2700$ cub. in. $- 2142$ cub. in. $= 558$ cub. in.

30. What is the cubical content of a box which measures on the outside 2 ft. $9\frac{1}{2}$ in. in length, 2 ft. $6\frac{3}{4}$ in. in breadth, and 2 ft. $3\frac{1}{4}$ in. in height, the sides and top being $\frac{3}{4}$ in. thick and the bottom 1 in. thick?

31. A box with a lid is made of planking $1\frac{1}{2}$ in. thick; if its external dimensions be 3 ft. 6 in., 2 ft. 6 in., and 1 ft. 9 in., find how many sq. feet of plank are used in its construction.

32. The external length, breadth, and depth of a closed box are 4 ft., 2 ft., and 16 in. respectively, and the wood of which it is made is an inch thick. Find (1) the number of cubic inches of wood in the box, and (2) the cost of lining it with metal at $4\frac{1}{2}d.$ per sq. foot.

CHAPTER VIII.

THE UNITARY METHOD. RATIO AND PROPORTION.

98. Let us consider the following examples:

Ex. 1. *If 23 yards of cloth cost £5. 15s., what is the cost of 31 yards of the same cloth?*

In such a question as this each yard costs the same as every other yard, so that the cost of 23 yards is found by multiplying the cost of 1 yard by 23.

So the cost of 1 yard is found by dividing the cost of 23 yards by 23.

Now 23 yards cost £5. 15s.; ∴ 1 yard cost $\frac{1}{23}$ of £5. 15s.;

∴ 1 yard costs $\frac{1}{23}$ of 115s., that is, 5s.;

∴ 31 yards cost 31 × 5s., that is, 155s., that is, £7. 15s.

Ex. 2. *If 7 tons 8 cwt. of iron cost £15. 8s., what is the cost of 12 tons 19 cwt.?*

7 tons 8 cwt. = 148 cwt. and 12 tons 19 cwt. = 259 cwt.

Thus 148 cwt. cost £15. 8s., that is, 308s.;

∴ 1 cwt. costs $\frac{308}{148}$s., that is, $\frac{77}{37}$s.;

∴ cost of 259 cwt. = $259 \times \frac{77}{37}$s. = $7 \times \cancel{37} \times \frac{77}{\cancel{37}}$ = 539s. = £26. 19s.

Ex. 3. *If 43 yards of silk cost £5. 7s. 6d., how many yards can be bought for £7. 7s. 6d.?*

£5. 7s. 6d. = 107s. 6d. = 1290d., and £7. 7s. 6d. = 147s. 6d. = 1770d.

Hence 1290d. is the cost of 43 yds.

∴ 30d. ,, ,, 1 yd.

∴ 1d. ,, ,, $\frac{1}{30}$ yd.

∴ 1770d. ,, ,, $\frac{1770}{30}$ yds.

i.e. £7. 7s. 6d. ,, ,, 59 yds. ∴ Ans. = 59 yds.

Ex. 4. *If a man's wages for 18 days be £2. 18s. 6d., in how many days will he earn £9. 11s. 9d.?*

Here £2. 18s. 6d. = 58s. 6d. = 702d.

Also £9. 11s. 9d. = 191s. 9d. = 2301d.

Hence he earns 702d. in 18 days,

that is, 39d. in 1 day, that is, 1d. in $\frac{1}{39}$ day.

He therefore earns 2301d. in $\frac{2301}{39}$ days,

that is, £9. 11s. 9d. in $\frac{2301}{39}$ days, that is, £9. 11s. 9d. in 59 days.

Ex. 5. *If a train takes* 24 *minutes to travel between two stations which are* 20 *miles apart, how far does it go in an hour?*

In 24 minutes the train travels 20 miles.

Hence in 1 minute ,, ,, $\frac{20}{24}$ miles.

i.e. ,, ,, ,, ,, $\frac{5}{6}$ miles.

$\therefore$ in 60 minutes ,, ,, $60 \times \frac{5}{6}$ miles.

$\therefore$ in 1 hour ,, ,, 50 miles.

Ex. 6. *Find a servant's wages for* 2 *months* 3 *weeks and* 3 *days at the rate of* £21 *per year, reckoning* 7 *days to a week, and* 4 *weeks to a month.*

Here 2 months 3 weeks and 3 days = 11 weeks and 3 days = 80 days.

The wages are £21 for 1 year, that is, £$\frac{21}{12}$ for one month,

that is, £$\frac{21}{12 \times 28}$ for one day, that is, £$\frac{21}{12 \times 28} \times 80$ for 80 days.

Hence the required answer $= \frac{\overset{7}{\cancel{21}}}{\underset{4}{\cancel{12}} \times \underset{4}{\cancel{28}}} \times 80 = £\frac{80}{16} = £5.$

99. In each of the preceding examples it will be seen that we first found the value or equivalent of *one thing*, and then of the number required.

This method is called the **Unitary Method.** It is generally more convenient to arrange the statement so that the quantity of the same kind as that to be found comes last.

EXAMPLES LIV.

1. If 9 yards of cloth cost £1. 18*s.* 3*d.*, what is the cost of 1 yard?

2. If 27 articles cost £3. 6*s.* 6*d.*, what do (1) 9 articles, (2) 108 articles cost?

3. If 20 sheep cost £41, what do (1) 5 and (2) 120 sheep cost?

4. If 5 tons 5 cwt. of coal cost £2. 16*s.*, what do (1) 5 cwt. and (2) 9 tons cost?

5. If 74 yards of silk cost £12. 19*s.*, what do 31 yards cost?

6. If 37 yards of silk cost £5. 1*s.* 9*d.*, how many yards can be bought for £8. 7*s.* 9*d.*?

7. What is the cost of 45504 pencils at 7*s.* 9*d.* a gross?

8. If the fare for 30 miles be 6*s.* 3*d.*, how many miles can I travel for £1. 3*s.* 4*d.*?

9. If 17 cwt. cost £6. 16*s.*, how much will 23 cwt. cost? Also, how many cwt. can be bought for £14. 16*s.*?

10. What is the cost of 2 tons 3 cwt. of coal if 6 cwt. cost 7*s.*?

11. If 18 oz. of silver cost £2. 4*s.* 3*d.*, how many oz. can be bought for £4. 13*s.* 5*d.*?

12. If a man earns £20. 16*s.* in 64 days, how long will he take to earn £28. 5*s.* 6*d.*?

13. If $10\frac{1}{7}$ oz. cost 21*s.*, what will $91\frac{2}{7}$ lbs. cost?

14. If 1 lb. of tea costs 1·875 shillings, what is the cost of 8·75 lbs.?

15. If 7 cwt. 1 qr. 13 lbs. of tea cost £52. 5*s.*, how much will 5 cwt. 3 qrs. 6 lbs. cost?

16. If 1 ton 2 cwt. 14 lbs. of copper cost £47. 18*s.* 9*d.*, how much can be obtained for £115. 2*s.* 1*d.*?

17. If 5 quarters 5 bushels of wheat cost £6. 6*s.* $6\frac{3}{4}d.$, how much wheat can be bought for £2. 13*s.* $5\frac{1}{4}d.$?

18. If $\frac{7}{8}$ of a yard of ribbon cost 3*s.* $2\frac{1}{2}d.$, what is the cost of $11\frac{5}{16}$ yards?

19. If a man walks $12\frac{3}{4}$ miles in 3 hours, how long would he be in walking 17 miles?

20. If $2\frac{7}{8}$ yards of calico cost 1*s.* $5\frac{1}{4}d.$, how much will $15\frac{3}{4}$ yards cost?

21. When oranges are sold at 18 for a shilling, how much must be paid for 75?

22. If three pounds of butter cost 3*s.* 9*d.*, how much will eleven pounds cost?

23. What is the cost of 115 lambs at 32*s.* per score?

24. A man's wages are at the rate of £59. 19*s.* 10*d.* per year of 313 working days; what should he be paid for 23 days' work?

25. If 1 ton 4 cwt. of coal cost £1. 7*s.*, what is the price of 10 tons 11 cwt.?

26. If 58 bushels of corn cost £9. 3*s.* 8*d.*, how many bushels can be bought for £5. 17*s.* 2*d.*?

27. If $5\frac{1}{2}$ yards of silk can be bought for £2. 8*s.* $1\frac{1}{2}d.$, how many yards can be bought for £20. 15*s.* $7\frac{1}{2}d.$?

28. If 24 articles cost £4. 4*s.*, how much will 37 such articles cost?

29. If 28 articles cost £5. 5s., how many such articles will be obtained for £8. 5*s.*?

30. If a train go 60 miles in one hour, how long will it take to go one-third of a mile?

31. If 15 yards of silk cost £1. 13*s.* 9*d.*, how much will 20 yards 1 foot cost?

32. If the price of 3000 copies of a book be £4725, how much will 1937 copies sell for?

33. If I earn £523. 5*s.* in 7 months, how much shall I earn in one year at the same rate?

34. If I earn £316. 5*s.* in 5 months, how long shall I take at the same rate to earn £695. 15*s.*?

35. If 25 men reap 15 acres in one day, how many men will reap 24 acres in a day?

36. If a man walk 54 miles in $4\frac{1}{2}$ hours, how long will he be in walking 45 miles?

37. If 5 lbs. of tea cost 8*s.* 4*d.*, and $2\frac{1}{2}$ lbs. of coffee cost 4*s.*, what is the cost of $11\frac{1}{2}$ lbs. of tea and 9 lbs. of coffee?

38. If the weight of a sovereign be 5 dwt. $3\frac{1}{2}$ grs., what is the value of 3 oz. 1 dwt. 18 grs. of standard gold?

39. When eggs are at 24 a shilling, how many must be given in payment of a debt of £1. 11*s.* $5\frac{1}{2}d.$?

40. A man agreed to buy a haystack, paying £5 for each waggon load. He found his waggon held 1 ton 3 cwt. 2 qrs., and that the whole stack weighed 10 tons 11 cwt. 2 qrs.; how much has he to pay?

41. *A* earns 30*s.* a week, and *B* earns £18. 15*s.* $9\frac{1}{4}d.$ whilst *A* earns £16. 11*s.* $6\frac{3}{4}d.$; how much does *B* earn per week?

42. If of every £1000 spent on University buildings, St. John's College had to pay £169, for how much of the £4014. 11*s.* 8*d.* spent on the Literary Schools was the College liable?

43. Find a servant's wages for five months three weeks and six days at one pound seven shillings and five pence per month, reckoning seven days to a week and four weeks to a month.

44. What should be a person's salary from April 5 to July 12, both inclusive, at the rate of £136. 17*s.* 6*d.* a year?

45. How much does a person spend in 35 days who, with an annual income of 400 guineas, is £18 in debt at the end of the year?

46. A party of 7 people spent 47 guineas in 12 days. At the same rate what would the same number of people spend from July 29 to August 13, both days being included?

47. A man has an income of £91. 5*s.* per annum. He saves one-fifth of it and spends the rest; how much does he spend in 125 days?

100. **Ex. 1.** *If* 16 *men can do a piece of work in* 27 *days of* 10 *hours each, in how many days of* 8 *hours each can* 18 *men do the same piece?*

Set down the statement so that 'days' (which have to be found) comes last, thus:

16 men working 10 hours a day finish in 27 days;

$\therefore$ 16 men working 1 hour a day finish in 27×10 days.

[Working 1 hour a day, they will take 10 times as long as if they worked 10 hours.]

$\therefore$ 1 man working 1 hour a day finishes in $27 \times 10 \times 16$ days.

[1 man will take 16 times as long as 16 men.]

$\therefore$ 1 man working 8 hours a day finishes in $\frac{27 \times 10 \times 16}{8}$ days.

[1 man working 8 hours a day takes $\frac{1}{8}$ of time he would take working 1 hour a day.]

$\therefore$ 18 men working 8 hours a day finish in $\frac{27 \times 10 \times 16}{18 \times 8}$ days.

[18 men will take $\frac{1}{18}$ as long as 1 man.]

$$Ans. = \frac{\overset{3}{\cancel{27}} \times 10 \times \overset{2}{\cancel{16}}}{\underset{2}{\cancel{18}} \times \cancel{8}} = 30 \text{ days.}$$

Ex. 2. *1500 men in a fortress have provisions for 48 days, but 500 men join them; how long will the provisions last?*

The provisions will last 1500 men for 48 days.

$\therefore$,, ,, 1 man for 48×1500 days.

[For 1500 men will eat in 1 day as much as 1 man in 1500 days.]

$\therefore$ the provisions will last 2000 men for $\dfrac{48 \times 1500}{2000}$ days.

[For 2000 men will take $\frac{1}{2000}$th as long to eat a certain quantity of provisions as 1 man will.]

$\therefore$ required answer $= \dfrac{\overset{1}{\cancel{48}} \times \overset{3}{\cancel{1500}}}{\underset{1}{\cancel{2000}}} = 36$ days.

In this case also, if the number of men be doubled (or multiplied by 3, 4, etc.), the time that the food lasts is clearly halved (or divided by 3, 4, etc.).

Ex. 3. *If $42\frac{3}{4}$ yards of cloth, 44 inches wide, cost £17. 12s. $8\frac{1}{4}d.$, how much should $33\frac{1}{4}$ yards of cloth of the same kind, 56 inches wide, cost?*

$42\frac{3}{4}$ yards of cloth 44 in. wide cost £17. 12s. $8\frac{1}{4}d.$

$\therefore$ $42\frac{3}{4}$,, ,, 1 ,, $\dfrac{1}{44}$ of £17. 12s. $8\frac{1}{4}d.$;

$\therefore$ 1 ,, ,, 1 ,, $\dfrac{1}{44 \times 42\frac{3}{4}}$ of £17. 12s. $8\frac{1}{4}d.$;

$\therefore$ 1 ,, ,, 56 ,, $\dfrac{56}{44 \times 42\frac{3}{4}}$ of £17. 12s. $8\frac{1}{4}d.$;

$\therefore$ $33\frac{1}{4}$,, ,, 56 ,, $\dfrac{56 \times 33\frac{1}{4}}{44 \times 42\frac{3}{4}}$ of £17. 12s. $8\frac{1}{4}d.$;

$$Ans. = \frac{56 \times 33\frac{1}{4}}{44 \times 42\frac{3}{4}} \text{ of £17. 12s. } 8\tfrac{1}{4}d. = \frac{\overset{14}{\cancel{56}} \times \overset{7}{\cancel{133}}}{\underset{11}{\cancel{44}} \times \underset{9}{\cancel{171}}} \text{ of £17. 12s. } 8\tfrac{1}{4}d.$$

$$= \frac{98}{99} \text{ of £17. 12s. } 8\tfrac{1}{4}d. = \text{£17. 9s. } 1\tfrac{1}{2}d.$$

EXAMPLES LV.

1. If 35 men can do a piece of work in 16 days, in how many days will 14 men do the same work?

2. If 91 men do a certain work in 55 days, how long will 65 men take to do the same work?

3. If 68 men can do a certain work in 48 days of 9 hours each, how many men will be required to do the same work in 36 days of $8\frac{1}{2}$ hours each?

4. If 110 men do a certain piece of work in 72 days, how many men will do the same work in 66 days?

5. If 135 men can do a certain piece of work in 36 days of $9\frac{1}{2}$ hours each, in how many days of 9 hours will 171 men complete the same work?

6. A man, walking at the rate of 4 miles an hour, goes from one place to another in 15 hours; how long would he take if he bicycled at the rate of 10 miles an hour?

7. A train goes from one place to another in 4 hours travelling at the rate of 42 miles per hour; how long would another train take which goes 28 miles per hour?

8. If 9 horses can be fed for 16 days at a cost of £8. 16*s.*, for how many days can 7 horses be fed for £3. 17*s.*?

9. If 21 oxen are fed for 15 days at a cost of £11. 16*s.* 3*d.*, what will be the cost of feeding 36 oxen for 27 days?

10. If 35 horses eat 21 pecks in 8 days, how many horses will 36 bushels feed for 96 days?

11. If $11\frac{1}{4}$ cwt. be carried 108 miles for £2. 0*s.* 6*d.*, what will be the cost of carrying 2 tons 7 cwt. a distance of 50 miles?

12. If 35 tons of manure be carried 24 miles for £3. 15*s.*, how many tons will be carried 42 miles for £5. 5*s.*?

13. If 33 men earn £577. 10*s.* in 56 days, how many men will earn £427. 10*s.* in 19 days?

14. If 30 bushels of corn would feed 50 horses for one week, for how long would they feed 35 horses?

15. A store of provisions would last 2100 men for 12 days; how long would it be sufficient for 2800 men?

16. 500 men in a fortress have food enough to last 21 days, but are joined by 200 more men; if the daily supply of each man be now reduced to two-thirds of what it was, for how long will the food suffice?

17. If it costs 12 guineas to supply food for 36 adults and 60 children for a week, how much will it cost to supply 51 adults and 45 children for ten days, supposing that 3 adults eat as much as 5 children?

18. If 2000 copies of a book of 18 sheets require 65 reams of paper, how much paper is required for 4500 copies of a book of 16 sheets?

19. If 3 ducks are worth 4 chickens and 3 geese are worth 10 ducks, find the value of a goose, a pair of chickens being worth 4*s.* 6*d.*

20. If 4 fowls are worth 3 ducks, 7 ducks are worth 2 geese, and 9 geese are worth 7 turkeys, what is the price of a fowl when a turkey is worth 18 shillings?

21. If $\frac{2}{11}$ of a chicken be worth 4*d.* and $\frac{3}{4}$ of a chicken be worth $\frac{11}{120}$ of a turkey, how much should be given for 50 turkeys?

22. If 17 tons of copper are worth 43 tons of zinc, find the value of 1 ton 12 cwt. 2 qrs. of copper when 5 cwt. 1 qr. 14 lbs. of zinc are worth £4. 12*s.* $9\frac{1}{2}$*d.*

23. If 13 cubic inches of copper balance 17 of iron, and 15 of iron balance 16 of tin, and 19 of tin balance 12 of zinc, how many cubic inches of zinc balance 1235 cubic inches of copper?

24. If 6 lbs. of tea be worth 48 lbs. of sugar, and 5 lbs. of coffee be worth 32 lbs. of sugar, how many lbs. of coffee are worth 8 lbs. of tea?

101. Ratio. The ratio of one quantity to another is the fraction that the first quantity is of the second.

Thus, since 3 miles $= \frac{3}{4}$ of 4 miles, the ratio of 3 miles to 4 miles $= \frac{3}{4}$. This ratio is written 3 : 4.

Also, since $2\frac{1}{2}$ hours $= \frac{2\frac{1}{2}}{3\frac{1}{2}}$ of $3\frac{1}{2}$ hours $= \frac{5}{7}$ of $3\frac{1}{2}$ hours, the ratio of $2\frac{1}{2}$ hours to $3\frac{1}{2}$ hours is $\frac{5}{7}$, or 5 : 7.

102. A ratio is therefore a **number,** fractional or whole. The numerator and denominator of the fraction that expresses the ratio are called the **terms** of the ratio.

The quantities forming the ratio must always be of the same kind. Thus there can be no ratio between 7 tables and 5 miles ; for 7 tables cannot be expressed as a fraction of 5 miles. Neither can we compare cwts. with years, or quarts with sovereigns.

But 3 furlongs : 5 miles is a possible ratio ; for the two quantities concerned, though not expressed in terms of the same denomination, are of the same kind.

103. To find the ratio between two compound quantities of the same kind we reduce both quantities to the same denomination, as in Art. 51.

Ex. *What is the ratio of* 3 *cwt.* 3 *qrs.* 20 *lbs. to* 7 *cwt.* 0 *qrs.* 8 *lbs.?*

3 cwt. 3 qrs. 20 lbs. = 15 qrs. 20 lbs. = 440 lbs.,

and 7 cwt. 0 qrs. 8 lbs. = 28 qrs. 8 lbs. = 792 lbs.

The required ratio thus

$$= \frac{3 \text{ cwt. } 3 \text{ qrs. } 20 \text{ lbs.}}{7 \text{ cwt. } 0 \text{ qrs. } 8 \text{ lbs.}} = \frac{440 \text{ lbs.}}{792 \text{ lbs.}} = \frac{55 \times 8}{99 \times 8} = \frac{55}{99} = \frac{5}{9} = 5 : 9.$$

EXAMPLES LVI.

Express in its most simple form the ratio of

1. 10 to 5. **2.** 12 to 3. **3.** 5 to 15. **4.** 6 to 16. **5.** 3 to $\frac{1}{3}$.

6. 4 to $\frac{1}{2}$. **7.** 2 tons to 5 cwt. **8.** £1 to 7*s.* 6*d.*

9. 4 yds. to 1 ft. **10.** 100 to $8\frac{1}{3}$. **11.** 6·75 to 5.

12. 8·75 to 3·25. **13.** £1. 4*s.* 3*d.* to 15*s.* 9*d.* **14.** 4 ac. 2 r. to 3 ac.

15. Find the sum of money which is to £7. 11*s.* 3*d.* as 7 to 11.

16. Find the area which is to 4 ac. 1 r. 9 sq. po. as 9 to 13.

104. When two ratios are equal, their equality is expressed by putting the symbol :: or = between them. Thus the equality of the ratios 9 : 11 and 45 : 55 is expressed,

9 : 11 :: 45 : 55, or 9 : 11 = 45 : 55,

and is read 9 *is to* 11 *as* 45 *is to* 55.

105. When two ratios are equal, the four terms of the ratios are said to be in **Proportion.** Thus, since $9 : 11 :: 45 : 55$, the numbers 9, 11, 45, and 55 are in proportion, or are proportionals. Again, since $105 : 66 :: 35 : 22$ the four numbers 105, 66, 35, and 22 are in proportion.

Since the first two terms of proportion form a ratio, and so also the second two terms, the first two must be quantities of the same kind, and the third and fourth terms must also be quantities of the same kind.

EXAMPLES LVII.

Are the following numbers and quantities in proportion?

1. 8, 9, 48, 54. **2.** 7, 11, 49, 76.

3. 23, 29, 92, 116. **4.** 16 lbs., 17 lbs. 8 oz., 48 lbs., 52 lbs. 8 oz.

Find the missing terms in the following proportions:

5. 12, 28, —, 49. **6.** 18, 42, 54, —. **7.** 22, —, 77, 161.

8. —, 12, 52, 39. **9.** $9\frac{1}{3}$, $4\frac{1}{5}$, —, $2\frac{1}{4}$. **10.** $\frac{4}{3}$, $\frac{1}{4}$, —, $\frac{3}{5}$.

11. £350, £630, —, 108 sheep. **12.** 12 hrs., —, 21 men, 35 men.

13. 3 tons 4 cwt., 1 ton 1 cwt., £4, —.

MISCELLANEOUS EXAMPLES LVIII.

Proportion.

1. If three persons are boarded four weeks for seven pounds, how many can be boarded thirteen weeks five days for one hundred and twelve pounds?

2. If 3000 copies of a book of 11 sheets require 66 reams of paper, how much paper is required for 5000 copies of a book of $12\frac{1}{2}$ sheets?

3. If 752 railway passengers can travel 159 miles for a certain sum of money at 1*d.* per mile, how many can travel 318 miles for three times as much at 2*d.* per mile?

4. If when meat is at 9*d.* per lb. it costs £11. 16*s.* 3*d.* to supply a family of 12 persons for 5 weeks, how much will it cost to supply a family of 18 persons for 7 weeks when meat is at 1*s.* per lb.?

5. If 172 yds. 2 qrs. of cloth, 1 yard wide, can be made from 60 lbs. of wool, how many yards of equal fineness, 5 qrs. wide, will 3 lbs. 12 oz. make?

6. If 4 lbs. of bread cost $5\frac{1}{2}$*d.* when wheat was 44 shillings a quarter, what should have been the cost of 6 lbs. of bread when wheat was 63 shillings a quarter?

7. When the price of wheat was 7*s.* $1\frac{1}{2}$*d.* per bushel, the fourpenny loaf weighed 2 lbs. 3 ozs.; what ought it to have weighed when the price of wheat was 7*s.* 11*d.* per bushel?

8. If, when the price of wheat was 8*s.* a bushel, one pennyworth of bread weighed 10 ozs., what should have been the weight of the 3*d.* loaf when wheat of the same quality cost 7*s.* 6*d.* a bushel?

9. How many articles worth £4. 18*s.* 6½*d.* each should be given in exchange for 2365 articles worth £3. 16*s.* 7½*d.* each?

10. If 27 men can perform a piece of work in 15 days, how many extra men must be put on if the work is to be completed in 9 days?

11. If 64 sheep can be grazed in a field for 21 days, for how many days can 96 sheep be grazed in it?

12. 1500 men in a fortress have provisions for 48 days, but 500 men join them; how long will the provisions now last?

13. 5000 men have provisions for 107 days, and after 17 days 500 of them go away; how much longer will the provisions now last?

14. An army of 12000 men was besieged in a fortress, and had provisions for 3 weeks; a certain number of men broke through the besieging force, and then the provisions lasted for 5 weeks; how many men broke through?

15. If with a capital of £1000 a tradesman gains £100 in seven months, what capital must he have in order to gain £60. 10*s.* in eleven months?

16. If a person with a capital of £750 gain £30 in 8 months, what should he gain in 5 months with a capital of £800?

17. The carpet which covers a room 25 ft. long and 21 ft. broad is worth £6. 11*s.* 3*d.*; what is the breadth of a room whose length is 29 ft., and the carpeting of which costs £10. 3*s.*?

18. 14 men can build in 6 days a wall 19 ft. long and 4 ft. high; how many men will be wanted to build a wall 38 ft. long and 3 ft. high in nine days?

19. In how many days can 25 men, working 12 hours a day, do a piece of work, which 18 men, working 10 hours a day, can do in 15 days?

20. If 6 men can do a piece of work in 30 days of 9 hours each, how many men will it take to do 10 times the amount, if they work 25 days of 8 hours each?

21. In how many days will the same amount of work be done by 75 men, working 8 hours a day, which 100 men will do in 9 days, working 10 hours a day?

22. If 24 men can do a piece of work in 4 days working 10 hours a day, in how many days can 20 men do a piece of work half as much again in quantity working 8 hours a day?

23. If five men dig a trench in 1⅓ days working 4½ hours a day, how long would one man take to dig a trench half as large again working 5 hours a day?

24. If 17 men can do a piece of work in 24 days of 10 hours each, in how many days of 8 hours can 18 men do the same piece?

25. If 100 men can make an embankment 6 miles long in 60 days, working 12 hours a day, how many hours a day must 80 men work in order to make an embankment 4 miles long in 48 days.

26. If 6 cwt. of parcels are carried 85 miles for 17*s.*, how much will the carriage of 15 tons 12 cwt. for 150 miles amount to at the same rate?

27. If the carriage of 17 cwt. 3 qrs. for $7\frac{1}{2}$ miles cost £1. 0*s.* $8\frac{1}{2}$*d.*, what weight, at the same rates, should be carried 20 miles for 16*s.* 4*d.*?

28. If £100 will pay for the railway travelling of 20 travellers for 5 days of 12 hours each, how long would it last 25 travellers, travelling 6 hours a day, the cost of travelling per hour being the same?

29. If 125 men can make an embankment 100 yds. long, 20 ft. wide, and 4 feet high in 4 days, working 12 hours a day, how many men must be employed to make an embankment 1000 yds. long, 16 ft. wide, and 6 ft. high in 3 days, working 10 hours a day?

30. If 12 horses are fed for 17 days at the cost of £11. 1*s.*, for how many days can 4 horses be fed for £11. 14*s.*?

31. If 5 acres of pasturage will support 10 oxen for $6\frac{1}{2}$ days, for how long will they support 26 oxen?

32. If 16 horses eat 12 pecks in 10 days, how long will 63 bushels last 56 horses?

33. If 7 bush. 2 pks. be consumed by 10 horses in 7 days, how many horses will consume 3 qrs. 6 bush. at the same rate in 10 days?

34. The cost of keeping 25 horses being at the rate of £11. 6*s.* $0\frac{1}{2}$*d.* per week, what will be the cost of keeping 13 horses from September 1st to October 31st?

35. A garrison of 1400 men has just food enough to allow 24 ounces of bread per day to each man for 36 days, but the garrison is increased by 400 men; how many ounces of bread must be allowed each man that they may be able to protract the siege for 20 days longer?

36. 7000 men besieged in a fort held out for 3 weeks on an allowance of 15 ounces a day; how long would the fort have held out if 2000 non-combatants had been sent away at the commencement of the siege, and the daily ration reduced to 9 ounces?

37. Four reapers begin to cut a field of barley, and it is found after working for 5 days that they have cut 10 acres. Two additional men are then employed, who work at the same rate, and the whole is cut in 3 days more; how many acres does the field contain?

38. A contractor engaged to finish 6 miles of railway in 200 days, but after employing 140 men for 60 days, he found that only $1\frac{1}{2}$ miles were completed; how many additional men must be engaged that the work may be finished within the given time?

39. If 15 horses and 148 sheep can be kept for 9 days for £75. 15*s.*, what sum will it cost to keep 10 horses and 132 sheep for 8 days, supposing 5 horses to eat as much as 84 sheep?

40. Each link of a surveyor's chain has stretched to one-tenth of an inch beyond its proper length; in what distance will the error amount to 50 feet?

41. A parish map is drawn on the scale of a foot to a mile; express as the decimal of a square inch the size on this map of a three-acre field.

42. The scale of a map is 6 inches to the mile; how many acres are there in an estate which occupies 50 square inches on the map?

CHAPTER IX.

PROPORTIONATE DIVISION.

106. Sometimes we are required to divide a given sum of money, or a given quantity of any commodity, into parts which are to one another in given ratios.

Such a division is called a proportionate division. All such divisions are more or less simple examples of proportion.

The following examples will illustrate the method of procedure:

Ex. 1. *Divide* £1342 *amongst three persons* A, B, *and* C, *so that their shares may be proportional to the numbers* 1, 3, *and* 7.

We have $1+3+7=11$.

Now for each portion that A has, B will have 3, and C will have 7.

Hence out of each 11 portions A will have 1, B will have 3, and C will have 7.

Therefore, if we divide the given sum into 11 parts, A will have 1 of them, B will have 3, and C will have 7.

$\therefore$ A's share $= \frac{1}{11}$ of given sum $= \quad £\frac{1}{11}\frac{3}{}\frac{4}{}\frac{2}{} = £122$

B's share $= \frac{3}{11}$,, $\quad = 3 \times £\frac{1}{11}\frac{3}{}\frac{4}{}\frac{2}{} = £366$

and C's share $= \frac{7}{11}$,, $\quad = 7 \times £\frac{1}{11}\frac{3}{}\frac{4}{}\frac{2}{} = £854$

£1342

Ex. 2. *Divide* £3. 18*s*. *among three persons* A, B, *and* C, *so that their shares may be in the ratios of the fractions* $\frac{1}{2}$, $\frac{1}{3}$, *and* $\frac{1}{4}$.

The L.C.M. of 2, 3, and 4 is 12.

These fractions are therefore equal to $\frac{6}{12}$, $\frac{4}{12}$, and $\frac{3}{12}$.

They are thus proportional to the numbers 6, 4, and 3.

Now $6+4+3=13$.

If then we divide the given sum of money into 13 equal parts, the shares will respectively be 6, 4, and 3 of such parts.

Thus A's share $= \frac{6}{13}$ of £3. 18*s*. $= 6 \times 6s. = £1.\ 16s.$

B's share $= \frac{4}{13}$ of £3. 18*s*. $= 4 \times 6s. = £1.\ 4s.$

and C's share $= \frac{3}{13}$ of £3. 18*s*. $= 3 \times 6s. =$ 18*s*.

£3. 18*s*.

Ex. 3. *A, B, C in an examination get* 1513 *marks between them; A gets* 4 *as often as B gets* 5, *and B gets* 6 *as often as C gets* 7. *How many did each get?*

Since A gets 4 while B gets 5, then A gets 4×6, *i.e.* 24,
while B gets 5×6, *i.e.* 30.
Since B gets 6 while C gets 7, then B gets 6×5, *i.e.* 30,
while C gets 7×5, *i.e.* 35.

$\therefore$ A gets 24, while B gets 30 and C gets 35. We have then to divide 1513 in ratio of 24 : 30 : 35. Now $24 + 30 + 35 = 89$.

$\therefore$ A's marks $= \frac{24}{89}$ of $1513 = 408$
B's marks $= \frac{30}{89}$ of $1513 = 510$
C's marks $= \frac{35}{89}$ of $1513 = 595$
1513

It should be noticed that we found the L.C.M. of the two numbers referring to B, and were thus able to express the proportion of A, B, and C in one statement.

EXAMPLES LIX.

Divide

1. 176 .....into three parts in the proportion of 2, 3, and 6.
2. 7072 ..... ,, five ,, ,, 1, 4, 6, 7, and 8.
3. £273 ..... ,, three ,, ,, 6, 7, and 8.
4. £20. 0*s.* 7*d.* ..... ,, three ,, ,, 3, 7, and 9.
5. £97. 18*s.* 8½*d.* ... ,, three ,, ,, 7, 9, and 13.
6. £62. 3*s.* 8*d.* ..... ,, four ,, ,, 4, 8, 5, and 11.
7. £352. 4*s.* 2*d.*..... ,, four ,, ,, 21, 37, 43, and 57.
8. £89. 3*s.* 10*d.*..... ,, four ,, ,, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$.
9. £280. 13*s.* 6½*d.*.... ,, four ,, ,, $1\frac{1}{2}$, $1\frac{1}{3}$, $1\frac{1}{4}$, and $1\frac{1}{5}$.
10. 1 cwt. 1 qr. 24 lbs. ,, four ,, ,, 11, 17, 23, and 31.
11. 5 tons 10 cwt. ... ,, three ,, ,, 4·5, 1·1, and 2·4.

12. 442 yds. 2 ft. 11 in. of wire into three parts in the proportion of $\frac{5}{6}$, $\frac{6}{7}$, and $\frac{7}{8}$.

13. Divide £4. 1*s.* 1*d.* between A, B, and C, so that A gets twice as much as B, and C one-half as much as B.

14. Divide £24. 8*s.* 7*d.* between A, B, and C, so that A gets $3\frac{1}{2}$ times as much as C, and B $2\frac{1}{3}$ times as much as C.

15. Divide £74. 5*s.* 9*d.* between A, B, and C, so that A gets $\frac{5}{7}$ of B's share, and C gets $\frac{3}{4}$ of what A gets.

16. In three class-rooms of a certain school are 222 boys; the numbers in the first and second are in the proportion of 3 : 5, and those in the second and third in the proportion of 7 : 11; find the number in each.

17. A body of 50,000 men is to be raised in four provinces which contain respectively 175,002, 82,434, 22,116, and 20,448 men fit for service; how many should each supply?

18. A, B, and C play cricket; A's runs are to B's and B's to C's as 3 : 2; they get between them 342 runs; how many does each get?

19. An examiner wishes to mark three questions in the proportion of the numbers 2, 3·2, and 4·8, the sum of the marks being 150; how must he distribute them?

20. Three men A, B, and C divide 169 cartridges among them in such a way that, as often as A takes 3 B takes 4, and as often as B takes 2 C takes 3; how many does each get?

21. Two little girls, aged 10 years and 8 years 9 months, have £10. 6*s*. 3*d*. in their common savings' box. If this be divided between them in proportion to their ages, what does each get?

22. Divide £945. 9*s*. between A, B, and C, so that A's share may be to B's share as 3 to 5, and B's to C's as 10 to 11.

23. There is in a city of five parishes a population of 207,318; three parishes have each 48075, and the fourth parish has twice as many as the fifth: find the number in the fifth.

24. Divide £10. 17*s*. 6*d*. between A, B, and C, so that for each sovereign given to A, B gets a guinea and C a half-crown.

25. Divide £157. 10*s*. among three persons, so that the first may have twice as much as the second, and the third half as much as the first and second together.

26. A man leaves £14,000 to be so divided among his three children that the eldest may have £1000 more than the second, and twice as much as the third; what is the share of each?

27. A man having three sons left £9656 to be divided between them in proportion to their ages at the time of his death; when he died their ages were 25, 22, and 21 years respectively: what was the share of each? Also, what would have been the difference in the share of each if the father had lived a year longer?

28. A train has first, second, and third class passengers in the proportion of 1 : 3 : 7; the total number is 572: find how many there are of each class.

29. *Divide* £7. 4*s*. *between* 3 *men*, 5 *women*, *and* 8 *boys*, *so that each man shall have as much as* 3 *women and each woman as much as* 2 *boys.*

1 woman's share = 2 boys' shares,
$\therefore$ 5 women's shares = 10 boys' shares;
and 1 man's share = 3 women's shares = 6 boys' shares,
$\therefore$ 3 men's shares = 18 boys' shares.

3 men, 5 women, and 8 boys have between them (18 + 10 + 8) or 36 boys' shares.

$\therefore$ 36 boys' shares = £7. 4*s*.
and each boy's share = $\frac{1}{36}$ of £7. 4*s*. = 4*s*.;
$\therefore$ each woman's share = 8*s*.
and each man's share = 24*s*.

30. *Three graziers* A, B, *and* C *rent a piece of land for one month.* A *puts in* 19 *bullocks for* 25 *days*, B 69 *bullocks for* 6 *days*, *and* C 153 *bullocks for* 3 *days. If the total expenses of the hiring be* £33. 14*s*., *how should this cost be divided between them?*

The cost of 1 bullock for 25 days = cost of 25 bullocks for 1 day;
$\therefore$,, 19 bullocks ,, = ,, 19×25 ,, ,,
= ,, 475 ,, ,,

So the cost of 69 bullocks for 6 days = cost of 69×6 bullocks for 1 day
= ,, 414 ,, ,,
and the cost of 153 bullocks for 3 days = cost of 153×3 bullocks for 1 day
= ,, 459 ,, ,,

Hence A, B, and C have had from the pasture what is equivalent to the feed of 475, 414, and 459 bullocks for 1 day.

The expenses should therefore be divided in the proportions of 475, 414, and 459. Now $475+414+459=1348$;

$\therefore$ A's share $= \frac{475}{1348} \times £33.\ 14s. = 475 \times 6d. = £11.\ 17s.\ 6d.$

B's share $= \frac{414}{1348} \times £33.\ 14s. = 414 \times 6d. = £10.\ 7s.\ 0d.$

and C's share $= \frac{459}{1348} \times £33.\ 14s. = 459 \times 6d. = £11.\ 9s.\ 6d.$

£33. 14s. 0d.

31. Divide £69. 7*s.* 6*d.* between 2 men, 4 women, and 6 boys, so that each man shall have as much as two women, and each woman as much as three boys.

32. 7 men, 8 women, and 11 children divide £71. 0*s.* 1½*d.* between them, so that each woman receives three times as much as a child, and each man as much as a woman and child together. How much does each man have?

33. A sum of money is to be divided between 11 men and 18 boys, and five men are to receive as much as nine boys. When three men and three boys have had their shares, what fraction of the whole remains?

34. A property is left as follows: One-fifth to A, and the rest to be divided among three others in the ratio of 1 : 2 : 3. If the least portion be £5375, what was the original property worth?

35. Two troops of cavalry hire a field in common for which they are to pay £23. 10*s.* One troop puts into the field 27 horses for 29 days; the other 19 horses for 33 days. How much should each pay?

36. A, B, and C rent a field jointly for £25. 12*s.* 5*d.* A puts in 15 sheep for 6 months, B 45 sheep for 8 months, and C 81 sheep for 4 months. What should they each pay towards the rent?

37. Two men rent a field for £40. The first, who pays £15, put in it 2 cows for every 5 that the other puts in. How long ought the latter to allow his cattle to graze, if the former has his in for 4 months?

38. A and B join in giving a picnic, each paying for the guests he invites. A invites fifteen and B ten. The whole cost is £6. 18*s.* 6½*d.* On the day, B pays for the carriages for the whole party at 2*s.* 6*d.* per head, and A pays all the other expenses. How must they afterwards settle accounts with one another?

39. Three persons A, B, and C agree to pay their hotel bill in the proportions 4 : 5 : 6. A pays the first day's bill which amounts to £2. 10*s.* 10*d.*, B the second which amounts to £3. 12*s.* 2*d.*, and C the third which amounts to £3. 17*s.* How must they settle accounts?

CHAPTER X.

PERCENTAGES.

107. Suppose we know that in a certain year the population of a certain town increased from 56350 at the commencement of the year to 58604 at the end of the year.

The actual increase $= 58604 - 56350 = 2254$; therefore 2254 is the increase on a population of 56350. What is the increase for each 100?

On 56350 there is an increase of 2254;

$\therefore$ „ 1 „ „ $\frac{2254}{56350}$;

$\therefore$ „ 100 „ „ $\frac{2254}{56350} \times 100$, *i.e.* 4.

Hence the increase is 4 for each 100 of its inhabitants at the commencement of the year.

This is expressed by saying that the increase was 4 per cent. [Per Cent. is an abbreviation for the Latin "Per Centum" and means "Per Hundred."]

The symbol % is often used as a further abbreviation for "per cent." Thus, if it were stated that, in a sovereign, $91\frac{2}{3}\%$ is pure gold, it is meant that $91\frac{2}{3}$ parts out of each 100 parts of the sovereign are pure gold.

The applications of the term "per cent." are very varied. Its meaning will be best apprehended from the examples given in this and subsequent chapters.

108. In some of the simple cases of percentage the student may find it convenient to remember the corresponding vulgar fractions. Thus

$2\% = \frac{2}{100} = \frac{1}{50}$, $4\% = \frac{4}{100} = \frac{1}{25}$, $5\% = \frac{5}{100} = \frac{1}{20}$,

$8\frac{1}{3}\% = \frac{8\frac{1}{3}}{100} = \frac{1}{12}$, $10\% = \frac{10}{100} = \frac{1}{10}$, $12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \frac{1}{8}$,

$16\frac{2}{3}\% = \frac{16\frac{2}{3}}{100} = \frac{1}{6}$, $33\frac{1}{3}\% = \frac{33\frac{1}{3}}{100} = \frac{1}{3}$, and $66\frac{2}{3}\% = \frac{66\frac{2}{3}}{100} = \frac{200}{300} = \frac{2}{3}$.

109. *To find the value of a given percentage of a given quantity.*

Ex. 1. *How much is* $7\frac{1}{2}$ *per cent. of* 500?

$7\frac{1}{2}$ per cent. of $500 = 7\frac{1}{2}$ hundredths of $500 = \frac{7\frac{1}{2}}{100} \times 500 = 7\frac{1}{2} \times 5 = 37\frac{1}{2}$.

Ex. 2. *A rent collector collects* £1245, *and is paid a commission at the rate of* 3 *per cent. on the amount he collects; what is his commission?*

For each 100 parts into which the rent can be divided he gets 3.

Thus he gets $\frac{3}{100}$ of the total rent.

Hence his commission $= \frac{3}{100} \times £1245 = £\frac{3\,7\,3\,5}{100} = £37\frac{7}{20} = £37.\ 7s.$

Ex. 3. *The population of Manchester at the beginning of the year* 1895 *was* 524,500, *and the number of births in that year was* 3·4 *per cent. of this population. Find the total number of births in Manchester in that year.*

For each 100 of the population the births were 3·4.

Hence the total number of births

$= \frac{3{\cdot}4}{100} \times$ total population $= \frac{3{\cdot}4}{100} \times 524500 = 3{\cdot}4 \times 5245 = 17833.$

In each case we see that *we multiply the given quantity by the rate per cent. and divide the result by* 100.

EXAMPLES LX.

How much is

1. 5 % of 1600 and of 2500?

2. 7 % of 850 and of 1350?

3. 6 % of 1843 and of 2839?

4. $7\frac{1}{2}$ % of 1500 and of 3750?

5. $5\frac{2}{5}$ % of 2050?

6. $3\frac{2}{3}$ % of 6387?

7. a commission of $2\frac{1}{2}$ % on (1) £240, (2) £375, (3) £682. 12*s.* 6*d.*?

8. 30 % of £153. 16*s.* 8*d.* and of £239. 15*s.* 10*d.*?

9. 7 % of £217. 14*s.* 2*d.* and of £889. 17*s.* 11*d.*?

10. $6\frac{2}{3}$ % of £20, of 30 yards, and of 15 acres?

11. 9 % of £30, of 150 yards, and of 35 tons?

12. 8 % of 7 tons 13 cwt. 3 qrs. 5 lbs.?

13. $5\frac{2}{3}$ % of 5 ac. 2 r. 25 sq. poles?

14. $12\frac{1}{2}$ % of £63. 5*s.* 8*d.* and of 5 days 4 hrs. 13 mins. 4 secs.?

15. $22\frac{1}{2}$ % of 5 tons 12 cwt. 16 lbs. and of 5 mi. 400 yds.?

16. $66\frac{2}{3}$ % of £75. 16*s.* 3*d.* and of 2 mi. 461 yds.?

17. $23\frac{3}{4}$ % of £54. 17*s.* $8\frac{3}{4}$*d.* to the nearest farthing?

18. A tank contained 2387 gallons, and lost 4 per cent. by leakage: how many gallons were left?

19. The population of a city was 1,000,000 and it increased by $2\frac{1}{2}$ per cent. in each of three successive years; what was the population at the end of the third year?

20. A tradesman allows 5 per cent. reduction for ready money; how much cash shall I have to pay for goods marked £5. 5*s.*?

21. An agent's commission of $3\frac{1}{2}$ % on a man's income of £850 is deducted before he receives it; how much does he actually get?

22. Goods which cost £111. 9*s.* 2*d.* are sold for 35 per cent. more than they cost; for how much are they sold?

23. Coal is sold at 16*s.* 8*d.* per ton, with an allowance of 5 % for ready money. How much cash do I pay for a load of coal weighing 6 tons 8 cwt.?

24. The population of a certain town increases every year by very nearly $7\frac{1}{2}$ per cent. If in April 1901 it was 704,000, what was it in April 1904?

25. At the beginning of a certain year the population of a town was 16,400, and the deaths during the year were 3 per cent. of the population, and the births were 3·75 per cent. What was the population at the end of the year, neglecting changes caused by removals?

26. A farmer's rent is £210, and his landlord lowers it by 15 per cent.; how much has he to pay?

27. What should be the yearly rent of a farm of 280 acres, worth £52 an acre, in order that the landlord may receive as rent 3 per cent. on the value of the farm?

28. Air is composed of 75·55 per cent. of nitrogen, 23·22 per cent. of oxygen, and 1·23 per cent. of carbonic acid. In a chamber containing 6548 cubic feet, how much is there of each gas?

29. Common salt contains sodium and chlorine in the ratio of 23 : 35·5. Find the weight of each in 468 lbs. of salt.

30. Saltpetre is composed of potassium, nitrogen, and oxygen in the ratio 39 : 14 : 48. Find the weight of each in 2727 tons of saltpetre.

110. *To find how much per cent. one quantity is of another.*

Ex. 1. *Find what per cent* 5 *is of* 16.

If x be the required rate per cent., then, as in Art. 109,

$$\frac{x}{100}\times 16 = 5,$$

$$i.e.\ x = \frac{5\times 100}{16} = \frac{125}{4} = 31\tfrac{1}{4}.$$

Ex. 2. *Standard silver is such that* 37 *parts out of every* 40 *are pure silver, and the remainder is inferior metal; what percentage of standard silver is pure silver?*

Out of every 40 parts 37 are pure silver.

$\therefore$,, ,, 1 part $\frac{37}{40}$ is ,, ,,

$\therefore$,, ,, 100 parts $\frac{37}{40}\times 100$ are pure silver.

Also $$\frac{37}{40}\times 100 = \frac{3700}{40} = 92{\cdot}5.$$

The answer is therefore 92·5 parts out of every hundred, that is, 92·5 per cent.

In each of these questions we see that the required answer is obtained *by taking the fraction whose numerator is the first quantity and whose denominator is the second quantity and multiplying the result by* 100.

EXAMPLES LXI.

What percentage are

1. 5 and 10 of 25? **2.** 8 and 15 of 24? **3.** $\frac{7}{8}$ and $\frac{5}{16}$ of 1?

4. £12 and £27 of £40? **5.** 2*s.* 6*d.* and 8*s.* 6*d.* of £1?

What percentage are

6. 3*s.* 4½*d.* and 15*s.* 7½*d.* of £1? **7.** 3*s.* and 5*s.* 6*d.* of 12*s.* 6*d.*?

8. 3*s.* 7½*d.* and 11*s.* 9¾*d.* of £5. 16*s.* 8*d.*? **9.** 24 lbs. of 2 cwt.?

What rates per cent. are equivalent to the fractions?

10. $\frac{1}{4}$, $\frac{3}{5}$, and $\frac{9}{10}$. **11.** $\frac{2}{3}$, $\frac{4}{9}$, and $\frac{7}{15}$. **12.** $\frac{3}{10}$, $\frac{7}{8}$, and $\frac{11}{16}$.

13. $1\frac{3}{5}$ and $2\frac{3}{4}$. **14.** ·5 and ·03.

15. A tank originally contained 3840 gallons, and 480 gallons leaked away; how much per cent. was lost by leakage?

16. A man owned 2451 acres and sold 827 of them: what percentage of his original estate has he left?

17. Out of 95 candidates for an examination 43 pass; what is the percentage of passes to two places of decimals?

18. In a village of 1000 people it is found that 515 are females; find the percentage of males.

19. At what rate % is an agent paid commission if he hand over to his employer £21,153. 16*s.* 6*d.* and retain £1715. 3*s.* 6*d.* for himself?

20. An arithmetic paper is worked by 2500 candidates, of whom one-fifth are girls and the rest boys; of the boys 5 per cent. fail, and of the girls 40 per cent.: find what percentage of the whole number of candidates succeed in passing.

21. The census return for London in 1881 was 4,766,661, and in 1891 it was 5,633,332; find the increase per cent.

22. The census returns for England and Wales in 1891 was 29,002,525, and in 1901 it was 32,527,843; find the increase per cent.

23. At what rate % is the deduction made when 19*s.* 10½*d.* is taken from an account of £39. 15*s.* in consideration of immediate payment?

24. An army of 120,000 men lost 15 per cent. by desertion to the enemy; what increase per cent. did the enemy gain, their numbers previously being 64,000?

25. In a school of 460 girls 320 passed an examination in reading, and in another of 112 girls only 65 passed; what percentage of the total number in the two schools passed?

26. In payment of a debt of £1864. 3*s.* 4*d.* a man accepts £1780. 5*s.* 7*d.*; at what rate per cent. is the deduction made?

27. An examination paper was answered by pupils of whom 40 per cent. were girls; 25 per cent. of the girls and 15 per cent. of the boys failed on the paper; what percentage of the whole number of candidates passed?

28. An order for books to the amount of £35. 15*s.* 4*d.* is subject to a reduction of 3*d.* in the shilling. What is the actual sum to be paid?

29. 122·5 gms. of chlorate of potash contain 39 gms. of potassium, 35·5 gms. of chlorine, and 48 gms. of oxygen. What percentage of the whole, to the second decimal place, does each form?

111. The following examples may be subjoined:

Ex. 1. *Of what sum is* £15 *equal to* $3\frac{3}{4}$ *per cent.?*

Here $\frac{3\frac{3}{4}}{100} \times$ required sum $=$ £15; $\therefore$ required sum $=$ £15 $\times \frac{100}{3\frac{3}{4}}$;

$\therefore$ required answer $=$ £15 $\times \frac{4\,0\,0}{1\,5} =$ £400.

Ex. 2. *A man gave away* $17\frac{1}{2}$ *per cent. of his income, and had* £736. 6*s.* 3*d. left; what was his income and how much did he give away?*

He gave away $\frac{17\frac{1}{2}}{100}$, that is, $\frac{35}{200}$, that is, $\frac{7}{40}$ of his income.

Thus he had left $1-\frac{7}{40}$, that is, $\frac{33}{40}$, of his income;

$\therefore$ $\frac{33}{40}$ of his income = £736. 6*s.* 3*d.*;

$\therefore$ his income = $\frac{40}{33}\times$ £736. 6*s.* 3*d.* = 40 × £22. 6*s.* 3*d.* = £892. 10*s.*;

$\therefore$ amount he gave away = £892. 10*s.* − £736. 6*s.* 3*d.* = £156. 3*s.* 9*d.*

Ex. 3. *The population of a town is* 4 *per cent. more than it was* 3 *years ago. It is now* 26312: *what was it then?*

Since the increase is 4 per 100,

there are now 104 persons instead of 100,

,, 1 ,, ,, $\frac{100}{104}$,

,, 26312 ,, ,, $\frac{100}{104}\times 26312$.

$\therefore$ *Ans.* = 25300.

Ex. 4. 5 % *of an army are killed in battle,* 10 % *of the rest die of their wounds, and* 3 % *of the survivors die from other causes, and then there are* 16,587 *left; what was the original strength of the army?*

5 per cent. die, so that 95 per cent., that is, $\frac{95}{100}$, that is, $\frac{19}{20}$, survive the battle.

Of these 10 per cent. die, and therefore 90 per cent. are left.

Hence the number that do not die from the battle = $\frac{90}{100}\times\frac{19}{20}$ of the original strength.

Of these 3 per cent. die, and thus $\frac{97}{100}$ are left;

$\therefore$ final number of survivors = $\frac{97}{100}\times\frac{90}{100}\times\frac{19}{20}$ of original number

= $\frac{165587}{200000}$ of original number;

$\therefore$ $\frac{165587}{200000}$ of original number = 16587, so that original strength = 20000.

MISCELLANEOUS EXAMPLES LXII.

Percentages.

1. Of what sum of money is £3 equal to 25 per cent.?

2. Of what sum is £36 equal to 15 per cent.?

3. Of what number is 72 equal to 3·6 per cent.?

4. Of what number is 27 equal to $2\frac{1}{2}$ per cent.?

5. Of what sums of money are £13. 6*s.* 8*d.* and £15 respectively equal to $12\frac{1}{2}$ per cent.?

6. Of what number is 1·385 equal to 5 per cent.?

7. Of what weight is 13 lbs. 9 oz. equal to 7 per cent.?

8. Of what length is 3 miles 400 yds. equal to 71 per cent.?

9. If $2\frac{3}{4}$ per cent. of the value of an estate be £1001, what is this value?

10. The population of a town is 3 per cent. more than it was two years ago, and now it is 14,111 ; what was it then ?

11. A man gives away 15 per cent. of his income, and has £901 left ; how much did he give away ?

12. In a campaign 18 per cent. of the soldiers die, and 9102 are left ; what was the original number of soldiers ?

13. A farmer loses 72 sheep ; if this number be $16\frac{2}{3}$ per cent. of his original flock, how many has he left ?

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14. Between the years 1841 and 1851 the population of England increased 14·1 per cent. In the latter year it was 21,121,290 ; what was it in the former year ?

15. The population of a town increased by 8·62 per cent. from 1891 to 1901, and in the latter year it was 45,637 ; what was it in 1891 ?

16. The population of a town increased, from 1891 to 1901, by 5·45 per cent., and this increase was 3462 people ; find its population in both 1891 and 1901.

17. A man estimates his expenses as follows : rent of house, one-tenth of his income ; expenses of living, 80 % of the remainder. He finds he will then save £126 per annum. What was his total income ?

18. In an examination *A* obtains 48 % of full marks, and *B* obtains 33 %. Together they get 567 marks. What were full marks ?

19. Seventy-five per cent. of the area of a farm is arable ; of the remainder 85 per cent. is pasture and the rest is waste—the area of the waste being 3 ac. 0 r. 20 sq. po. : what is the area of the farm ?

20. An army lost $\frac{2}{17}$ths of its numbers by sickness and other causes, and then lost 15 per cent. of its remaining strength in a battle ; if now there are 17,901 men left, what was the original strength ?

21. At an examination in which full marks were 1000, *A* got 20 % more than *B* ; *B* 10 % more than *C*. If *A* obtains 660 marks, what percentage of full marks did *C* get ?

22. *A* makes an article and sells it to *B* at a profit of 20 per cent., and *B* sells it to *C* at a profit of 25 per cent. *C* paid 25*s.* for the article. What did it cost ?

23. A publisher buys books of the author thus : he reckons a shilling as $8\frac{1}{2}d.$, pays for 25 copies as 24, and then deducts 10 per cent. : what percentage of the nominal price does the author get ?

24. From a sum of money $7\frac{1}{2}$ per cent. is deducted, and then 5 per cent. of the remainder is also deducted ; if the sum now left be £228. 9*s.* 6*d.*, what was the original sum ?

25. The cost of repairs of a house amounts to 15 per cent. of the rent paid, and the amount the landlord has left after this cost is paid is £76. 10*s.* ; how much does his tenant pay ?

26. If the price of candles $8\frac{1}{2}$ inches long be 9*d.* per half-dozen, and that of candles of the same thickness and quality, but $10\frac{1}{2}$ inches long, be 11*d.* per half-dozen, what kind do you advise a person to buy, and what would be the saving per cent. if your advice be followed ?

CHAPTER XI.

PROFIT AND LOSS.

112. When a merchant, or tradesman, enters into business he hopes to sell the goods in which he deals at a price higher than that at which he bought them.

If the price at which he sells the goods—called the Sale Price—be higher than the price at which he bought the goods —called the Cost Price—together with his expenses, then he makes a Profit.

The profit therefore = Total Sale Price of the goods less the total of the Cost Price of the goods and the expenses connected with their sale, or more shortly,

Profit = Sale Price − Total Cost Price.

It must be carefully noted that unless otherwise stated, the profit is reckoned as so much per cent. on the outlay (that is, the total cost price) and *not* on the Sale Price.

113. *Given the Cost Price and the Sale Price, to find the Profit or the Profit per cent.*

Ex. 1. *If* 127 *lbs. of tea are bought at* 1*s.* 3½*d. per lb. and sold at* 1*s.* 9*d.*, *what is the profit?*

Here and
Sale Price = 127 × 1*s.* 9*d.* = £11. 2*s.* 3*d.*
Cost Price = 127 × 1*s.* 3½*d.* = £8. 4*s.* 0½*d.*
∴ Profit = £2. 18*s.* 2½*d.*

Otherwise thus: the profit on *each* pound sold = 1*s.* 9*d.* − 1*s.* 3½*d.* = 5½*d.*;
∴ total profit = $127 \times 5\frac{1}{2} = 127 \times \frac{11}{2}d.$
$= \frac{1397}{2}d. = 698\frac{1}{2}d. = 58s.\ 2\frac{1}{2}d.$
= £2. 18*s.* 2½*d.*

Ex. 2. *Goods bought at* £48. 7*s.* 6*d. per ton are sold at* £43 *per ton; find the loss per cent.*

£48. 7*s.* 6*d.* − £43 = £5. 7*s.* 6*d.*

Hence on an outlay of £48. 7*s.* 6*d.* (= £$48\frac{3}{8}$) there is a loss of £5. 7*s.* 6*d.* (= £$5\frac{3}{8}$). We want to know what would be the loss on £100.

On £$48\frac{3}{8}$ there is a loss of £$5\frac{3}{8}$.

∴ ,, £1 ,, ,, £$\frac{5\frac{3}{8}}{48\frac{3}{8}}$;

∴ ,, £100 ,, ,, £$\frac{5\frac{3}{8}}{48\frac{3}{8}} \times 100$, *i.e.* £$11\frac{1}{9}$.

EXAMPLES LXIII.

1. A grocer buys 572 lbs. of tea at 2*s.* 2¼*d.* per lb., and sells them at 2*s.* 6*d.* a pound; how much does he gain?

2. A dealer buys 15 horses at £28. 13*s.* each, and sells 8 of them at £35, and the rest at £42. 10*s.* each; find his total gain.

3. I buy 45 articles at 1*s.* 6½*d.* each, and 55 others at 1*s.* 10½*d.* each. If I sell them all at 1*s.* 9*d.* each, how much do I gain or lose?

4. A man buys 520 books at 3*s.* 6*d.* each, and sells half of them at 3*s.* 7½*d.* each, and the other half at 3*s.* 2½*d.* each; how much does he gain or lose?

5. A man bought 1763 yards of cloth at 5*s.* 3½*d.* per yard, and sold it at 6*s.* 11*d.* per yard; what was his profit?

6. A man buys 497 sacks of potatoes for £339, and sells 248 sacks for 17*s.* 9*d.* per sack, and the remainder for 18*s.* 3*d.* per sack; find his gain.

7. A farmer pays 30*s.* an acre rent on a farm of 215 acres. His working expenses in addition amount to £589. 17*s.* 4¾*d.*, and his receipts are £1305. 6*s.* 8*d.*; how much does he gain by his year's farming.

8. If a plot of land is bought for £3500, and a man who owns two-fifths of it sells half his share for £800, how much does he gain by the transaction?

9. If a cask of wine containing 84 gallons cost £112. 5*s.*, what is gained by selling it at 31*s.* 6*d.* a gallon?

10. A grocer buys a chest of tea containing 180 lbs. at 1*s.* 2½*d.* per lb. If 10 lbs. be spoilt, what profit does he make by selling the remainder at 1*s.* 9*d.* per lb.?

11. The cost of 1500 copies of a certain book was £202. 9*s.* 1*d.*; the publisher allows the booksellers 30 % off the nominal price, 7*s.* 6*d.*, gives them one extra on every 24 copies, and finally takes 5 % off their bill for cash payment. What did the profits amount to?

Find the profit, or loss, per cent. when the cost price and sale price are respectively

12. 12*s.* and 15*s.* **13.** 5*s.* and 12*s.* **14.** 6*s.* and 5*s.* **15.** 12*s.* and 10*s.* 6*d.*

16. £5. 10*s.* and £6. 15*s.* **17.** £120 and £100. **18.** £100 and £80.

19. £1. 2*s.* 6*d.* and £1. 10*s.* 6*d.* **20.** 15*s.* 9*d.* and 17*s.* 6*d.*

21. £1. 10*s.* 6*d.* and £2. 10*s.* 6*d.* **22.** £4612. 10*s.* and £4976. 7*s.* 6*d.*

23. £15. 6*s.* 3*d.* and £11. 15*s.* 9¾*d.*

24. 27*s.* 6*d.* per cwt. and £25 per ton.

25. 6½*d.* per lb. and £4. 10*s.* per cwt.

26. £1. 12*s.* per cwt. and 4½*d.* per lb.

27. 24*s.* per ton and 1*s.* 6*d.* per cwt.

28. 17*s.* 4*d.* per cwt. and 2½*d.* per lb.

29. £5. 5*s.* per cwt. and 6½*d.* per lb.

30. £28 per ton and 4*d.* per lb.

31. £24 per cwt. and £2000 for 3 tons 13 cwt. 1 qr.

32. By selling a quantity of tea for £100 I gain £25; what is my profit per cent.?

33. By selling tea for £100 I lose £25; what is my loss per cent.?

34. I buy 48 horses at £25 each, and sell 14 at £20, 10 at £22 10*s.*, 9 at £32, and 15 at £45. If my expenses are £1 per horse, how much per cent. do I gain?

35. A draper bought 600 yards of silk at 3*s.* 4½*d.* per yard, and having sold 360 yards at 4*s.* 6*d.* per yard, and 81 yards at 3*s.* 9*d.*, he was robbed of the rest; find his gain or loss per cent.

36. *What does a person gain or lose per cent. by selling butter at 8½d. per lb. which cost him £5. 5s. per cwt.?*

1 cwt. = 112 lbs., so that selling price of 1 cwt. = $112 \times 8\frac{1}{2}d. = 952$ pence.

But cost price of 1 cwt. = £5. 5*s.* = 105*s.* = 1260 pence.

Also $1260 - 952 = 308$.

Hence on an outlay of 1260 pence he loses 308 pence.

$\therefore$ on 1*d.* he loses $\frac{308}{1260}d.$

$\therefore$ on 100*d.* he loses $\frac{308}{1260} \times 100d.$, *i.e.* $24\frac{4}{9}$.

His loss is therefore $24\frac{4}{9}$ per cent.

37. A merchant buys a 50-gallon cask of wine for £62. 10*s.*, and sells it at 26*s.* per gallon; what is his gain per cent.?

38. A refiner buys sugar at the rate of £12. 10*s.* a ton. The cost of refining is 10*s.* 10*d.* per cwt. He sells the refined sugar at the rate of $2\frac{3}{4}d.$ per lb. How much per cent. does he gain?

39. A druggist buys a certain drug at 3*d.* per oz. Avoirdupois, and sells at 6*d.* per oz. Troy; what is his gain per cent.?

40. A grocer buys sugar at £1. 5*s.* 4*d.* per cwt., and sells it at 5*d.* per lb., taking off 5 per cent. from his bills for cash payments; what percentage of profit does he make?

41. Find the value of 353 dozens of plants at £3. 2*s.* 6*d.* per gross (of 12 dozens). If 150 dozens be sold for 5*s.* per dozen, 180 dozens at 9*s.* 3*d.* per dozen, and the remainder be spoilt, what is the gain or loss?

42. A person buys coffee at £5. 12*s.* 6*d.* per cwt., and chicory at £2. 5*s.* 5*d.*, and mixes them in the proportion of 2 of chicory to 5 of coffee. If he retails the mixture at 1*s.* 3*d.* a pound, what is his gain per cent.?

43. If 57 lbs. of tea bought at 1*s.* 9*d.* per lb. be mixed with 9 lbs. bought at 3*s.* 7*d.* per lb., what is the cost of the mixture per lb.? And if the whole be sold at 2*s.* 3*d.* per lb., what is the gain per cent.?

44. If 12 gallons of wine are bought at 84*s.* a gallon, and 60 gallons are bought at 48*s.*, and the whole when mixed sold at 72*s.*, find the profit per cent.

45. *If eggs are bought at* 15 *for* 1*s. and sold at* 10 *for* 9*d.*, *what is the gain per cent. on the outlay?*

10 eggs are sold for 9*d.*, so that 1 egg is sold for $\frac{9}{10}d.$

$\therefore$ 15 eggs are sold for $15 \times \frac{9}{10}d.$, that is, for $13\frac{1}{2}d.$

$\therefore$ the buying price of 15 eggs is 12*d.*, and the selling price is $13\frac{1}{2}d.$

$\therefore$ profit on 1*s.* outlay $= 13\frac{1}{2}d. - 12d. = 1\frac{1}{2}d.$
$\therefore$ on an outlay of 100*s.* the profit is $100 \times 1\frac{1}{2}d.$,
that is, $150d.$, that is, $12\frac{1}{2}s.$

The gain is therefore $12\frac{1}{2}$ per cent.

It should be noticed that we may consider *the cost price and the sale price of any other number of eggs* and we shall get the same result; *e.g.* 30 eggs would cost 24*d.* and would sell for 27*d.*;

$\therefore$ profit on $24d. = 3d.$, giving us $12\frac{1}{2}\%$ as before.

46. A man buys eggs at 1*s.* 3*d.* per dozen and sells them at 11*s.* 8*d.* per hundred; find his gain per cent.

47. A man buys eggs at 10*d.* per dozen and sells them at the rate of 100 for 7*s.* $9\frac{3}{4}d.$; find his gain per cent.

48. A woman buys eggs at 4 for 3*d.* and sells them at 7 for 6*d.*; what does she gain per cent.?

49. If oranges are bought at 11 for 10*d.* and sold at 10 for 11*d.*, what is the gain per cent.?

50. A schoolboy buys oranges at the rate of 3 for 2*d.*, and sells them at the rate of 2 for $1\frac{1}{2}d.$; what does he gain or lose per cent.? If the buying and selling prices were reversed, what would the percentage of loss or gain be?

51. A costermonger sells his cabbages so as to get for four what he paid for five; at what rate per cent. is his profit?

52. A woman buys a certain number of eggs at 21 a shilling and the same number at 19 a shilling; she mixes them together, and sells them at 20 a shilling; what does she gain or lose per cent. by the transaction?

53. If to every quart of milk a milkman adds half a pint of water, and if he sells the mixture at the same price per quart that he gave for the milk alone, what is his profit per cent.?

54. How much is gained or lost per cent. by buying a number of oranges at five for twopence, and selling half of them at two a penny and half at three a penny?

55. A woman buys eggs at 14 for a shilling, and an equal number at 18 for a shilling; she mixes them and sells them at 15 for a shilling: how much does she gain or lose per cent.?

114. *Given the Cost Price and the total gain, or gain per cent., to find the Sale Price.*

It must be noted that, if the gain per cent. is given, it means the gain per cent., *calculated on the cost price*, that is, on the sum of money laid out. This cost price includes not only the actual price paid for the goods, but also any expenses that may be incurred in dealing with the goods.

Ex. 1. *A horse is bought for* 50 *guineas and sold at a profit of* 20 *per cent.; what is this profit, and for how much is the horse sold?*

Profit $= \frac{20}{100} \times$ cost price $= \frac{20}{100} \times 50$ guineas $= 10$ guineas;
$\therefore$ sale price $= 50$ guineas $+ 10$ guineas $= 60$ guineas.

Ex. 2. *A dealer bought a ton of sugar for* £23. 6*s.* 8*d.*; *at what price per lb. must he sell it to gain* 10%?

1 ton, or 2240 lbs., cost £23. 6*s.* 8*d.*,

$\therefore$ 1 lb. cost $\frac{1}{2240}$ of £23. 6*s.* 8*d.*, *i.e.* $2\frac{1}{2}d.$

Now what cost 100*d.* must be sold for 110*d.* to gain 10%,

$\therefore$,, 1*d.* ,, ,, $\frac{110}{100}d.$,,

,, $2\frac{1}{2}d.$,, ,, $\frac{110}{100} \times 2\frac{1}{2}d.$,, *i.e.* $2\frac{3}{4}d.$ *Ans.*

EXAMPLES LXIV.

Find the Sale Price in the following cases:

1. Cost price, 15*s.*; gain, 10%. **2.** Cost price, £10; loss, 5%.

3. Cost price, £2. 12*s.* 6*d.*; loss 5 per cent.

4. Cost price, 2*s.* $8\frac{1}{2}d.$ per yard; gain 15 per cent.

5. Cost price, £97. 4*s.*; gain $5\frac{5}{9}$ per cent.

6. If I buy bay at £4. 8*s.* per ton, at what price do I sell it if I lose $12\frac{1}{2}$ per cent.?

7. I buy a horse for £1500, and sell it again at a profit of 25 per cent.; how much do I receive if the expenses of the sale are 5 per cent. on the sale price?

8. A table costs £5. 10*s.* to make, and the expenses of sale, etc., are 30 per cent. upon this; if the profit is $27\frac{3}{11}$ per cent. on the total outlay, for how much is it sold?

9. A man buys copper at £35 per ton; at what price per lb. must he sell it so as to gain 20 per cent.?

10. A grocer buys a cwt. of tea for £9. 16*s.*; if 4 lbs. of it are spoilt, at what price per lb. must he sell the rest to gain £3. 14*s.* on the whole?

11. A dairyman bought 63 gallons of milk at 2*s.* per gallon, but 15 gallons were lost by leakage; at what price per gallon must he sell the remainder so as to gain 25 per cent. on the whole prime cost?

12. A person buys a quantity of wine at 16*s.* per gallon; 20 per cent. of it is loss: find at what price per gallon he must sell the rest so as to gain 20 per cent. on his outlay.

13. A grocer mixes 12 lbs. of tea at 1*s.* 6*d.* per lb. with 2 lbs. of tea at 2*s.* 8*d.* per lb.; at what price per lb. must he sell the mixture in order that he may gain 25 per cent.?

14. A tradesman forms a mixture of tea by adding 1 lb. at 3*s.* and $\frac{1}{2}$ lb. at 4*s.* 6*d.* to every 2 lbs. at 2*s.*; at what price must he sell it per lb. in order to gain 10 per cent.?

15. An innkeeper buys 10 gallons of spirits at 12*s.* per gallon, 15 at 4*s.* 6*d.*, and 18 at 5*s.* 9*d.*; how must he sell the mixture per gallon that he may gain £2. 2*s.* 3*d.* on his outlay?

16. A grocer buys coffee at the rate of £8. 10*s.* per cwt. and chicory at £2. 10*s.* per cwt., and mixes them in the proportion of 5 parts chicory to 7 coffee; at what rate per lb. must he sell the mixture so as to gain $£16\frac{2}{3}$ per cent. on his outlay?

115. *Given the Sale Price and the gain, or the gain per cent., to find the Cost Price.*

Ex. *What was the cost price of an article which was sold for* £1. 4*s.* 6*d.*, *and on which there was a profit of* $16\frac{2}{3}$ *per cent.?*

Since there was $16\frac{2}{3}$ % profit,

we sell for $116\frac{2}{3}$ what cost 100;

$\therefore$,, ,, 1 ,, ,, $\frac{100}{116\frac{2}{3}}$;

$\therefore$,, ,, $1 \times$ £1. 4*s.* 6*d.* ,, ,, $\frac{100}{116\frac{2}{3}} \times$ £1. 4*s.* 6*d.*

$\therefore$ *Ans.* $= \frac{100}{116\frac{2}{3}} \times$ £1. 4*s.* 6*d.* $= \frac{6}{7} \times$ £1. 4*s.* 6*d.* = £1. 1*s.*

EXAMPLES LXV.

Find the cost price in the following cases:

1. Sale price, £125; profit, £47. 10*s.* 6*d.*

2. Sale price, £372. 6*s.* 5*d.*; profit, £53. 17*s.* 8*d.*

3. Sale price, £90; profit, 20%. **4.** Sale price, £35; loss, $16\frac{2}{3}$%.

5. Sale price, £35; gain, $16\frac{2}{3}$%.

6. Sale price, £11. 11*s.* 3*d.*; profit, 11%.

7. Sale price, £150. 10*s.*; profit, $9\frac{1}{11}$%.

8. Sale price, 3*s.* 7*d.* per lb.; profit, $7\frac{1}{2}$%.

9. Sale price, £45; loss, 10%. **10.** Sale price, £18. 13*s.*; loss, $6\frac{3}{4}$%.

11. Sale price, $3\frac{3}{4}$*d.* per lb.; loss, 10%.

12. Sale price, 4*s.* $7\frac{1}{2}$*d.* per lb.; gain, 11%.

13. Sale price, 18*s.* 3*d.* per cwt.; loss, 27%.

14. What is the cost of coal per ton if the sale of 15 cwt. for 21*s.* gives a profit of $16\frac{2}{3}$ per cent.?

15. If 5 per cent. profit be made by selling 125 yards of cloth for £95, what was the cost price per yard?

16. A plumber sold 96 cwt. of lead for £107. 12*s.* 6*d.* and gained $2\frac{1}{2}$ per cent.; what did it cost him per cwt.?

17. A man sells 10 cwt. of sugar at $2\frac{3}{4}$*d.* a pound and thus gains 11*s.* 8*d.*; what is his profit per cent.?

18. A dealer bought 120 oxen and expected to make £252 profit by selling them at 12 guineas each, but 15 of them died; to what rate per cent. were his profits reduced?

19. A grocer sells tea at 2*s.* 1*d.* per lb. and takes off 5% for cash payment; find what it cost him per cwt. if he make a profit of 33%.

20. *A makes an article and sells it to B, B sells it to C, and C sells it to D for* 35*s.* *A gains* 20 *per cent., B* 25 *per cent., and C* 40 *per cent. on the money they respectively lay out. What did it cost A?*

What cost A 100 he sells for 120.

$\therefore$ A's selling price, that is, B's buying price $= \frac{120}{100} \times A$'s cost price.

So B's selling price, that is, C's buying price

$= \frac{125}{100} \times B$'s buying price $= \frac{125}{100} \times \frac{120}{100} \times A$'s cost price.

So $35s.$ = C's selling price $= \frac{140}{100} \times C$'s buying price

$= \frac{\overset{7}{\cancel{140}}}{\underset{5}{\cancel{100}}} \times \frac{\overset{5}{\cancel{125}}}{\underset{4}{\cancel{100}}} \times \frac{\overset{6}{\cancel{120}}}{\underset{5}{\cancel{100}}} \times A$'s cost price $= \frac{21}{10} \times A$'s cost price ;

$\therefore$ A's cost price $= \frac{10}{\underset{3}{\cancel{21}}} \times 35s. = \frac{50}{3}s. = 16\frac{2}{3}s. = 16s.\ 8d.$

21. If the tradesman's profit is 25 per cent. and the manufacturer's 16 per cent., what is the cost to the manufacturer of an article for which the consumer pays £9. 13*s.* 4*d.*?

22. Goods pass successively through the hands of three dealers, each of whom in selling adds as profit 10 per cent. of the price he paid; if the third dealer sells the goods for £33. 5*s.* 6*d.*, how much did the first dealer pay?

23. A sells to B at a profit of 5 per cent., B sells to C at a profit of 10 per cent., and C to D at a profit of 5 per cent.; if D bought the goods for £404. 5*s.*, what did they cost A?

24. The wholesale and retail profits on an article which was sold to a customer for 6*s.* $1\frac{1}{2}d.$ were 5 per cent. and $16\frac{2}{3}$ per cent. respectively; what was the manufacturing price?

116. In the foregoing articles of this chapter the three fundamental problems of Profit and Loss have been discussed. We now give a set of Miscellaneous Examples, in which specimens of the more typical kinds are worked out.

MISCELLANEOUS EXAMPLES LXVI.

Profit and Loss.

1. *A man sold a field for* £138. 12*s. and by so doing lost* $5\frac{1}{2}$ *per cent. on what it cost him; find what per cent. he would have gained had he sold it for* £184. 5*s.*

Since he lost $5\frac{1}{2}\%$ or $\frac{5\frac{1}{2}}{100}$ of its cost, he sold it for $\frac{94\frac{1}{2}}{100}$ of its cost;

$\therefore$ $\frac{94\frac{1}{2}}{100}$ of cost $= £138\frac{3}{5}$, *i.e.* cost $= £138\frac{3}{5} \times \frac{100}{94\frac{1}{2}} = £146.\ 13s.\ 4d$

Had he sold the field for £184. 5*s.*, the gain would have been

£184. 5*s.* − £146. 13*s.* 4*d.*, that is, £37. 11*s.* 8*d.*, that is, $£37\frac{7}{12}$.

On an outlay of $£146\frac{2}{3}$ the gain is $£37\frac{7}{12}$;

,, £1 ,, $£\frac{37\frac{7}{12}}{146\frac{2}{3}}$;

,, £100 ,, $£\frac{37\frac{7}{12}}{146\frac{2}{3}} \times 100$;

$\therefore$ *Ans.* $= \frac{£\frac{451}{12}}{£\frac{440}{3}} \times 100 = £\frac{\overset{41}{\cancel{451}}}{\underset{4}{\cancel{12}}} \times \frac{3}{\underset{40}{\cancel{440}}} \times 100 = £\frac{410}{16} = £25\frac{5}{8}$ per cent.

2. If a grocer gains 10% by selling tea at 2*s.* 3*d.* per lb., what percentage will he gain by selling it at 2*s.* 9*d.*?

3. By selling an article at 22*s.* 6*d.* I lose 10%; what should I gain or lose per cent. by selling it at 25*s.*?

4. Goods are sold at £39 so as to gain 17%; what would be the gain per cent. if they were sold for £36?

5. By selling goods at £1400 a merchant gains 12%; find the cost price and the gain per cent. if he had sold for £1500.

6. If by selling a horse for £33. 16*s.* I gain 4%, find what percentage I should have lost by selling at £32. 3*s.* 6*d.*

7. By selling a carriage at £29. 18*s.* 9*d.* a man lost one-sixth of what it cost him; how much would he have lost or gained by selling it at £36?

8. A man was doubting whether to sell a horse at a price which would have made him lose $8\frac{1}{3}$ per cent. when he gets another bid of 218 guineas, and thereby cleared 9 per cent.; what was the first offer made?

9. *By selling a carriage for* £73. 3*s. I should lose* 5 *per cent.; at what price must I sell to gain* 15 *per cent.?*

What cost him 100 he sold for 100 – 5, that is, 95. If he had made a profit of 15 per cent. then what cost him 100 should have been sold for 115. Hence what he sold for 95 he should have sold for 115;

$\therefore$,, £1 ,, ,, $£\frac{115}{95}$;

$\therefore$,, $£73\frac{3}{20}$,, ,, $£\frac{115}{95} \times 73\frac{3}{20}$,

i.e. for £88. 11*s.*

10. If a man sell a watch for £19, and by so doing lose 5%, at what price should he sell it in order to gain 25%?

11. If $6\frac{1}{2}$% be gained by selling a horse for £85. 4*s.*, at what price must it be sold to gain 15%?

12. If 4% be lost by selling silk at 10*s.* per yard, at what price per yard should it be sold to gain 5%?

13. A grocer by selling sugar at $4\frac{1}{2}d.$ per lb. loses 10%; at what price per cwt. must he sell to gain 15%?

14. By selling goods at 4*s.* 4*d.* per lb. 9% is lost; what advance must be made in the selling price to gain 5%?

15. A grocer having 180 lbs. of tea sold 60 lbs. at 4*s.* 3*d.* per lb., and gained thereby $6\frac{1}{4}$ per cent.; at what price must he sell the rest so as to gain 10 per cent. on the whole?

16. I have two horses of equal value: by selling one of them for £127. 10*s.* I find I lose 15 per cent.; for what sum must I sell the remaining one in order to gain 8 per cent. on the whole transaction?

17. *A man A sold a watch at a loss of* 5 *per cent.; had he sold it for* £3. 15*s. more, he would have gained* 10 *per cent.: what did the watch cost him? Also what was the cost of manufacture if the manufacturer sold the watch to A at a profit of* 25 *per cent.?*

Here an addition of £3. 15*s.* to the price makes up for the 5 per cent. loss, and gives also a profit of 10 per cent.

$\therefore$ 15% of the cost to A, *i.e.* $\frac{15}{100}$ of the cost to $A = £3\frac{3}{4}$.

$\therefore$ cost to $A = £3\frac{3}{4} \times \frac{100}{15} = £25$.

Again, since the manufacturer gained 25 %,
he sold at £125 what cost him £100,
∴ ,, £25 ,, ,, £20.
∴ *Ans.* = £20.

18. A person sold a horse at a loss of 20 %; if he had received £10 more for it, he would have gained 10 %: find the cost of the horse.

19. By selling goods for a certain sum a man gains 5 per cent.; if he had sold them for 3 shillings more he would have gained 6 per cent.: find their cost price.

20. If 3 per cent. more be gained by selling a horse for £83. 5*s.* than by selling him for £81, how much did the horse cost?

21. What is the cost of wheat when an additional profit of 5 per cent. raises the price by 1*s.* 2*d.* per quarter?

22. *A man buys oranges at sixpence a dozen and an equal number at ninepence a score; he sells them at ninepence a dozen and thus makes a profit of* 6*s.* 5*d.: how many oranges did he buy?*

Let us suppose that he bought 60 oranges of each kind.

The cost of 60 oranges at sixpence a dozen $= \frac{60}{12} \times 6\text{d.} = 30d.$

The cost of 60 oranges at ninepence a score $= \frac{60}{20} \times 9d. = 27d.$

∴ the total cost of the 120 oranges $= 30d. + 27d. = 57d.$

He sells these 120 oranges for $\frac{120}{12} \times 9d.$; that is, 90*d.*

∴ 90*d.* − 57*d.*, *i.e.* 33*d.*, is the profit on 120 oranges,
∴ 1*d.*, ,, ,, $\frac{120}{33}$,,
∴ 77*d.*, ,, ,, $\frac{120}{33} \times 77$, *i.e.* 280 oranges.

We might have started with any number of oranges other than 60. But 60 was the most convenient number; for we thus avoided fractions, since 60 is a whole number of dozens and also a whole number of scores. In fact, 60 is the L.C.M. of 12 and 20.

23. Eggs are bought at three for twopence, and sold at four for threepence; how many must be sold to realize a profit of two shillings?

24. Oranges were bought at 5*d.* a dozen, and sold at 6 for 3*d.*; if the profit on them was £1. 0*s.* 7½*d.*, how many oranges did I buy?

25. A woman buys pears at 11*d.* a dozen, and sells them at 13 for 1*s.*; how many must she sell to gain £1?

26. If a tradesman marks his goods 15 per cent. above the cash price, what will he take for goods marked 28*s.* 9*d.*?

27. A tradesman, who is selling off, makes a reduction of 10 per cent. from the marked price; what is the marked price of goods for which he receives 15*s.* 9*d.*?

28. What nominal price must a publisher put upon a book for which, after allowing the bookseller 33⅓ per cent., he wishes to receive 5*s*?

29. A man marks his goods 20% above cost price, but makes a reduction of 10% on the marked price for ready money; what is his gain per cent.?

30. A tradesman's charges are 25% over cost price; if he allow his customers 12% off their bills for cash payments, what profit does he make?

CHAPTER XII.

PARTNERSHIPS.

117. Sometimes two, or more, people put their money together and enter on a business hoping to make a profit on their money. Such an association of two, or more, persons in order to carry on a business is called a **Partnership.**

The question arises, How should the profits, or losses, of such a partnership be shared amongst the partners?

Clearly, in proportion to the amount of capital supplied by each partner. The solutions of such questions are thus examples of Proportional Division.

Ex. *A puts into a business* £2500, *B puts* £1500, *and C puts* £1000. *At the end of a year the profits amount to* £625; *how should this be divided?*

The capital = £2500 + £1500 + £1000 = £5000;

$\therefore$ A's share $= \frac{2500}{5000} \times$ profits $= \frac{1}{2} \times$ £625 = £312. 10*s.*,

B's share $= \frac{1500}{5000} \times$ profits $= \frac{3}{10} \times$ £625 = £187. 10*s.*,

and C's share $= \frac{1000}{5000} \times$ profits $= \frac{1}{5} \times$ £625 = £125.

EXAMPLES LXVII.

1. Two partners put £1500 and £2550 into a business; how should a profit of £1125 be divided?

2. A, B, and C invest capital to the extent of £2250, £1500, and £1000 respectively in a business; if a profit of £145. 13*s.* 4*d.* be divided between them, how much will each receive?

3. Three partners invest £3000, £1200, and £2000 in a business, and at the end of a year the profits are £744; what is the share of each?

4. Three men enter into partnership, puting into the business £300, £400, and £500 respectively; how much should each receive out of each £100 of profits?

5. Three partners A, B, and C invest £4730, £3680, and £2840 respectively in a business; how should they divide a profit of £595. 6*s.* 3*d.*?

6. Three partners subscribe £415. 15*s.*, £379. 13*s.*, and £1000; how do they divide a profit of £897. 14*s.*?

7. A, B, and C are partners; the capitals of A and B amount to £900, and those of B and C amount to £1900: if the total capital is £2200, how should they divide a profit of £88?

8. Three partners A, B, and C own a business; B's capital is twice A's, and C's twice B's; their profits, which are 10 per cent. on the whole capital, amount to £2331: what capital has each partner in the business?

118. Cases like the preceding, in which the capital of the partners is invested for the same time, are often called examples of Simple Partnership. In all such cases the profits are simply divided in proportion to their shares of the capital. When the shares of the partners are not invested in the business *for the same time*, the question to be solved is one of Compound Partnership.

Ex. *A, B, and C divide a profit of* £590. 10*s.* *at the end of a year's business; A has had* £1160 *invested for the* 12 *months, B* £750 *for* 5 *months; and C* £850 *for* 7 *months: what share should each receive of the profits?*

£1160 in 12 months earns as much profit as $12 \times$ £1160 in 1 month, that is, as much as £13920 in 1 month.

So £750 in 5 months earns as much profit as $5 \times$ £750 in 1 month, that is, as much as £3750 in 1 month.

Similarly, £850 in 7 months earns as much profit as $7 \times$ £850 in 1 month, that is, as much as £5950 in 1 month.

The profits should therefore be shared in the proportions of 13920, 3750, and 5950.

Now $13920 + 3750 + 5950 = 23620$;

$\therefore$ *A*'s share $= \frac{13920}{23620} \times$ £590. 10*s.* $= \frac{696}{1181} \times$ £$\frac{1181}{2}$ = £$\frac{696}{2}$ = £348.

So *B*'s share $= \frac{3750}{23620} \times$ £590$\frac{1}{2}$ $= \frac{375}{2362} \times$ £$\frac{1181}{2}$ = £$\frac{375}{4}$ = £93. 15*s.*

and *C*'s share $= \frac{5950}{23620} \times$ £$\frac{1181}{2}$ = £$\frac{595}{4}$ = £148. 15*s.*

Often in a business all the partners are not working partners, that is, partners who give their personal attention to the business. A partner who does not attend to the business, but who only provides capital, is called a Sleeping Partner.

EXAMPLES LXVIII.

1. *A*, *B*, and *C* enter into partnership. *A* advances £1080 for 5 months, *B* £960 for 3 months, and *C* £1120 for 6 months. They gain £992 altogether. Find the share of each.

2. *A* and *B* engage in trade, and gain £521. 8*s.* 2*d.*; *A* put in £250 for 6 months, *B* £300 for 5 months, and *C* £400 for 12 months; required each partner's share of the gain.

3. *A* commences business with a capital of £4000, and after 4 months takes *B* into partnership with a capital of £3000. Two months later they take *C* into partnership with a capital of £5000. At the end of the year their net profits amount to 16 per cent. on the whole capital then invested. What should each receive of the profits?

4. Two people enter into partnership with £3000 and £4500 respectively. At the end of 8 months the partner with smaller capital advances £2500 more. Seven months later they close the business with a profit of £1040. How should this profit be divided?

5. A and B each invest a certain sum of money in a business. The sum which A invests is $\frac{2}{3}$ of that which B invests. At the end of 7 months A withdraws $\frac{1}{4}$ of his capital, and at the end of 9 months B withdraws $\frac{1}{4}$ of his. The profits at the end of the year are £132. 12*s.*; how ought this sum to be divided?

6. A contributed £3000, B £4000, and C £5000 to a business; B withdraws his money at the end of 9 months, but A and C continued the business for 8 months longer, when the total profits were found to be £2150; how should the profits be divided?

7. Three partners in a business have gained £3650. The first advanced one-third of the capital for one-fourth of the time, the second advanced one-fourth of the capital for one-half of the time, and the third the remainder of the capital. Find the respective shares of the profits.

8. A contributed £600 to the partnership firm of A and B for 5 months, and received $\frac{2}{5}$ of the whole profit; B's share of the capital was £500. How long was B's money in the business?

9. X, Y, and Z go into partnership in a business, in which X invests £300, Y £360, and Z £480. After two and a half years Y withdraws, but X and Z continue the business for a year longer, and then wind it up. The total profit is found to be £1320. How much should each receive?

10. A began business with £450, and is joined afterwards by B with £1350; when did B join, if the profits at the end of the year are divided equally?

11. A and B are partners in business; A contributes $\frac{1}{3}$ of the capital for 18 months, and B receives $\frac{3}{4}$ of the total profits: for how long was B's money in the business?

12. A, B, and C engage in business; A puts in £5000 for the first 4 months and afterwards doubles it; B puts in £3000 at first and trebles it at the end of six months; C has £3500 in for 8 months and then withdraws it; the first year's profits amount to £4000: how should they be divided?

13. A is a working and B a sleeping partner in a business. A owns £1200 of the capital and B £2000. A receives 10% of the profits for managing, the rest being proportionately divided; find their respective shares of £800 profit.

14. A and B are partners in a business in which A has invested £4900 and B £1400; B being the working partner receives 6% of all the profits; the rest is divided in proportion to their capitals: what share does each receive out of £450 profit?

15. A and B put respectively £6750 and £1500 into a business, and B receives 15% of the profits for managing it, and the remainder is divided between the partners in proportion to their capitals; what does each get out of a profit of £726. 9*s.* 2*d.*?

CHAPTER XIII.

BANKRUPTCIES, TAXES, PREMIUMS.

119. In the present chapter we shall consider some examples of problems, in which payments are made at a given rate in the £, or at a given rate per cent.

For example, when we say that there is paid 2*s.* 6*d.* in the £ on £65, we mean that for each £ in £65 a sum of 2*s.* 6*d.* is paid, and that therefore the total sum paid is 65 times 2*s.* 6*d.*

120. Sometimes a trader makes mistakes in his business, and incurs such heavy losses that the money that he possesses is not sufficient to pay all the money that he owes. In such a case the trader becomes a **Bankrupt.**

All his property is then sold and turned into cash, and the total value of this property is divided amongst those to whom he owes money, in proportion to the money he owes them.

[The persons to whom a man owes money are called his **Creditors;** those who, on the other hand, owe him money are called his **Debtors.**]

Thus suppose his total debts to be £1000, and that the total value of all his property is £500.

Since £500 is just one-half of £1000, each creditor will get just one-half of what is due to him, so that, for each £1 due to him, the creditor will receive 10*s.* This is expressed by saying that each creditor will receive 10*s.* in the £.

Similarly, if a bankrupt's estate is sufficient to pay 13*s.* in the £, then for each £1 due to a creditor he will receive 13*s.* The bankrupt is then said to pay a dividend of 13*s.* in the £.

The total amount of money that a man owes is called his **Liabilities.** The total amount of money that a man has, or that his property will sell for, is called his **Assets.** When a man's liabilities exceed his assets, then he is insolvent.

121. *Given a bankrupt's liabilities and also his assets, to find what dividend he can pay in the £.*

Ex. *A bankrupt's liabilities are* £423. 6*s.* 8*d.*, *and his assets are* £296. 6*s.* 8*d.*; *what dividend can he pay?*

On £423. 6*s.* 8*d.* he can pay £296. 6*s.* 8*d.*;

$\therefore$ on £$423\frac{1}{3}$,, £$296\frac{1}{3}$;

$\therefore$ on £$\frac{1270}{3}$,, £$\frac{889}{3}$;

$\therefore$ on £1 ,, £$\frac{889}{1270}$,

that is £$\frac{7}{10}$, that is 14*s.*

EXAMPLES LXIX.

How much in the £ can a bankrupt pay in the following cases?—

1. Debts, £843. 15*s.*; assets, £284. 15*s.* $3\frac{3}{4}$*d.*

2. Debts, £13,901. 6*s.* 8*d.*; assets, £10,252. 4*s.* 8*d.*

3. Debts, £7843. 5*s.* 6*d.*; assets, £4575. 4*s.* $10\frac{1}{2}$*d.*

4. Debts, £9089. 1*s.* 4*d.*; assets, £5254. 12*s.* 4*d.*

5. A bankrupt's debts are £5374. 15*s.* 9*d.* (including £106. 5*s.* 9*d.* for rent, taxes, and wages which have to be paid in full), and his assets are £2444. 3*s.* $8\frac{1}{4}$*d.*; what dividend in the £ can he pay his creditors?

6. The realized value of an estate is £5364. 15*s.* It is encumbered with debts amounting to £3218. 17*s.* over and above this value; what dividend in the £ do the creditors receive?

122. *Given the debt and the dividend in the £, to find the amount received by a creditor.*

Ex. *The amount owing to a creditor is* £1346. 6*s.* 8*d.*, *and the dividend is* 15*s.* $7\frac{1}{2}$*d. in the £; find the amount that the creditor receives.*

This is a simple example of Practice.

	£.	*s.*	*d.*			
	1346 .	6 .	8	= amount owing.		
10*s.* = $\frac{1}{2}$ of £1	673 .	3 .	4	= dividend at 10*s.*		in the £.
5*s.* = $\frac{1}{2}$ of 10*s.*	336 .	11 .	8	= ,,	5*s.*	,,
6*d.* = $\frac{1}{10}$ of 5*s.*	33 .	13 .	2	= ,,	6*d.*	,,
$1\frac{1}{2}$*d.* = $\frac{1}{4}$ of 6*d.*	8 .	8 .	$3\frac{1}{2}$	= ,,	$1\frac{1}{2}$*d.*	,,
	£1051 .	16 .	$5\frac{1}{2}$	= ,,	15*s.* $7\frac{1}{2}$*d.*	,,

EXAMPLES LXX.

Find to the nearest farthing the dividend on

1. £865. 2*s.* 6*d.* at 7*s.* 10*d.* in the £.

2. £3549 at 17*s.* 6*d.* in the £. **3.** £5471 at 5*s.* $9\frac{3}{4}$*d.* in the £.

4. £721. 13*s.* 6*d.* at 14*s.* 2*d.* in the £.

5. £1463. 15*s.* at 4*s.* 7*d.* in the £. **6.** £278 at 18*s.* $9\frac{1}{2}$*d.* in the £.

7. £3758. 17*s.* 6*d.* at 13*s.* $7\frac{1}{2}$*d.* in the £.

8. £3458. 17*s.* 5*d.* at 13*s.* $8\frac{1}{4}$*d.* in the £.

9. A bankrupt fails for £12,500, and his estate realizes £3906. 5*s.*; what dividend does he pay, and what does a creditor receive whose claim is £798. 10*s.*?

10. A bankrupt's debts amount to £4875. 5*s.* and his assets to £3250. 3*s.* 4*d.*; what will a creditor receive whose claim is £574. 10*s.*?

11. A creditor received on a debt of £300 a dividend of 12*s.* 6*d.* in the £; he afterwards gets a further dividend of 3*s.* 6*d.* in the £ on the deficiency: how much does he receive in all?

12. A bankrupt owes to one creditor 1000 guineas, to each of two others £500, and to each of three others £150; his assets are £600: how much does he pay in the £, and what does the first creditor receive?

123. *Given the assets and the dividend in the £, to find the total amount of the bankrupt's liabilities.*

Ex. 1. *A bankrupt's assets were* £1589. 15*s.* 6*d.*, *and he paid* 11*s.* 9*d.* *in the* £; *what were his debts?*

We have £1589. 15*s.* 6*d.* = £1589·775, and 11*s.* 9*d.* = £·5875. (Art. 77.)

Now on a debt of £1 he pays £·5875;

$\therefore$,, £$\frac{1}{\cdot 5875}$,, £1;

$\therefore$,, £$\frac{1589\cdot775}{\cdot5875}$,, £1589·775;

$\therefore$ his total debt = £$\frac{1589\cdot775}{\cdot5875}$ = £2706.

EXAMPLES LXXI.

In the following cases find the amount of the debts:

1. Assets, £2210; dividend, 14*s.* 2*d.* in the £.

2. Assets, £753. 13*s.* 4*d.*; dividend, 14*s.* in the £.

3. Assets, £210. 16*s.*; dividend, 12*s.* 9*d.* in the £.

4. A bankrupt's estate paid 12*s.* 9*d.* in the £; had the assets been £35. 11*s.* 5*d.* more, the dividend would have been 13*s.* in the £; what were his liabilities?

5. A creditor received 16*s.* 3*d.* in the £, and thereby lost £135. 10*s.*; how much was due to him?

6. The largest creditor in a bankrupt's estate receives £1202. 10*s.* 10*d.* in discharge of his claim, and the remaining creditors receive altogether £6012. 14*s.* 2*d.* between them. The total indebtedness being £10822. 17*s.* 6*d.*, what does the largest creditor lose?

124. Income and other Taxes.

The amount of a tax or rate is always stated as so much in the £.

Ex. *Find the amount paid as income tax by a man whose gross income is* £1143. 8*s.* 6*d.*, *when the tax is at the rate of* 8*d.* *in the* £.

A man's gross income is his income *before* the tax is deducted; his net income is the amount that remains *after* the tax has been paid.

The amount of the tax is always calculated to the nearest penny.

8*d.* = $\frac{1}{30}$ of £1. Thus the tax is one-thirtieth of the gross income.

$\therefore$ Income tax on £1143. 8*s.* 6*d.* = $\frac{1}{30}$ × £1143. 8*s.* 6*d.*

= $\frac{1}{10}$ × £381. 2*s.* 10*d.* = £38. 2*s.* $3\frac{2}{5}$*d.* = £38. 2*s.* 3*d.*;

for $\frac{2}{5}$*d.* is neglected since it is less than $\frac{1}{2}$*d.*

$\therefore$ net income = £1143. 8*s.* 6*d.* − £38. 2*s.* 3*d.* = £1105. 6*s.* 3*d.*

EXAMPLES LXXII.

Fill up the gaps in the following table:

	Gross Income.	Rate of Income Tax.	Amount Paid.	Net Income.
1.	£1115. 10*s.*	1*s.* in the £.		
2.	£984. 7*s.* 6*d.*	8*d.* ,,		
3.	£478. 10*s.*	10*d.* ,,		
4.		1*s.* ,,	£75	
5.		6½*d.* ,,	£126. 17*s.* 2*d.*	
6.		10*d.* ,,	£27. 15*s.*	
7.	£1350.			£1288. 2*s.* 6*d.*
8.	£500.			£483. 6*s.* 8*d.*
9.	£545.			£529. 2*s.* 1*d.*
10.		10*d.* ,,		£645. 18*s.* 4*d.*
11.		7*d.* ,,		£1632. 18*s.* 10*d.*
12.		1*s.* ,,		£1441. 12*s.* 6*d.*
13.		1*s.* 4*d.* ,,	£11,000,000.	

14. When the income tax is 11*d.* in the £, a man pays £5. 5*s.* less than when it was 1*s.* in the £; what is his income?

15. When the income tax was 8*d.* in the £, a person's net income was £1421; what is it now that the tax is 1*s.* in the £?

16. After paying a tax of 5*d.* in the £ a man's net income was £733. 7*s.* 11*d.*; what is it when the tax is 5½*d.* in the £?

17. A person lends £1280 at 4½% and £975 at 5%; what is his net income after deducting income tax at 9*d.*?

18. The net rental of an estate, after deducting 7*d.* in the £ for income tax and 5% of the remainder for the cost of collection, is £959. 3*s.* 8*d.*; what is the gross rental?

19. What sum must be bequeathed to a man so that after a legacy duty of 10% has been paid he may receive £2700?

20. A man pays £75 a year as rent for his house; his rates are two poor rates of 11d. in the £, a lighting rate of 7d. in the £, and a highway rate of 9d. in the £. If the rates be calculated on $\frac{4}{5}$ths of his rent, what is the total cost of his house for rent and taxes?

21. Find the whole annual outlay for a residence of which the rent is £96, the poor rate 3*s.* 4*d.* in the £, the gas rate two-thirds of the poor rate, and the highway rate three-fifths of the gas rate.

22. The property in a certain parish is assessed at £54,000; what must be the rate in the £ so that a sum of £8550 may be raised?

What will be the total amount of the rate on a college which is assessed at £5200?

23. What is the ratable value of a parish if a rate of 3*s.* 5½*d.* in the £ produces a sum of £14,352. 1*s.* 8*d.*?

24. A rate of 2*s.* 7½*d.* in the £ on a parish produces £745. 1*s.* 3*d.*; what is its ratable value?

25. In a certain parish a rate of 1*s.* $4\frac{1}{2}$*d.* in the £ brings in £2227. 10*s.*; what rate will bring in £2970?

26. What is the gross income of an estate, if, after an income tax of $2\frac{1}{2}\%$ and rates at 3*s.* $6\frac{1}{2}$*d.* in the £ are paid, the net income is £8809?

125. Premiums on Insurance Policies. When a ship is sent to a foreign country, there is always danger that the ship and goods will be lost, either through a storm, or by fire, or through other casualties. In general, the owner of the ship does not take all this risk upon himself.

He therefore agrees with men, called Underwriters, that on payment of a certain sum, called the **Premium,** they shall make good to him any loss that may be incurred.

The Premium is usually calculated as so much per cent. on the sum insured.

Similarly, any property may be insured against loss to the owner by fire. So also a man may assure to his family a sum of money to be paid to them on his death.

Ex. *A man insures his property for* £9650; *what is the premium at* $3\frac{1}{2}\%$?

$$\text{It} = 3\tfrac{1}{2}\% \text{ of } £9650 = \frac{3\frac{1}{2}}{100} \times £9650$$

$$= \tfrac{7}{200} \times £9650 = £337.\ 15s.$$

EXAMPLES LXXIII.

1. A man insures his life for £1750; what is the annual premium at £2. 16*s.* 2*d.* per cent.?

2. What is the premium on a policy of assurance for £1258. 5*s.* at $6\frac{1}{4}\%$?

3. If a ship worth £3484 be insured for only $91\frac{2}{3}\%$ of its value, what would a person who owns $\frac{2}{5}$ of it lose if it became a total wreck?

4. A ship which cost £9600 is wrecked; what did a person lose who owned $\frac{3}{16}$ of it, the ship being insured for 95% of its real value?

5. A man insures a ship for £1245, and pays a premium at the rate of $3\frac{1}{2}\%$; what is the premium?

The ship being lost, the man recovers the value of his ship and also the premium; what was the value of the ship?

6. A cargo of goods was insured at $3\frac{1}{8}\%$ on the price they were expected to fetch, which was 20% more than the price paid for them. The assurance came to £117. 10*s.*; what was the cost price?

7. A man's yearly income is £250; when the income tax is at 1*s.* in the £, find what he should pay, after deducting from his income an abatement of £160 and also the annual premium on a life-policy of £1000 at the rate of 4% on this amount before calculating the tax.

CHAPTER XIV.

SIMPLE INTEREST.

126. Money which is paid for the loan of money is called **Interest,** and is usually expressed as so much per Cent. per Annum.

Thus, suppose that a man borrows £100 for a year, and agrees to repay the £100 at the end of the year, and also £4 for its loan, he is said to pay Interest at the rate of 4 per cent. per annum.

The sum of money which it is agreed shall be paid for the loan of *each* £100 for one year is called the **Rate per Cent. per Annum,** or more shortly, the **Rate per Cent.**

The sum of money borrowed, or lent, is called the **Principal.** The sum of money which is obtained by adding the interest in any given time to the Principal is called the **Amount** in that given time.

Thus, in the example of the last article, the Principal is £100, the Rate per Cent. is £4, and the Amount, which is the total sum the man had to repay at the end of the year,

$$= \text{Principal} + \text{Interest} = £100 + £4 = £104.$$

127. At Simple Interest, the interest for two, three ... years is two, three ... times the interest for one year.

The interest for a fraction of a year is calculated thus: It bears the same ratio to the interest for one year that the given fraction of the year bears to one year.

Thus the interest for 6 months is half the interest for 1 year.

The interest for 1 month is $\frac{1}{12}$ of the interest for 1 year.

The interest for 7 weeks is $\frac{7}{52}$ of the interest for 1 year.

The interest for 153 days is $\frac{153}{365}$ of the interest for 1 year.

Thus, if the fraction of the year be given in months, we take a year as equal to 12 months.

If it be given in weeks, we take a year as 52 weeks.

If it be given in days, we take a year as 365 days.

128. **Ex.** *Find the interest on* £698 *for* $3\frac{3}{4}$ *years at* £4 *per cent. per annum.*

Interest on £100 for one year = £4.

$\therefore$,, £1 ,, $= £\frac{4}{100}$.

$\therefore$,, £698 ,, $= £698 \times £\frac{4}{100} = £\frac{698 \times 4}{100}$.

Now interest for 3 years = 3 times interest for 1 year

and " $\frac{3}{4}$ " $= \frac{3}{4}$ " " "

$\therefore$ " $3\frac{3}{4}$ " $= 3\frac{3}{4}$ " " "

$\therefore$ interest on £698 for $3\frac{3}{4}$ " $= 3\frac{3}{4} \times$ £$\frac{6\,9\,8 \times 4}{1\,0\,0}$

$= £\frac{698 \times 4 \times 3\frac{3}{4}}{100} = \frac{\text{principal} \times \text{rate per cent. per annum} \times \text{time}}{100}$.

This will be found to be true in all cases, so that if I be the Interest, P the Principal, r the rate per cent., and n the number of years, then

$$I = \frac{P \times r \times n}{100}.$$

Hence to obtain the simple interest **we multiply the principal by the rate per cent., and the product by the time expressed in years and fractions of a year, and divide the result by 100.**

129. *Given the principal, the rate per cent., and the time, to find the simple interest.*

We multiply the principal by the rate per cent., and the number of years (fractional or otherwise) in the time, and divide the result by 100.

Ex. 1. *Find the simple interest on* £855 *for* 3 *years at* 4 *per cent. per annum.*

£.	s.	d.	
855	0	0	
			4 = rate per cent.
3420	0	0	
			3 = number of years.
100) 102,60	0	0	
20			
12,00			*Ans.* = £102. 12*s.*

Or thus: By the rule of Art. 128,

$$\text{Interest} = \frac{£855 \times 4 \times 3}{100} = \frac{£10260}{100} = £102\tfrac{6}{10} = £102.\ 12s.$$

Ex. 2. *Find the simple interest on* £1024. 7*s.* 6*d.* *for* $4\frac{3}{4}$ *years at* 4 *per cent. per annum.*

$$\text{Interest} = \frac{£1024.\ 7s.\ 6d. \times 4 \times 4\frac{3}{4}}{100} = \frac{£1024.\ 7s.\ 6d.}{100} \times 19$$

$$= \frac{£102.\ 8s.\ 9d.}{10} \times 19 = £10.\ 4s.\ 10\tfrac{1}{2}d. \times 19 = £194.\ 12s.\ 7\tfrac{1}{2}d.$$

To get the amount of a given sum in a given time we add the interest to the principal. Thus, in this example, the amount

$$= £1024.\ 7s.\ 6d. + £194.\ 12s.\ 7\tfrac{1}{2}d. = £1219.\ 0s.\ 1\tfrac{1}{2}d.$$

Ex. 3. *Find the simple interest on* £45. 6*s.* 8*d. for* 17 *months at* $3\frac{3}{4}$ *per cent. per annum.*

$$17 \text{ months} = \tfrac{17}{12} \text{ year},$$

$$£45.\ 6s.\ 8d. = £45\tfrac{1}{3} = £\tfrac{136}{3}.$$

Hence the required interest

$$= \frac{£\frac{136}{3} \times \frac{17}{12} \times 3\frac{3}{4}}{100} = £\frac{\overset{17}{\cancel{136}} \times \overset{3}{\cancel{15}}}{\underset{3}{\cancel{3}} \times \underset{4}{\cancel{12}} \times \underset{20}{\cancel{100}}} = £\tfrac{289}{120} = £2\tfrac{49}{120} = £2.\ 8s.\ 2d.$$

EXAMPLES LXXIV.

[*Throughout this Chapter fractions of a penny may be neglected in answers.*]

Find the simple interest for one year on

1. £400 at 4%. **2.** £500 at 3%. **3.** £700 at 6%. **4.** £850 at 3%.
5. £350 at 4%. **6.** £6350 at 3%. **7.** £865 at 4%. **8.** £750 at $2\frac{1}{2}$%.
9. £350 at $3\frac{1}{4}$%. **10.** £1000 at $4\frac{1}{8}$%. **11.** £950 at $2\frac{3}{4}$%. **12.** £389 at 5%.
13. £923 at 3%. **14.** £845. 10*s.* at 5%. **15.** £588. 10*s.* at 4%.
16. £647. 6*s.* 8*d.* at 3%. **17.** £2783. 10*s.* at $4\frac{1}{2}$%.

Find the simple interest on

18. £500 for 3 years at 5%. **19.** £750 for 6 years at 4%.
20. £875 for 7 years at 3%. **21.** £625 for 6 years at $4\frac{1}{2}$%.
22. £375 for 4 years at $3\frac{1}{2}$%. **23.** £950 for 4 years at $2\frac{3}{4}$%.
24. £175 for $3\frac{1}{2}$ years at 4%. **25.** £2350 for $2\frac{1}{2}$ years at 3%.

Find the amount at simple interest of

26. £429. 3*s.* 4*d.* in 4 yrs. at $2\frac{1}{2}$%. **27.** £976. 10*s.* in 16 months at $2\frac{3}{4}$%.
28. £1111. 13*s.* 4*d.* in $3\frac{1}{2}$ years at 5%.
29. £425 in 1 yr. 219 days at $3\frac{1}{2}$%. **30.** £3695. 15*s.* in 1 yr. at $4\frac{1}{4}$%.
31. £468. 11*s.* $10\frac{1}{2}$*d.* in 4 yrs. at 4%.
32. £4775 in $3\frac{1}{2}$ years at $3\frac{3}{4}$%. **33.** £320. 15*s.* in 5 years at $2\frac{1}{2}$%.
34. £480 in 7 months at 3%. **35.** £317. 4*s.* in 5 months at $3\frac{3}{4}$%.
36. £346. 11*s.* 3*d.* in 13 months at 4%.
37. £452. 7*s.* 6*d.* in 1 year 73 days at $6\frac{1}{4}$%.

Find the simple interest on

38. £4327. 5*s.* 6*d.* for 2 years at $3\frac{1}{2}$%.
39. £6395. 11*s.* 8*d.* for 3 years at $4\frac{1}{2}$%.
40. £237. 8*s.* 2*d.* for $2\frac{3}{4}$ years at $4\frac{1}{4}$%.
41. £625. 6*s.* 3*d.* for $3\frac{3}{4}$ years at $2\frac{1}{2}$%.
42. £8375. 6*s.* 10*d.* for 17 months at 3%.
43. £7843. 15*s.* 6*d.* for 2 years 2 months at $3\frac{1}{2}$%.

44. Compare the simple interest on £247. 10*s.* at 5% for two years with that on £375 at $3\frac{3}{8}$% for the same time.

45. *Find the simple interest on* £253. 12*s. from March* 16*th to August* 12*th at* 3 *per cent. per annum.*

In counting the number of days we do not count the first of the days mentioned, because the first day for which interest is counted has not elapsed till March 17th. The number of days in March during which interest is reckoned thus = 31 − 16 = 15.

March April May June July August

Thus the total number of days = 15 + 30 + 31 + 30 + 31 + 12 = 149.

∴ the required interest = £253. 12*s.* $\times \frac{3}{100} \times \frac{149}{365}$.

In such a question as this it is better to work in decimal fractions.

Now £253. 12*s.* = £253·6

∴ required interest = £2·536 × 3 × $\frac{149}{365}$

= £3·1057

= £3. 2*s.* 1*d.* to the nearest penny.

The decimal in such a case need only be worked accurately to the third or, at the most, the fourth place.

Find the simple interest on

46. £7000 for 85 days at 4 %.

47. £5000 for 97 days at 2½ %.

48. £387. 6*s.* 3*d.* for 310 days at 3 %.

49. £275 for 1 year 25 days at 3½ %.

50. £10,000 for 1 day at 3 %.

51. £150 from March 16th to May 31st at 4 %.

52. £500 from March 1st to June 10th at 2½ %.

53. £847 from May 15th to September 10th at 4 %.

54. £3685 from November 14th to March 4th at 5 %.

55. £2834. 13*s.* 9*d.* from June 15th to October 8th at 2½ %.

56. £837. 6*s.* 3*d.* from February 7th to April 10th at 2¾ %.

57. £34. 10*s.* from August 10th to October 21st at 4½ %.

58. £186. 10*s.* from June 9th, 1901, to August 31st, 1903, at 4 %.

59. Find the simple interest on £1345 borrowed on Jan. 13th, 1904, and repaid on Sept. 25th, 1904, at 5 per cent. per annum.

60. My banker lends me £250 on May 2nd, and £375 on July 14th, on condition that I repay the whole on December 31st, with simple interest at the rate of 5 per cent. per annum ; what sum of money have I to pay him on that day?

61. A sum of £1250 was borrowed on the 1st of April and repaid on the 25th of August of the same year, with interest at the rate of 3¾ per cent. per annum ; what sum had to be repaid?

130. *Given the Rate per Cent., the Time, and also the Interest (or the Amount), to find the Principal.*

Ex. 1. *On what sum of money is* £10. 17*s.* 7*d. the simple interest for* 2⅓ *years at* 3½ *per cent.?*

We have £10. 17*s.* 7*d.* = principal $\times \frac{3\frac{1}{2}}{100} \times 2\frac{1}{3}$ = principal $\times \frac{10}{300} \times \frac{7}{3}$

= principal $\times \frac{7}{90}$;

∴ principal = $\frac{90}{7}$ × £10. 17*s.* 7*d.* = 90 × £1. 11*s.* 1*d.* = £139. 17*s.* 6*d.*

Or thus: Interest on £100 for $2\frac{1}{3}$ years at $3\frac{1}{3}$ per cent. $= £\frac{7}{3} \times \frac{10}{3} = £\frac{70}{9}$;

$\therefore$ $£\frac{70}{9}$ is the interest on £100;

$\therefore$ £1 ,, ,, $£100 \times \frac{9}{70}$;

$\therefore$ £10. 17*s*. 7*d*., *i.e.* $£10\frac{211}{240}$,, ,, $£100 \times \frac{9}{70} \times 10\frac{211}{240}$,

i.e. £139. 17*s*. 6*d*.

If the amount be given, we must find the amount of £100 for the given time at the given rate per cent., and then use the Unitary Method.

Ex. 2. *Find the sum of money which will in* 224 *days amount to* £287. 15*s*. 3*d*. *at* $3\frac{3}{4}$ *per cent. per annum.*

Interest on £100 in 224 days at $3\frac{3}{4}$ per cent. $= £\frac{\overset{56}{\cancel{224}}}{\underset{73}{\cancel{365}}} \times \frac{\overset{3}{\cancel{15}}}{\cancel{4}} = £\frac{168}{73}$;

$\therefore$ amount of £100 $= £100 + \frac{168}{73} = £\frac{7468}{73}$.

Also £287. 15*s*. 3*d*. $= £287{\cdot}7625$.

Thus $£\frac{7468}{73}$ is the amount of £100;

i.e. £1 ,, ,, $£100 \times \frac{73}{7468}$;

$\therefore$ $£287{\cdot}7625$,, ,, $£100 \times \frac{73}{7468} \times 287{\cdot}7625$,

i.e. of £281. 5*s*. $9\frac{1}{4}$*d*. nearly.

EXAMPLES LXXV.

On what sum will the simple interest amount to

1. £75 in 3 years at 4%?

2. £63 in 4 years at $3\frac{1}{2}$%?

3. £40 in 1 year at 3%?

4. £33 in 18 months at $2\frac{3}{4}$%?

5. £945 in $3\frac{1}{2}$ years at $4\frac{1}{2}$%?

At simple interest, what sum will amount to

6. £392 in 3 years at 4%?

7. £275 in 2 years at 5%?

8. £855 in 4 years at $3\frac{1}{2}$%?

9. £308 in 4 years at 3%?

10. £2775. 19*s*. in $6\frac{1}{3}$ years at $7\frac{1}{2}$%?

On what sum will the interest amount to

11. £4. 9*s*. 7*d*. in 5 mths. at 5%?

12. £6. 3*s*. 9*d*. in 219 days at $3\frac{3}{4}$%?

13. £127. 11*s*. in 5 years at 3%?

14. £121. 15*s*. 5*d*. in 2 years 1 month at $5\frac{3}{5}$%?

At simple interest, what sum, to the nearest penny, will amount to

15. £159. 12*s*. 9*d*. in $5\frac{1}{2}$ years at $6\frac{1}{4}$%?

16. £444. 10*s*. in 146 days at 4%?

17. £4704. 1*s*. $3\frac{3}{4}$*d*. in 3 years at $2\frac{1}{2}$%?

18. £39. 12*s*. 6*d*. in 16 months at $4\frac{1}{2}$%?

19. What sum lent at 4% per annum will in 3 years produce as much, at simple interest, as £800 lent at $4\frac{1}{2}$% will produce in 5 years?

20. Reckoning simple interest, what sum of money, lent out at 5 per cent. per annum, will produce in 2 years the same amount of interest that £450, lent out at 4 per cent. per annum, will produce in 3 years?

21. The rental of an estate is £365. 15*s.*, and a man buys it at such a price that he obtains $3\frac{1}{2}$ per cent. per annum on the purchase-money; how much did he give for the estate?

22. If the interest on a certain sum of money for two-fifths of a year at $4\frac{1}{2}$ per cent. be 11 guineas, what will be the interest on the same sum for three-quarters of a year at $3\frac{1}{2}$ per cent.?

131. *Given the Principal, the Interest (or the Amount), and the Time, to find the Rate per Cent.*

Ex. 1. *At what rate % will* £1045 *amount in* 6 *years to* £1264. 9*s.?*

The interest = £1264. 9*s.* − £1045. = £219. 9*s.* = £$219\frac{9}{20}$.

$$\text{Since interest} = \text{principal} \times \frac{\text{rate per cent.}}{100} \times \text{time},$$

$$\therefore\ 219\tfrac{9}{20} = 1045 \times \frac{\text{rate per cent.}}{100} \times 6\,;$$

$$\therefore\quad \text{rate per cent.} = \frac{\overset{7}{\cancel{2}\cancel{1}\cancel{9}\cancel{3}}}{\underset{2}{\cancel{2}\cancel{0}\cancel{9}}} \times \frac{\overset{3}{\cancel{2}\cancel{0}\cancel{9}}}{\underset{2}{\cancel{2}\cancel{0}\cancel{9}} \times \cancel{5} \times \cancel{3}} = \frac{7}{2} = 3\tfrac{1}{2}.$$

Otherwise thus: The interest on £1045 for 6 years is £$219\frac{9}{20}$.

$\therefore$ the interest on £1045 for 1 year is £$\frac{4389}{20} \div 6$, that is, £$\frac{1463}{40}$.

$\therefore$,, £1 ,, is £$\frac{\overset{7}{\cancel{2}\cancel{1}\cancel{9}\cancel{3}}}{40 \times \underset{5}{\cancel{2}\cancel{0}\cancel{9}\cancel{3}}}$, that is, £$\frac{7}{200}$.

$\therefore$,, £100 ,, is £$\frac{7}{200} \times 100$, that is, £$\frac{7}{2}$.

Hence rate per cent. $= \frac{7}{2} = 3\frac{1}{2}$.

Ex. 2. *At what rate per cent. simple interest will a sum of money double itself in* 20 *years?*

Here interest on £100 in 20 years is £100;

$\therefore$,, ,, 1 year is £$\frac{100}{20}$, that is, £5;

$\therefore$ required rate per cent. = 5.

EXAMPLES LXXVI.

At what rate per cent. per annum will the simple interest on

1. £150 be £12 in 2 years?
2. £300 be £27 in 3 years?
3. £450 be £81 in 4 years?
4. £25 be £3. 18*s.* 9*d.* in $3\frac{1}{2}$ years?
5. £4373. 6*s.* 8*d.* be £246 in $1\frac{1}{4}$ years?
6. £2575. 7*s.* 6*d.* be £193. 3*s.* $0\frac{3}{4}$*d.* in $2\frac{1}{2}$ years?
7. £1557. 16*s.* 3*d.* be £128. 15*s.* 7*d.* in 2 years 7 months?
8. £500 be £8. 5*s.* in 219 days?
9. Half-a-crown be three farthings in a month?
10. A shilling be a farthing in a month?

At what rate per cent. simple interest will the amount of

11. £200 be £224 in 3 years?
12. £500 be £545 in 3 years?
13. £3570 be £4694. 11*s.* in 7 years?

At what rate per cent. simple interest will the amount of

14. £3775 be £4105. 6*s.* 3*d.* in $2\frac{1}{2}$ years?

15. £427. 7*s.* 6*d.* be £470. 2*s.* 3*d.* in $2\frac{1}{2}$ years?

16. £2100 be £5250 in $37\frac{1}{2}$ years?

17. £325. 16*s.* 8*d.* be £374. 6*s.* $0\frac{1}{4}$*d.* in $3\frac{1}{2}$ years?

18. £540 be £734. 8*s.* in 9 years?

19. At what rate per cent. simple interest will a sum of money double itself in 25 years?

20. What is the rate per cent. on a given sum of money if its interest in 4 years be $\frac{3}{20}$ths of the given sum?

21. A money lender charged 30*s.* for the loan of £25 for 1 month; what interest per cent. per annum is this?

132. *Given the Principal, the Interest (or the Amount), and the Rate per Cent., to find the Time.*

Ex. *In how many years will the interest on* £1111. 13*s.* 4*d.* *amount to* £194. 10*s.* 10*d.* *at 5 per cent.?*

£1111. 13*s.* 4*d.* $= £1111\frac{2}{3} = £\frac{3335}{3}$ and £194. 10*s.* 10*d.* $= £194\frac{13}{24} = £\frac{4669}{24}$;

$$\therefore \ £\frac{4669}{24} = £\frac{3335}{3} \times \frac{5}{100} \times \text{time};$$

$$\therefore \ \text{time} = \frac{\cancel{4}^{\,7}\cancel{6}\cancel{6}\cancel{9}}{\cancel{24}\ \cancel{8}_{\,2}} \times \frac{\cancel{3}}{\cancel{3335}_{\,\cancel{5}}} \times \frac{\cancel{4}^{\,\cancel{2}}\cancel{6}\cancel{6}}{\cancel{5}} = \frac{7}{2}; \quad \therefore \ \text{required time} = 3\tfrac{1}{2} \text{ years.}$$

Or thus: Interest on £1111. 13*s.* 4*d.* for 1 year at 5 per cent.

$$= £\tfrac{5}{100} \times £1111.\ 13s.\ 4d. = £\tfrac{1}{20} \times £1111\tfrac{2}{3} = £\frac{3335}{60} = £\frac{667}{12};$$

$$\therefore \ \text{required number of years} = £194\tfrac{13}{24} \div £\frac{667}{12} = \frac{\cancel{4}^{\,7}\cancel{6}\cancel{6}\cancel{9}}{\cancel{24}\ \cancel{8}_{\,2}} \times \frac{\cancel{4}\cancel{2}}{\cancel{6}\cancel{6}\cancel{7}} = 3\tfrac{1}{2}.$$

EXAMPLES LXXVII.

In what time will the simple interest on

1. £865 be £173 at 5%? **2.** £2000 be £315 at $4\frac{1}{2}$%?

3. £180 be £18. 18*s.* at $4\frac{1}{2}$%? **4.** £387. 10*s.* be £58. 15*s.* 5*d.* at $3\frac{1}{2}$%?

5. £1275 be £274. 11*s.* at $3\frac{4}{5}$%? **6.** £285 be £84. 15*s.* 9*d.* at $3\frac{1}{2}$%?

7. £588. 10*s.* 10*d.* be £18. 16*s.* 8*d.* at 4%?

At simple interest, in what time will the amount of

8. £250 be £265 at 3%? **9.** £625 be £700 at 4%?

10. £175 be £199. 10*s.* at $3\frac{1}{2}$%? **11.** £768. 17*s.* 6*d.* be £1230. 4*s.* at 5%?

12. £450 be £564. 3*s.* 9*d.* at $3\frac{1}{2}$%? **13.** £615 be £2152. 10*s.* at $7\frac{1}{2}$%?

14. £345 be £454. 2*s.* $1\frac{1}{2}$*d.* at $2\frac{3}{4}$%?

15. £583. 13*s.* 4*d.* be £656. 12*s.* 6*d.* at $4\frac{1}{2}$%?

Find, to the nearest day, in what time the amount of

16. £345. 10*s.* will be £354. 15*s.* at 5%.

17. £967 will be £1000 at $3\frac{3}{8}$%.

18. £2873. 5*s.* will be £2934. 4*s.* at 4%.

19. In what time, at simple interest, will a sum of money double itself at 5 per cent. ?

20. What sum of money, lent out at $3\frac{1}{4}$ per cent. simple interest, will amount to £546. 6*s*. 8*d*. in 9 months, and in what further time will it amount to £563. 13*s*. 4*d*. ?

21. If in 5 years £6520 amount to £7824, in what time at the same rate will £450 amount to £531 ?

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22. A man borrows £10,000 on which the interest is at the rate of 4 per cent. ; each year he pays £800 for interest and in part payment of the debt ; what amount does he owe at the end of three years ?

23. The debt of a certain country is £589,000,000, upon which $2\frac{1}{2}$ per cent. is paid annually for interest. If the sum of £25,000,000 be annually set aside for this purpose and to reduce the debt, find to the nearest pound the amount of the debt at the end of three years.

24. A sum of £20,833. 6*s*. 8*d*. is borrowed at 5 per cent. per annum interest. The borrower at the end of each year pays one-fifth of the amount he then owes ; what will be his debt at the end of three years ?

25. What must a tradesman charge for goods that cost him £45 in order that after giving 9 months' credit, reckoning 5 per cent. interest, he may make 20 per cent. profit ?

26. A dealer buys goods for £1050 and sells them six months later for £867 ; how much did he lose reckoning his money to be worth to him 8 per cent. per annum ?

27. A man has £4000 ; he lends £500 at $3\frac{1}{2}$ per cent., and £2000 at 4 per cent. At what rate must he lend the remainder of his money so that his total income may be $4\frac{1}{2}$ per cent. on his £4000 ?

28. A man puts out two equal sums of money at interest, the one at $2\frac{1}{2}$ per cent. and the other at 3 per cent. At the end of $5\frac{1}{2}$ years he has received £18. 7*s*. $1\frac{1}{2}d$. more from the latter investment than from the former ; find the sums invested. [Use x.]

29. Two sums of money amounting to £1946. 5*s*. were invested, the smaller at 4 per cent. and the larger at $4\frac{1}{2}$ per cent. per annum. At the end of 18 months the simple interest on the two sums amounted to £125. 4*s*. $10\frac{1}{2}d$. ; what were these two sums ? [Use x.]

30. A man puts out a certain sum of money at $3\frac{1}{2}\%$ interest ; he also invests £375. 6*s*. 3*d*. at 5 %. At the end of a year the total interest is £26. 5*s*. $11\frac{1}{2}d$. Find the amount invested at $3\frac{1}{2}\%$.

31. A man bought 4 houses for £9800. What must be the rent of each house so that, after deducting one-sixth of it for repairs, he may get 5 % on his outlay ?

32. A man lent £2500 at 5 % for 18 months. What additional sum must he lend for five-sixths of this time at $3\frac{3}{4}\%$, so that the total interest may be £450 ?

CHAPTER XV.

COMPOUND INTEREST.

133. When the interest is not paid at the end of each year by the borrower, but is added to the Principal instead, so that the Principal at the beginning of each year is greater than at the commencement of the previous year, the Principal is said to be accumulating at Compound Interest.

The amount due by the debtor is thus continually increasing, and the amount of interest that he ought to pay each year is also continually increasing.

Ex. *Find the amount at compound interest of* £200 *in* 4 *years at* 5 %.

The interest in any year $= \frac{5}{100}$ths $= \frac{1}{20}$th of the principal at the beginning of that year.

£200	= principal at beginning of the 1st year.
10	= interest in 1st year $= \frac{1}{20}$th of previous line.
210	= principal at beginning of 2nd year.
10·5	= interest in 2nd year $= \frac{1}{20}$th of previous line.
220·5	= principal at beginning of 3rd year.
11·025	= interest in 3rd year $= \frac{1}{20}$th of previous line.
231·525	= principal at beginning of 4th year.
11·57625	= interest in 4th year $= \frac{1}{20}$th of previous line.
243·10125	

20

2·02500

12

·300

Ans. = £243. 2*s.* 0·3*d.*

134. It will be observed that at 5 % the Amount of a sum of money in one year = Principal + $\frac{5}{100}$ × Principal (by Art. 128)

= Principal + ·05 × Principal = 1·05 × Principal.

Hence in any year the amount at its end = 1·05 × the amount at the commencement of that year.

If therefore the Principal at the commencement of any year be expressed as a decimal, we obtain the amount at the end of the year by multiplying the Principal by 5, and setting down the result two decimal places to the right; on adding the result to the Principal, we have the amount at the end of the year.

Thus, in the example given, the Principal at the beginning of the third year is £220·5. This, on being multiplied by 5, and set down two places to the right of the 220·5, gives 11·025.

The Principal at the beginning of the fourth year is 231·525. This, on being multiplied by 5, and being set down two places to the right, gives 11·57625.

It is clear that the above process is merely an abbreviated form of successive multiplication by 1·05.

135. Similarly, if the interest be at the rate of 3, 4, 6, ... per cent., the amount at the end of any year is obtained by multiplying the amount at the commencement of that year (expressed as a decimal) by 3, 4, 6, ... and setting the result down two places to the right, and then adding.

Ex. *Find the compound interest on* £346. 15*s.* 6*d.* *for* 3 *years at* 4 *per cent.*

By Art. 77 we have £346. 15*s.* 6*d.* = £346·775.

	£346·775	
4 % = ·04	13·8710	0 = 1st year's interest.
	360·646	
4 % = ·04	14·4258	4 = 2nd ,, ,,
	375·0718	4
4 % = ·04	15·0028	736 = 3rd ,, ,,
	390·0747	136
	20	
	1·4940	Amount = £390. 1*s.* 6*d.*, to the nearest penny.
	12	Principal = £346. 15*s.* 6*d.*
	5·928	Interest = £43. 6*s.* 0*d.*

Since it is unnecessary to calculate interest more accurately than, at the most, to the nearest farthing, we only want the answer correct to three places of decimals.

We need therefore only calculate the interest to four decimal places, or to five, at the most, if there be several years' interest to be calculated.

136. If the time given be not a whole number of years, we calculate the amount for the complete number of years, and then add on the simple interest for the portion of the year remaining.

Suppose again that the interest is to be reckoned at a rate per cent. which is not integral (say $4\frac{1}{2}$).

Since $4\frac{1}{2} = 4{\cdot}5$, $\therefore\ 1 + \frac{4\frac{1}{2}}{100} = 1{\cdot}045$, so that the multiplier of Art. 134 in this case becomes 1·045.

So if the interest were at $3\frac{1}{4}$ per cent.,

$$\text{the multiplier} = 1 + \frac{3\frac{1}{4}}{100} = 1 + \frac{3{\cdot}25}{100} = 1{\cdot}0325.$$

EXAMPLES LXXVIII.

Find, to the nearest penny, the compound interest on

1. £50 for 2 years at 4%.
2. £270 for 2 years at 3%.
3. £275 for 2 years at 5%.
4. £100 for 3 years at 5%.
5. £4500 for 2 years at $4\frac{1}{4}$%.
6. £3725 for 3 years at 4%.
7. £133. 6*s.* 8*d.* for 3 years at 5%.
8. £2500 for 4 years at 5%.
9. £235 for $2\frac{1}{2}$ years at 4%.

At compound interest find, to the nearest penny, the amount of

10. £500 for 2 years at 5%.
11. £2000 for 2 years at 3%.
12. £6000 for 3 years at 4%.
13. £400 for 3 years at 2%.
14. £3680 in 3 years at 4%.
15. £2540 in 3 years at 4%.
16. £2476. 10*s.* in 3 years at $3\frac{1}{2}$%.
17. £5600 in 2 years at $3\frac{3}{4}$%.
18. 100 guineas in 3 years at $4\frac{1}{4}$%.
19. £690 in 3 years at $4\frac{1}{2}$%.
20. £16,000 in 3 years at $2\frac{1}{4}$%.

What is the difference, to the nearest penny, between the simple and compound interest on

21. £1750 for 3 years at 4%?
22. £787. 10*s.* for 3 years at $3\frac{1}{2}$%?
23. £415. 10*s.* for 4 years at $2\frac{1}{4}$%?

At compound interest, find the amount, to the nearest penny, of

24. £750 in 2 years at 5%, payable half-yearly.
25. £835. 16*s.* 8d. in 18 months at 3%, payable half-yearly.
26. £1347. 8*s.* 6*d.* in 2 years at 4%, payable half-yearly.
27. £10,000 in 1 year at $2\frac{3}{4}$, payable quarterly.

28. Find the amount of £520. 16*s.* 8*d.* at 4% per annum compound interest for a year and a half, interest being due half-yearly.

29. What will £3255. 4*s.* 2*d.* amount to in a year and a half if it be put out at compound interest at 8% per annum, the interest being supposed to be added at the end of each half year?

30. Find the difference between the compound interest on £10,000 for 3 years at 4%, according as the interest is payable yearly or half-yearly.

31. A man buys a farm worth £26,333. 6*s.* 8*d.*, and pays £20,000 down; find what additional sum he should pay at the end of 2 years, reckoning compound interest at $3\frac{1}{2}$%.

32. A boy puts £30 in the Post Office Savings Bank on his sixteenth birthday, and the amount accumulates at $2\frac{1}{2}$% per annum compound interest; find, to the nearest penny, the amount on his twentieth birthday.

CHAPTER XVI.

PRESENT VALUE AND DISCOUNT.

137. Suppose that A has engaged to pay B £1040 at the end of one year from now, and that he proposes to pay B now instead. Clearly he should not pay as much as £1040, because B could have the use of the money and put it out at interest during the year.

If interest be allowed at the rate of 4 per cent., what should he pay B?

The interest of £100 in 1 year is £4;
$\therefore$ the amount of £100 ,, ,, £104;
$\therefore$,, ,, £1000 ,, ,, £1040;

Hence the sum that A should pay B now is £1000.

For, if this sum be lent at interest for one year it would become £1040 at the end of the year, and so would then discharge the debt of A to B.

Hence B would be just as well off if he took £1000 now as he would be if he waited one year and then received £1040. The £1000 is called the present value, or present worth, of the £1040 due at the end of the year. The £40, taken off the debt on account of the present payment, is called the **True Discount.**

We thus have £1000 + £40 = £1040;

i.e. **Present Value + True Discount = Sum due,**

i.e. **Present Value + Interest on Present Value = Sum due,**

so that True Discount = Interest on Present Value.

Ex. *Find the present value of* £969 *due* 6 *months hence at* 4 *per cent.*

We want the sum of money which will in 6 months at 4 per cent. interest become £969.

In 6 months the interest on £100 is £2;
$\therefore$,, amount of £100 will be £102;
$\therefore$,, ,, £$\frac{100}{102}$,, £1;
$\therefore$,, ,, £$\frac{100}{102} \times 969$ will be £969.

Now £$\frac{100}{102} \times 969 = £950$, so that the present value is £950.

Also the true discount = £969 − £950 = £19.

EXAMPLES LXXIX.

[*Answers should be worked to the nearest penny.*]

At simple interest find the true discount in the following cases:

	Sum.	Due in	Rate °/₀		Sum.	Due in	Rate °/₀
1.	£101	3 mths.	4.	**2.**	£1025	6 mths.	5.
3.	£419	1 yr. 7 m.	3.	**4.**	£63. 11*s.*	6 mths.	5.
5.	£2240. 9*s.* 4*d.*	4 yrs.	3.	**6.**	£51. 15*s.* 10*d.*	$4\frac{1}{3}$ yrs.	3.
7.	£142. 1*s.* 9*d.*	18 mths.	$3\frac{1}{2}$.	**8.**	£663. 17*s.*	6 mths.	4.
9.	£3143. 6*s.* 8*d.*	3 yrs. 9 m.	4.	**10.**	£420	9 mths.	5.
11.	£721. 13*s.* 8*d.*	73 days	$3\frac{1}{2}$.	**12.**	£201. 4*s.* $7\frac{1}{2}$*d.*	25 days	$3\frac{1}{2}$.

Find the present value in the following cases:

	Sum.	Due in	Rate °/₀		Sum.	Due in	Rate °/₀
13.	£700	3 yrs.	4.	**14.**	£491	7 yrs.	$3\frac{1}{4}$.
15.	£81. 8*s.*	5 mths.	$4\frac{1}{5}$.	**16.**	£574	8 mths.	$3\frac{3}{4}$.
17.	£1836	4 yrs.	5.	**18.**	£2478. 17*s.* 6*d.*	1 yr.	$4\frac{1}{3}$.
19.	£450. 13*s.* 4*d.*	2 yrs.	2.	**20.**	£1616. 2*s.* 8*d.*	3 yrs. 9 m.	4.
21.	£578. 1*s.* 4*d.*	3 yrs. 4 m.	$4\frac{1}{2}$.	**22.**	£1842. 15*s.*	3 mths.	5.
23.	£5747	9 mths.	$3\frac{1}{2}$.	**24.**	£160	7 mths.	4.
25.	£396. 14*s.* 4*d.*	30 days	4.	**26.**	£214. 13*s.* 4*d.*	60 days	5.

27. If the present worth of £218 due 2 years hence be £200, what is the present value of £1000 due 6 years hence at the same rate of simple interest?

28. *A* offers £20,000 now for an estate and *B* offers £22,500 to be paid in 2 years; assuming 4 per cent. as the rate of simple interest, which is the higher offer, and by how much per cent.?

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29. *If the true discount on* £766. 9*s.* 7*d.*, *due* 97 *days hence be* £6. 1*s.* 3*d.*, *at what rate of interest is the discount reckoned?*

The present value = £766. 9*s.* 7*d.* − £6. 1*s.* 3*d.* = £760. 8*s.* 4*d.*, so that this sum and its interest in 97 days amount to £766. 9*s.* 7*d.*

$\therefore$ £6. 1*s.* 3*d.* is the interest on £760. 8*s.* 4*d.* for 97 days;

$\therefore$ £$6\frac{1}{16}$,, ,, £$760\frac{5}{12}$,, 97 ,,

$\therefore$ £$6\frac{1}{16} \times \frac{365}{97}$,, ,, £$760\frac{5}{12}$,, 365 ,,

$\therefore$ £$\frac{97}{16} \times \frac{365}{97}$,, ,, £$\frac{9125}{12}$,, 1 ,,

$\therefore$ £$\frac{\cancel{3}\cancel{8}\cancel{5}}{\cancel{1}\cancel{6}_{4}} \times \frac{\overset{3}{\cancel{1}\cancel{2}}}{\cancel{9}\cancel{7}\cancel{2}\cancel{5}_{5}}$,, ,, £1 ,, 1 ,,

$\therefore$ interest in one year on £100 = $100 \times$ £$\frac{3}{4 \times 25}$ = £3.

30. The true discount on £258. 11*s.* 11*d.* due 18 months hence is £14. 12*s.* 9*d.*; at what rate is the simple interest reckoned?

31. If a person give £255. 5*s.* in payment of a debt of £280. 15*s.* 6*d.* due a year hence, at what rate per cent. simple interest is the discount calculated?

32. If I accept £750 as present payment for £795 due 16 months hence, at what rate % simple interest do I calculate the discount?

What is the rate of interest when

33. the present value of £1120 due in 16 months is £1050?

34. the discount on £2598. 18*s.* 3*d.* due in 3 months is £28. 18*s.* 3*d.*?

35. The present value of a bill of £442. 15*s.* is £385; find how long the bill has to run, allowing $4\frac{1}{2}$ per cent. simple interest.

36. At $4\frac{1}{2}$ per cent., if £40 be the present value of £42. 8*s.*, how long is it before the latter sum was due?

When is the amount due if

37. the amount be £137. 19*s.* 9*d.*, and its present value at 4 per cent. be £136. 3*s.* $5\frac{1}{4}d.$?

38. the amount be £735. 19*s.*, and the discount at $3\frac{1}{2}$ % be £5. 19*s.*?

39. The present worth of a sum due $4\frac{1}{2}$ years hence at 4 per cent. simple interest is £750; find the sum.

40. What is the sum due 16 months hence whose present value at 5 per cent. is £1050?

BANKER'S OR COMMERCIAL DISCOUNT.

138. Suppose *B* owes *A* £500, he can give him a promise in writing to pay after a certain time. Such a promise is called a **Bill of Exchange** or, briefly, a **Bill.**

If *A* wants the money sooner he gets his banker to give him cash for the Bill. The banker will deduct the interest on £500 for the remainder of the time that the Bill has yet to run, and pay *A* £500 less this interest. A Bill treated thus is said to be **discounted.**

A's banker afterwards presents the Bill to *B*'s banker, and receives £500 from him.

Hence Banker's or Commercial Discount differs from True Discount in being the Interest on the Sum Due, not that on the Present Value.

Thus £100 due in 1 year at 5 % will be found to have £$4\frac{16}{21}$ as its True Discount.

The Banker's Discount on £100 due in 1 year at 5 % is 5 % of £100, *i.e.* £5.

139. Days of Grace. By English Law a Bill is *legally* due three days after it is *nominally* due, and these three days are called the Days of Grace. In calculating the time at which a bill is discounted before it is due these three days must always be reckoned.

Thus, suppose a bill for three months to be dated Jan. 17th; it is nominally due on April 17th, but it is not legally due until April 20th. If, therefore, it be discounted on April 1st, this date is 19 days (not 16 days) before it is due, and the discount must be calculated for 19 days, not 16 days.

140. Again, when in a bill the term Month is used, a Calendar Month is always meant. Thus a bill for three months drawn on Jan. 31st is nominally due on the last day of April, that is, on April 30th. Including the three days of grace it is payable on May 3rd. Also a bill for three months drawn on either Nov. 28th, 29th, or 30th is nominally due on the last day of February, that is, on February 28th in an ordinary year. It is therefore payable three days later, namely, on March 3rd.

141. **Ex.** *Find the banker's discount on a bill for* £75 *drawn March* 16*th*, 1905, *for* 3 *months, and discounted on April* 7*th, at* $2\frac{1}{2}$ *per cent.*

The bill is nominally due on June 16th, and is legally payable on June 19th.

April May June

Now, from April 7th to June 19th = 23 + 31 + 19 = 73 days.

The banker's discount = interest on £75 for 73 days at $2\frac{1}{2}$ per cent.

$$= \frac{73}{365} \times \frac{2\frac{1}{2}}{100} \times £75 = \frac{1}{5} \times \frac{5}{200} \times £75 = £\frac{75}{200} = £\frac{3}{8} = 7s.\ 6d.$$

EXAMPLES LXXX.

[*Answers should be worked to the nearest penny.*]

Find the commercial discount on the following bills of

1. £3041. 13*s.* 4*d.* paid 41 days before legally due at $2\frac{3}{4}$ %.
2. £1825 ,, 59 ,, ,, ,, $2\frac{1}{4}$ %.
3. £608. 6*s.* 8*d.* ,, 82 ,, ,, ,, $2\frac{1}{2}$ %.
4. £98. 8*s.* 6*d.* ,, 53 ,, ,, ,, 3 %.
5. £347. 11*s.* 8*d.* ,, 77 ,, ,, ,, 4 %.

Find the commercial discount on the following bills:

	Date of Bill.	Length of Time for which it was Drawn.	Amount of Bill.	Rate of Discount.	Date when Discounted.
6.	June 8th.	3 months.	£2240.	4 %.	June 30th.
7.	June 3rd.	3 months.	£1120.	4 %.	June 25th.
8.	March 14th.	9 months.	£6132. 10*s.*	$3\frac{1}{4}$ %.	May 12th.
9.	August 4th.	3 months.	£97. 17*s.* 8*d.*	$3\frac{3}{4}$ %.	Sept. 13th.
10.	June 23rd.	6 months.	£649.	$3\frac{1}{4}$ %.	July 8th.
11.	April 20th.	3 months.	£500.	$5\frac{3}{4}$ %.	June 5th.
12.	Nov. 30th.	3 months.	£865. 12*s.* 9*d.*	3 %.	Jan. 7th.
13.	Dec. 15th.	4 months.	£785. 15*s.* 6*d.*	$2\frac{1}{2}$ %.	Feb. 15th.

Find the difference between the simple interest (that is, the commercial or banker's discount) and the true discount on

14. £5700 for 4 months at 4 %. **15.** £610 for 4 months at 5 %.

16. £264. 10*s.* for 3 years at 5 %. **17.** £1671. 17*s.* 6*d.* for 21 mths. at 4 %.

18. £100 for 5 months at 5 %. **19.** £4275 for 4 months at 5 %.

20. £500 for 3 months at 5 %.

21. A bill drawn on Jan. 1st, 1905, for 6 months for £772. 14*s.* 1*d.* was discounted on May 5th at $2\frac{1}{2}$ per cent.; find the true discount.

---

22. A bill of £1400 is discounted 3 months before it is due at 4 per cent.; how much is the commercial discount, and by how much does it exceed the true discount?

23. If the difference between the interest and the true discount for 3 months on a sum of money at $3\frac{1}{2}$ % be £1. 9*s.* 9*d.*, what is the sum?

24. Prove that at 5 % the banker's discount on £650 due in 3 months is equal to the true discount on £495. 12*s.* 6*d.* due in 4 months' time.

25. A publisher sells 3000 copies of a book at 41 shillings, and takes in exchange a bill at 6 months; how much would he gain or lose by selling them instead for cash at 39 shillings, commercial discount being at 5 per cent.?

26. If 5 per cent. be the current rate, whether of interest or discount, what will be the value on May 19th of two bills for £273. 15*s.* and £380. 16*s.* 8*d.*, due on May 1st and May 31st respectively?

27. The difference between the true and banker's discount on a sum of money due 12 months hence at 5 % interest is £30; find the sum.

28. *What rate of interest per cent. per annum does a banker get on his money when he discounts a 4 months' bill at 5 per cent.?*

[Off a bill of £100 he takes £$1\frac{2}{3}$, and thus gives £$98\frac{1}{3}$ for it, that is, he charges £$1\frac{2}{3}$ for the loan of £$98\frac{1}{3}$ for 4 months, and thus £5 for the loan of £$98\frac{1}{3}$ for 1 year. Thus the rate % per an. $= 100 \div 98\frac{1}{3} \times 5 = 5\frac{5}{59}$.]

29. If a banker discounts a 3 months' bill at 4 per cent., prove that the interest he gets on his money is $4\frac{4}{59}$ per cent. per annum.

30. A tradesman deducts 13*s.* 3*d.* from a bill of £22. 1*s.* 8*d.* due 6 months hence in consideration of cash payment; at what rate does he allow discount?

CHAPTER XVII.

STOCKS AND SHARES.

142. A Public Company is a partnership of a number of persons who provide between them the money required for the purpose of starting and equipping any business. These persons are called the Shareholders of the Company, and the money which they provide or "subscribe" is called the **Capital** of the Company. The Profits of the Company in each year are divided among the Shareholders in proportion to the amount of the money they have subscribed.

The Capital of a Company is often called its stock, and the profits on this capital, which are divided amongst the Shareholders, are called the **Dividends** on its Stock.

143. A shareholder can, in general, sell his shares to whomsoever he pleases. The price that he will receive for his share is hardly ever the same as the money that he originally subscribed. The price that he actually obtains is dependent on many circumstances, but chiefly on the important point whether the Company makes large or small profits.

Thus, on June 1st, 1905, a person who possessed a share in the London and North-Western Railway which originally cost £100 could have sold it for £153.

Similarly, a person who possessed in the Great Eastern Railway a share which originally cost £100 could have sold it on that date for £87.

Again, a person who possessed a share in the London, Chatham, and Dover Railway, which originally cost £100, could only have sold it on the same date for £17.

The chief reason for this great difference in prices is that in 1904 the profits which were paid to the owner of a £100 share in these three railways were respectively £5$\frac{3}{4}$, £3$\frac{1}{4}$, and nothing.

The actual selling price of any Stock is always quoted at so much per nominal £100 of the stock, that is, at so much for that portion of the capital of the company for which £100 was originally paid.

144. When the price of a Stock is over 100, it is said to be **"at a premium."** When the selling price is less than 100 it is said to be **"at a discount."** When the price is exactly 100 the Stock is said to be **"at par."**

Thus in the examples of Art. 143, London and North-Western Railway Stock is "at a premium of 53 per cent."

Great Eastern Stock is "at a discount of 13 (that is, 100 – 87) per cent.," and London, Chatham, and Dover Railway Stock is "at a discount of 83 (that is, 100 – 17) per cent."

145. The expression "£100 of Stock" thus means that quantity of Stock of which the *nominal* value is £100, that is, that quantity of the Stock which *originally* cost £100. It does not mean that quantity of Stock which would sell for £100. This point must be carefully noted by the student. *Its neglect is the cause of most of the difficulties found in questions on Stocks.*

146. Sometimes the capital of a company is not spoken of as so much Stock, but it is instead divided into shares, each of some given value. Thus the capital of a company may consist of 200,000 shares, each of £10.

147. British Public Debt. Consols. The main portion of the British Public Debt consists (1905) of a sum of about £590,000,000 Consols. The word "Consols" is an abbreviation for "Consolidated Annuities," and is so called because they were formed by consolidation into one stock of various debts, which consisted of sums of money borrowed under various conditions. On this sum the interest paid is $2\frac{1}{2}$ per cent.

148. Stocks and Shares are in general sold through the agency of men called Brokers, and the charge they make for their services is called Brokerage. In the case of British Funds it is, in general, 2*s.* 6*d.* (= £$\frac{1}{8}$) per £100 of stocks.

In the case of stock being bought the brokerage is **added** to the cost. (For the buyer has to pay the price of the stock and *also* to pay the broker for his trouble.)

In the case of stock being sold, the brokerage is **subtracted** from the sale price. (For the seller has to receive the selling price of the stock, and also has to pay the broker for his trouble. Hence he receives in all the selling price of the stock less the brokerage.)

Thus, suppose that the price of Consols is £91. 10*s.*; a person who buys £100 of Consols would pay to his broker £91. 10*s.* + 2*s.* 6*d.*, that is, £91. 12*s.* 6*d.*

So a person who sold £100 Consols at £91. 10*s.* would receive from his broker £91. 10*s.* – 2*s.* 6*d.*, that is, £91. 7*s.* 6*d.*

Brokerage is usually given as so much per cent. Thus "brokerage $\frac{1}{8}$ per cent." or "brokerage $\frac{1}{8}$" means that a broker-

age of $£\frac{1}{8}$, that is, 2*s.* 6*d.*, is to be reckoned on each nominal £100 of stock. It must be carefully noted that it does *not* mean so much per cent. on the cash paid or received by the customer.

149. We proceed to give some examples of the different kinds of problems on Stocks and Shares.

Case I. *To find the cost of a given quantity of stock bought at a given quoted price, and the resulting income.*

Ex. *Find the cost of* £1550 $2\frac{1}{2}$ *per cent. Consols at* 91, *and the income derived therefrom.*

[By £100 Consols we mean a nominal £100, that is, the amount for which the Government will ultimately refund £100.]

£100 of the stock cost £91 ;

$\therefore$ £1550 ,, ,, $£\frac{91}{100} \times £1550$;

$$\therefore \text{ cost} = £\frac{91}{\cancel{100}_{2}} \times \overset{31}{\cancel{1550}} = £\frac{2821}{2} = £1410.\ 10s.$$

Also the income from £100 Consols is $£2\frac{1}{2}$.

$\therefore$ the income from £1550 Consols $= £\frac{1550}{100} \times 2\frac{1}{2} = £\frac{31}{2} \times \frac{5}{2} = £\frac{155}{4}$

$= £38.\ 15s.$

With Brokerage. If brokerage be allowed at the rate of $\frac{1}{8}$ per cent., the cost of £100 of stock $= £91 + £\frac{1}{8} = £91\frac{1}{8}$.

$\therefore$ the cost of £1550 of stock $= 91\frac{1}{8} \times \frac{1550}{100} = \frac{729}{8} \times \frac{31}{2}$

$= \frac{22599}{16} = £1412\frac{7}{16} = £1412.\ 8s.\ 9d.$

EXAMPLES LXXXI.

[*Unless otherwise stated the brokerage is throughout the chapter supposed to be included in the price of the stock. In the final answers fractions of a penny may be omitted.*]

Find the cost of, and the income derived from,

1. £500 $2\frac{1}{2}$ per cent. Consols at $90\frac{1}{4}$.
2. £1000 G.-W. Railway Stock (dividend $5\frac{1}{2}$ per cent.) at $138\frac{1}{2}$.
3. £10,000 L. N.-W. Railway Stock (dividend $5\frac{3}{4}$ per cent.) at $152\frac{1}{4}$.
4. £700 Railway Stock at $169\frac{1}{8}$ (dividend 6 per cent.).
5. £4800 4 per cent. railway debentures at 110.
6. £4500 Great Eastern Ordinary at $82\frac{7}{8}$ (dividend $3\frac{1}{8}$ per cent.).
7. £1550 4 per cent. railway stock at $130\frac{5}{8}$.
8. £1666. 13*s.* 4*d.* L. S.-W. Railway 3 per cent. debenture at $117\frac{1}{2}$.
9. £3846. 5*s.* India 3 per cents. at 110.
10. £150 5 per cent. stock at $161\frac{3}{4}$ (brokerage $\frac{1}{2}$ per cent.).
11. £5346. 13*s.* 4*d.* G. E. R. 4% Preference Stock at $146\frac{1}{2}$ (brokerage $\frac{1}{4}$%).

12. A man buys £500 stock at 66, and afterwards 500 more of the same stock at 69. He sells the whole when the price has risen to 89; find the increase in his capital.

150. Case II. *Given the sum of money to be invested in a given stock at a given quoted price, to find the amount of stock obtained and the income derived from the investment.*

Ex. *A man invests* £10,000 *in the purchase of Railway Stock at* 160*; how much stock does he purchase, and what is the income he derives, supposing the dividend on the stock to be* 6 *per cent.?*

£160 cash will buy £100 stock.

$\therefore$ £10000 cash will buy £$\frac{100\times10000}{160}$ stock;

$\therefore$ amount of stock bought = £$\frac{100\times125}{2}$ = £6250.

Also the dividend on each £100 of stock = £6;

$\therefore$,, ,, £6250 of stock = £$\frac{\overset{125}{\cancel{6250}}}{\underset{2}{\cancel{200}}}\times\overset{3}{\cancel{6}}$ = £375.

In this question no brokerage is charged; if it had to be reckoned, say at $\frac{1}{4}$ per cent., then the cost of each £100 of stock would have been £160$\frac{1}{4}$ instead of £160.

EXAMPLES LXXXII.

Neglecting fractions of a penny, what income is derived from investing

1. £250 in a 3 % stock at 66$\frac{2}{3}$? **2.** £13,000 in a 3$\frac{1}{2}$ % stock at 91?

3. £8415 in a 3$\frac{3}{4}$ % stock at 137$\frac{1}{2}$? **4.** £13,000 in a 4 % stock at 96?

5. £12,002. 12*s.* 4$\frac{1}{2}$*d.* in a 3 % stock at 93?

6. £2000 in a 3 % stock at 93$\frac{1}{4}$?

7. £1250 in a 11$\frac{1}{2}$ % gas stock at 229?

8. £6937. 10*s.* in a 2$\frac{1}{2}$ % stock at 93$\frac{1}{2}$ (brokerage $\frac{1}{4}$ %)?

9. £2150 in the 2$\frac{3}{4}$ per cents. at 108$\frac{3}{8}$ (brokerage $\frac{1}{8}$ %)?

10. £2436. 13*s.* 4*d.* in the 2$\frac{1}{2}$ % Consols at 91$\frac{1}{4}$ (brokerage $\frac{1}{8}$ %)?

151. Case III. *To find the sum of money that must be invested in a given stock to obtain a given income.*

Ex. *Find what sum of money must be invested in a* 3 *per cent. stock at* 104$\frac{3}{4}$*, to bring in an income of* £100 *per year.*

By a 3 % stock is meant a stock the dividend on which is 3 % per annum.

In this case it is unnecessary to find the amount of stock bought.

For £104$\frac{3}{4}$ cash will buy £100 stock, and hence give an income of £3.

$\therefore$ £$\frac{104\frac{3}{4}}{3}$ will give an income of £1;

i.e. £$\frac{104\frac{3}{4}}{3}\times100$,, ,, ,, £100;

$\therefore$ *Ans.* = £$\frac{100\times104\frac{3}{4}}{3}$ = £$\frac{100}{3}\times\frac{419}{4}$ = £$\frac{41900}{12}$ = £3491. 13*s.* 4*d.*

EXAMPLES LXXXIII.

What sum of money must a man invest in order to obtain an income of

1. £300 a year from a railway stock at 173¼ which pays 9⅝ per cent.?

2. £1153 a year from the 2¾ per cents. at 105?

3. £25 per quarter from the 2¾ per cents. at 105¾ (brokerage ⅛ per cent.)?

4. £100 a year from the 2½ per cent. Consols at 90½?

5. £6883. 10*s.* by investing half his capital in the 3 per cents. at 90, and the other half in the 5 per cents. at 110?

6. £455. 13*s.* 4*d.* by investing one-third of his capital in the 5 per cents. at 142½, and two-thirds in the 4 per cents. at 113¾?

152. Case IV. *Given the income derived from the investment of a given sum of money in a given stock, to find the price of the stock.*

Ex. 1. *A man derives an income of* £100 *a year from the investment of* £2666. 13*s.* 4*d. in a* 3 % *stock; at what price was the stock quoted when he bought?*

Here £2666. 13*s.* 4*d. cash* produces £100 *income.*

∴ £8000 ,, ,, £300 ,,

∴ £80 ,, ,, £3 ,,

But £100 *stock* ,, £3 ,,

∴ £100 *stock* was worth £80 *cash*; ∴ price of the stock = £80.

Ex. 2. *By investing in a* 5 % *stock I get* 3⅛ % *on my money; what is the price of the stock?*

Here £100 cash produces £3⅛, that is, £$\frac{25}{8}$ income;

∴ £$\frac{800}{25}$,, ,, £1 income;

∴ £$\frac{800}{25}\times 5$, that is, £160 cash produces £5 income.

But £100 stock produces £5 income;

∴ £100 stock is worth £160 cash; ∴ price = £160.

EXAMPLES LXXXIV.

1. Find the price of a 3 per cent. stock when an investment of £434. 12*s.* 6*d.* cash in it produces an income of £14. 5*s.*

2. By investing £1365 in the 2½ % Consols, and paying ⅛ brokerage, a man obtains an income of £37. 10*s.*; at what price did he buy?

3. A person investing in a 4 % stock receives 4⅜ % on his money; what is the price of the stock?

4. What must be the price of a 6 % stock so that a man may get 4½ % on his money by investing in it?

5. What is the price of a 4 % stock from which a man would derive the same income as from an investment in a 4½ % stock at 108?

6. A man has an income of £415 derived from capital invested in a 4 per cent. stock. He sells out this stock at 119, and reinvests the proceeds in a 5 per cent. stock. What is the price of the latter stock if his new income is £425?

7. At what price must I buy a 4 % stock with the proceeds of the sale of £9600 3 % stock at $86\frac{3}{8}$ in order that I may have no change in my income, $\frac{1}{8}$ % brokerage being allowed on each transaction?

153. Case V. *To find which of two or more given stocks is the more profitable investment.*

Ex. *Which is the more profitable investment, a $3\frac{1}{2}$ per cent. stock at $93\frac{1}{2}$, or a 4 per cent. stock at $107\frac{1}{4}$?*

This question can be solved by finding what income would be obtained by investing any—the same—sum in each stock.

The income derived from the investment of £100 in the first stock

$$= £\frac{100}{93\frac{1}{2}} \times 3\tfrac{1}{2} = £\frac{100}{\frac{187}{2}} \times 7 = £\frac{700}{187} = £3\frac{139}{187} = £3{\cdot}74 \ldots.$$

The income derived from the investment of £100 in the second stock

$$= £\frac{100}{107\frac{1}{4}} \times 4 = £\frac{1600}{429} = £3\frac{313}{429} = £3{\cdot}72 \ldots.$$

The first income is therefore greater than the second income.

Hence the first stock is the more profitable.

EXAMPLES LXXXV.

Which is the more profitable investment—

1. A 3 per cent. stock at 98, or a $3\frac{1}{4}$ per cent. stock at 101?

2. A $2\frac{1}{2}$ per cent. stock at 75, or a 4 per cent. stock at 120?

3. A 5 per cent. stock at $142\frac{1}{2}$, or a 4 per cent. stock at $113\frac{3}{4}$?

4. A $3\frac{1}{2}$ per cent. stock at $80\frac{5}{8}$, or a 4 per cent. stock at $93\frac{1}{2}$?

5. A 3 per cent. stock at $83\frac{1}{4}$, a $3\frac{1}{2}$ per cent. stock at $98\frac{7}{8}$, or a 4 per cent. at $107\frac{1}{2}$, and which is the least profitable?

6. A 3 per cent. stock at $92\frac{3}{4}$, or a 4 per cent. stock at $123\frac{1}{4}$?

Find the difference in income obtained from the investment of

7. £37800 in a 4 % stock at $112\frac{1}{2}$, and a $2\frac{1}{2}$ % stock at 75.

8. £4850 in a $3\frac{1}{2}$ % stock at 97, and a $3\frac{3}{4}$ % stock at par.

9. Which will pay the better to invest in, a $2\frac{3}{4}$ per cent. stock at $108\frac{1}{2}$, or a $3\frac{1}{4}$ per cent. stock at $129\frac{3}{8}$? and what must be the sum of money invested for the difference to be £3. 5*s.*?

10. A 3 per cent. stock is at $92\frac{3}{4}$ and a 4 per cent. stock at $123\frac{1}{4}$. In which should one invest? How much is one investing when the difference of income is 1*s.*?

154. Case VI. *A given quantity of a given stock is sold, to find how much of a second given stock can be bought with the proceeds, and the resulting change of income.*

Ex. *A man sells* £28,000 *of a* $2\frac{3}{4}$ *per cent. stock at* $106\frac{5}{8}$ (*brokerage* $\frac{1}{8}$), *and buys with the proceeds a* 4 *per cent. railway stock at* $141\frac{1}{2}$ (*brokerage* $\frac{1}{2}$); *find the change in his income, and the amount of the new stock that he buys.*

Income from the first stock $= £\frac{28000}{100} \times 2\frac{3}{4} = £770$.

Each £100 of the first stock sells for $£106\frac{5}{8} - £\frac{1}{8}$, that is, $£106\frac{1}{2}$.

Hence sale price of £28,000 of the first stock

$$= £\frac{28000}{100} \times 106\frac{1}{2} = £280 \times \frac{213}{2} = £140 \times 213.$$

The cost of each £100 of the second stock $= £141\frac{1}{2} + £\frac{1}{2} = £142$.

Hence each £142 cash invested in the new stock brings £4.

$\therefore$ total income from the new stock $= \frac{140 \times 213}{142} \times \frac{4}{1} = £840$;

$\therefore$ gain in income $= £840 - £770 = £70$.

Also, since each £142 cash buys £100 of the new stock,

$\therefore$ amount of the new stock bought $= £\frac{70 \times 213}{142} \times 100 = £21,000$.

EXAMPLES LXXXVI.

Find the change in a man's income if he transfer

1. £4750 from a 4 % stock at 88, to a 5 % stock at $104\frac{1}{2}$.

2. £5000 from a 4 % stock at 108, to a 5 % stock at 120.

3. £6000 from a 5 % stock at 95, to a 4 % stock at 114.

4. £4500 from a $2\frac{1}{2}$ % stock at 75, to a $3\frac{1}{2}$ % stock at 105.

5. £10000 from the $2\frac{1}{2}$ % Consols at 91, to a 4 % railway stock at 128.

6. £23400 from the $4\frac{1}{2}$ per cents. at $96\frac{1}{2}$ to the $3\frac{1}{4}$ per cents. at 73. [Brokerage $\frac{1}{8}$ % in each case.]

7. £1475 from a 3 % stock at 81, to a 6 % stock at $129\frac{3}{8}$.

8. A man invests £4500 in a 6 per cent. stock at 98. Two years later he sells out at par, and invests both principal and accrued interest in a $3\frac{1}{2}$ per cent. railway stock at 80; find his new income.

9. I invest £2200 in buying 6 per cent. stock at $137\frac{1}{2}$. When the price rises to $142\frac{1}{2}$ I sell out and invest in a 5 per cent. stock at 120; what is the difference in my income from the two investments?

10. I invest £3000 in buying a 4 per cent. stock at 90. When the price rises to 95 I sell out and invest the proceeds in a 5 per cent. stock at 110; what is the difference in my income from the two investments?

11. I invest £1050 in the 3 per cents. at $82\frac{1}{2}$, and sell out at 85, and with the proceeds buy railway 6 per cents. at 115; what is the difference in my income?

155. Case VII. *To find the rate of interest derived from the investment of money in a given stock, or stocks, at a given price.*

If a man invests in a 4 per cent. stock at 105, it is clear that he does not get an interest of 4 per cent. on his money. For to

buy £100 of stock, and thus to get £4 per annum income, he must pay £105. Thus £105 cash will bring an income of £4.

Hence £100 cash will bring something less than £4. The amount it will give him as income $= £\frac{100}{105} \times 4 = £\frac{80}{21} = £3\frac{17}{21}$.

Ex. *What is the rate per cent. of interest that a man gets on money invested in a 4 per cent. stock, the price of which (including brokerage)* is £119⅜?

On £119⅜ cash the income from this stock is £4.

∴ ,, £1 ,, ,, ,, $£\frac{4}{119\frac{3}{8}}$.

∴ ,, £100 ,, ,, ,, $£\frac{4 \times 100}{119\frac{3}{8}}$.

∴ *Ans.* $= \frac{£100 \times 4}{£119\frac{3}{8}} = £\frac{\overset{20}{\cancel{100}} \times 4 \times 8}{\underset{191}{\cancel{959}}} = £\frac{640}{191} = £3.\ 7s.$ per cent. nearly.

EXAMPLES LXXXVII.

What rate of interest does a man get on his money if he invest

1. In a 3 per cent. stock at 90? **2.** In a 4½ per cent. stock at 112½?

3. In a 5 per cent. stock at 130?

4. £1980 in a 3½ % stock at 99, and £3220 in a 4½ % stock at 105?

5. £990 in a 3½ % stock at 99, and £1610 in the 4½ per cents. at 105?

6. One-third in a 2¾ % stock at 99, and the rest in a 3½ % stock at 105?

7. By investing £2509 in a certain stock at 144¾ a man obtains an income of £112. 13*s.* 4*d.*; what per cent. is the stock paying?

8. A man invests his money in the 2½ % Consols at 90, and at the end of 4½ months, after receiving a half-year's dividend, sold out at 91½. At what rate per cent. per annum did he receive interest on his capital?

156. We add an example where income tax has to be deducted from the income obtained.

Ex. *If the 2⅝ % Consols be at* 91 *(including brokerage), what sum of money must be invested to produce a net income of* £200 *per annum after an income tax of* 1*s. in the* £ *has been deducted?*

£91 will buy £100 of stock on which the dividend is £2½. Also the income tax on £2½ = 2½ shillings = £⅛.

Hence the net income = £2½ − £⅛ = £2⅜ = $£\frac{19}{8}$.

∴ $£\frac{19}{8}$ is the net income from an investment of £91.

∴ £1 ,, ,, ,, ,, $£91 \times \frac{8}{19}$.

∴ £200 ,, ,, ,, ,, $£91 \times \frac{8}{19} \times 200$.

∴ *Ans.* $= £\frac{91 \times 8 \times 200}{19} = £\frac{145600}{19} =$ £7663. 3*s.* 2*d.* to the nearest penny.

EXAMPLES LXXXVIII.

What sum must be invested to produce an income of

1. £47. 10*s.* at 4 % interest after deducting income tax at 1*s.*?

2. £306. 13*s.* 4*d.* from a 3 % stock at 87 after paying income tax at 10*d.*?

3. £146. 5*s.* from a 4 % stock at 105 after paying income tax at 6*d.*?

What income arises from the investment of

4. £7000 in a 4 % stock at $109\frac{3}{8}$ after paying income tax at 9*d.*?

5. £3040 in $2\frac{1}{2}$ % Consols at 91 after deducting income tax at 1*s.*?

6. £8064. 7*s.* in a 3 % stock at $102\frac{3}{8}$ after deducting income tax at 11*d.*?

What is the market price of

7. A 3 % stock if it yields $3\frac{1}{3}$ % after paying an income tax at 10*d.*?

8. A $4\frac{1}{2}$ % stock if it yields $3\frac{3}{4}$ % after paying income tax at 9*d.*?

9. A 6 % stock if it yields 5 % after paying income tax at 10*d.*?

10. $2\frac{1}{2}$ % Consols if they yield $2\frac{3}{4}$ % after paying income tax at 1*s.*?

11. Which is the better investment: $2\frac{1}{2}$ per cent. Consols at 100, subject to an income tax of 1*s.*; or railway stock at 171, paying $4\frac{1}{4}$ per cent. free of income tax?

MISCELLANEOUS EXAMPLES LXXXIX.

1. If $4\frac{1}{2}$ shares in a mine cost £190. 2*s.* 6*d.*, what is the cost of 21 shares?

2. If I buy £800 stock at $92\frac{7}{8}$ per cent. on January 1, but defer payment until the following March 15, what sum shall I have to pay, including interest at $4\frac{1}{2}$ per cent. per annum on the deferred payment?

3. A man invests £2500 in a 7 per cent. Japanese loan, which was issued at 98 per cent., at $1\frac{1}{2}$ per cent. discount; how much stock did he buy, and what was the income he derived?

4. If a 3 per cent. stock is at $98\frac{1}{4}$, how much money must be invested to yield an income of £120 per annum? Find also to the nearest penny the annual income from the same amount of money invested in a 4 per cent. stock at $127\frac{1}{8}$.

5. A man invests £25,935 in a 3 per cent. stock at 90. If the first year's dividend be invested in the same stock at 91, and the whole dividend for the second year at 95, what will be his income for the third year?

6. A sum of money invested in a stock at 95 produces £45 a year; what must be the price of this stock if the same sum of money would by being invested in it produce an income of £47. 10*s.*?

7. A man invests his capital, half in $3\frac{1}{2}$ per cent. and half in 4 per cent. stocks, and derives the same revenue from each. The $3\frac{1}{2}$ per cents. being at 98, what is the price of the 4 per cents.?

8. A man invests half his capital in a 4 per cent. stock, and half in a $4\frac{1}{2}$ per cent. stock, and derives the same income from each. The former stock being at 108, what is the price of the latter?

9. I invest £3740 in stock at $93\frac{1}{2}$. When the stock is at $95\frac{1}{8}$ I sell out half, and when at $92\frac{7}{8}$ I sell out the remainder; find the alteration in my capital, and the price at which the same stock must be if on reinvesting my capital I get the same income as before.

10. A person invests £1365 in the 3 per cents. at 91; he sells out £1000 stock when they have risen to $93\frac{1}{2}$, and the remainder when they have fallen to 85: how much does he gain or lose by the transaction?

11. A certain sum of money produces £666. 13*s.* 4*d.* when invested in $2\frac{1}{2}$ per cent. Consols, and £720 when invested in a railway 3 per cent. debenture at par; find the sum and the price of Consols.

12. What income does a man get by investing £3220 in the $3\frac{1}{2}$ per cents. at $80\frac{1}{2}$? If the stock rises to 92 and he then sells, at what price must he invest in a 4 per cent. stock to get £10 more yearly income?

13. The receipts of a railway company are applied as follows: 69% for working expenses, 10% for the reserve fund, a guaranteed dividend of 5% on one-fifth of the capital, and the remainder £40,000 for division among the holders of the rest of the stock, being a dividend of 4% per annum; find the capital and the receipts.

14. The capital of a railway is five millions. The average quarterly income is £222,892. 13*s.* 4*d.*, and the average weekly expenditure is £12,818. 13*s.* 4*d.*; at what rate per cent. is the dividend which the shareholders receive?

15. The gross receipts of a railway company in a certain year were apportioned as follows: 41% to pay the working expenses, 56% to give the shareholders a dividend at the rate of $3\frac{1}{2}$% on their shares, and the remainder, £15,000, is reserved; find the capital of the company.

16. A man sold $2\frac{1}{2}$ per cent. Consols at 85, and invested the proceeds in a 5 per cent. stock at $127\frac{1}{2}$, and thereby increased his income by £20 a year; what was his original income?

17. A man finds that he can obtain £5 more annual income by investing in the $3\frac{1}{2}$ per cents. at 96 than in the 3 per cents. at 88; how much has he to invest?

18. A man invests £12,000 partly in a 4 per cent. stock at 88, and partly in a 6 per cent. stock at 108, so that his income from each stock is the same; what is his income?

[If £x be the amount invested in the first stock, then

$$\frac{x}{88}\times 4=\frac{12000-x}{108}\times 6, \text{ giving } x.]$$

19. A man invests £9256. 10*s.* partly in 5 per cent. stock at 150, and partly in a $2\frac{3}{4}$ per cent. stock at 99; what sum must he invest in each stock in order that his income from each may be the same?

20. I invest £7000 partly in a 3 per cent. stock at 97, and the remainder in a 4 per cent. stock at 104; find how much I must invest in each in order that I may have equal returns from the two sources.

21. How must the sum of £1560 be divided so that, if one portion be invested in a 3% stock at 104 and the other in a 4% stock at 130, the net incomes, after an income tax of 6*d.* in the £ has been deducted in each case, may be as 3 : 2?

CHAPTER XVIII.

SQUARE ROOT.

157. If we multiply any number by itself, the result is called the square of that number: also this square is denoted by placing a small 2 to the right of, and slightly higher than, the given number.

Thus 3×3 is 9. This is expressed by saying that the square of 3 is 9, that is, that $3^2 = 9$. Similarly $4^2 = 16$, $7^2 = 49$, etc.

The square root of a given number is that number which, when multiplied by itself, is equal to the given number.

Thus, since $3 \times 3 = 9$, the square root of 9 is 3.

Also, ,, $13 \times 13 = 169$, ,, 169 is 13.

The symbol $\surd$, or $\sqrt{\ }$, is used to denote the square root of a number. Thus $\surd 4$ = the square root of 4 = the number which, when multiplied by itself, gives 4 = 2, since $2 \times 2 = 4$.

So $\surd 9 = 3$, since $3 \times 3 = 9$. Also, $\sqrt{196} = 14$, since $14 \times 14 = 196$.

[The symbol $\surd$ is really an abbreviation of the letter *r*, the initial letter of the Latin word *radix*, which means Root.]

158. By ordinary multiplication we have

$1^2 = 1$,	$5^2 = 25$,	$9^2 = 81$,	$13^2 = 169$,	$17^2 = 289$,
$2^2 = 4$,	$6^2 = 36$,	$10^2 = 100$,	$14^2 = 196$,	$18^2 = 324$,
$3^2 = 9$,	$7^2 = 49$,	$11^2 = 121$,	$15^2 = 225$,	$19^2 = 361$,
$4^2 = 16$,	$8^2 = 64$,	$12^2 = 144$,	$16^2 = 256$,	$20^2 = 400$.

[The square of any number of two digits may be found mentally as follows: Suppose 23 is the number: find the nearest multiple of 10: this is 20: take the number as much above 23 as 20 is below 23: this will be 26: Multiply 26 by 20 and add 3^2 (the square of the difference between the given number and the nearest multiple of 10): this gives us the required result. Thus $23^2 = 26 \times 20 + 3^2 = 529$.

Similarly $46^2 = 42 \times 50 + 4^2 = 2116$.

$65^2 = 60 \times 70 + 5^2 = 4225$.

The proof of this rule is Algebraic.]

159. Conversely, the table of the preceding article will give us some square roots.

Thus $\sqrt{289} = 17$, $\sqrt{324} = 18$, $\sqrt{361} = 19$, etc.

We should thus expect that the square root of any number between 289 and 324, say 305, would lie between 17 and 18.

We also see that, since there is no whole number between 17 and 18, the square root of 305 cannot be a whole number.

160. An integer is said to be a **perfect square** when its square root is a whole number. Thus 25, 49, 81, 121, 225, ... are perfect squares, because their square roots are 5, 7, 9, 11, 15, ... respectively.

161. In the case of some numbers the square root may easily be found by separating the number into factors which are themselves squares. We should try, in succession, and see whether any such number is divisible by 4, 9, 16, 25, 36, ... which are the squares of the small numbers 2, 3, 4, 5, 6, ...

Ex. 1. $2916 = 4 \times 729 = 4 \times 9 \times 81 = 2^2 \times 3^2 \times 9^2$;

$\therefore \sqrt{2916} = 2 \times 3 \times 9 = 54.$

Ex. 2. $7056 = 4 \times 1764 = 4 \times 4 \times 441 = 4 \times 4 \times 9 \times 49 = 4^2 \times 3^2 \times 7^2$,

so that $\sqrt{7056} = 4 \times 3 \times 7 = 84.$

Ex. 3. $148225 = 25 \times 5929 = 25 \times 7 \times 847 = 25 \times 7 \times 7 \times 121 = 5^2 \times 7^2 \times 11^2$,

so that $\sqrt{148225} = 5 \times 7 \times 11 = 385.$

EXAMPLES XC.

By breaking into factors, find the square roots of

1. 225. **2.** 324. **3.** 576. **4.** 784. **5.** 1024. **6.** 1225.
7. 1764. **8.** 2304. **9.** 4096. **10.** 5625. **11.** 17424.
12. 28224. **13.** 46656. **14.** 65536. **15.** 97344. **16.** 112896.

162. *To tell, by inspection, how many digits there are in the square root of a given number, which is a perfect square.*

Since $1^2 = 1$, $\therefore \sqrt{1} = 1$;

$10^2 = 100$, $\therefore \sqrt{100} = 10$;

$100^2 = 10000$, $\therefore \sqrt{10000} = 100$;

$1000^2 = 1000000$, $\therefore \sqrt{1000000} = 1000$;

$10000^2 = 100000000$, $\therefore \sqrt{100000000} = 10000$;

We observe that the square root of any number between 1 and 100 has **one** digit in it, *i.e.* the square root of a number containing **one** or **two** digits contains **one** digit. Similarly the square root of a number of **three** or **four** digits contains **two** digits and so on.

163. The simplest method of finding the number of digits in the square root of any number may be seen by considering such a number as 3845521.

Place a dot over the number 1 in the units place, and over every alternate number towards the left, that is, on the 5, 4, and 3. The number of digits in our number is 7, and hence by the preceding article the number of digits in its square root is 4, which is equal to the number of dots we have placed.

Similarly consider the number $9\dot{8}8\dot{0}3\dot{6}$. The number of digits in it is 6; hence by the last article the number of digits in its square root is 3, and this is the number of dots, if we place a dot over the 6 and over each alternate figure towards the left.

So the number of digits in the square root of $6\dot{3}9\dot{4}2\dot{7}3\dot{1}1\dot{6}\dot{1}$ is six, and in that of $6\dot{7}4\dot{0}4\dot{1}$ the number is three.

We shall now show how to find the square root of a number. The proof of the method is best omitted until the student has mastered the elements of Algebra.

164. **Ex. 1.** *Find the square root of* 222784.

Take the first two figures 22 as far as the first dot; 4 is the largest number whose square is less than 22, set its square under the 22, subtract and we have 6.

Bring down the next two figures 27 as far as the next dot, and we have 627. Set the double of 4, that is 8, to the left of the 627. Now 8 goes into 62 seven times. Take this 7 as the trial number. Place it after the 4 in the first line, and also after the 8 in the third line.

Multiply the 87 by the 7, and we have 609, which we place under the 627 and subtract. We thus have 18 and the last two figures of the original number on being brought down, make 1884.

$$\begin{array}{r|l} 2\dot{2}2\dot{7}8\dot{4} & (472 \\ 16 & \\ \hline 87) & 627 \\ & 609 \\ \hline 942) & 1884 \\ & 1884 \\ \hline \end{array}$$

Double the part found, that is 47, and put it, as 94, to the left of the 1884. Now 94 divides into 188 twice; set a 2 to the right of the 94 and to the right of the 47; multiply 942 by 2 and place it in the fifth line. On subtraction there is no remainder.

Thus 472 is the required square root.

Ex. 2. *Find the square root of* 4338889.

On setting down the square of 2 and subtracting it from 4 we have a remainder 0; 33 is then brought down.

On doubling the 2 we have 4; and 4 into the first figure, 3 of the 33, will not divide as much as 1; the next digit in the square root is therefore 0. We set down the 0 next the 2 and next the 4 and bring down the next two figures 88.

We thus have 3388 and 40 at its left. Now 40 goes into 338 more than 8 times. Thus 8 is the next figure of the root, etc.

$$\begin{array}{r|l} 4\dot{3}3\dot{8}8\dot{8}9 & (2083 \\ 4 & \\ \hline 408) & 3388 \\ & 3264 \\ \hline 4163) & 12489 \\ & 12489 \\ \hline \end{array}$$

EXAMPLES XCI.

Find the square root of

1. 289.	**2.** 441.	**3.** 729.	**4.** 1024.
5. 1521.	**6.** 1849.	**7.** 2209.	**8.** 3481.
9. 7569.	**10.** 7921.	**11.** 9216.	**12.** 43264.
13. 65536.	**14.** 82369.	**15.** 95481.	**16.** 207936.
17. 452929.	**18.** 546121.	**19.** 549081.	**20.** 622521.
21. 797449.	**22.** 3312400.	**23.** 32239684.	**24.** 63409369.
25. 1079521.	**26.** 574560900.	**27.** 1371739369.	**28.** 10074538384.

SQUARE ROOTS OF DECIMAL FRACTIONS AND OF MIXED DECIMALS.

165. Consider the following examples:

$$\sqrt{\cdot 81} = \sqrt{\tfrac{81}{100}} = \tfrac{9}{10} = \cdot 9;$$

$$\sqrt{\cdot 1296} = \sqrt{\tfrac{1296}{10000}} = \tfrac{36}{100} = \cdot 36;$$

$$\sqrt{\cdot 0169} = \sqrt{\tfrac{169}{10000}} = \tfrac{13}{100} = \cdot 13;$$

$$\sqrt{\cdot 0064} = \sqrt{\tfrac{64}{10000}} = \tfrac{8}{100} = \cdot 08;$$

$$\sqrt{147\cdot 1369} = \sqrt{\tfrac{1471369}{10000}} = \tfrac{1213}{100} = 12\cdot 13.$$

It is clear from these examples that the same rule holds for decimals as for integers. We set down the number, and *put a point on the figure in the units place*, and also a point on each alternate figure to the right as well as to the left.

Proceed just as if there were no decimal point, but as soon as any portion of the decimal part of the given number is brought down, insert a decimal point in the square root.

Ex. 1. *Find the square root of* $\dot{1}47\dot{\cdot}1\dot{3}6\dot{9}$.

$$\begin{array}{l} \dot{1}47\dot{\cdot}1\dot{3}6\dot{9}\ (\ 12\cdot 13 \\ \underline{1} \\ 22\ )\ 47 \\ \quad\ \underline{44} \\ 241\ )\ 313 \\ \quad\ \underline{241} \\ 2423\ )\ 7269 \\ \quad\ \underline{7269} \end{array}$$

Ex. 2. *Find the square root of* ·018769.

For the purpose of pointing we may write this decimal as $\dot{0}\cdot 0\dot{1}8\dot{7}6\dot{9}$.

The points will then be over the 0 of the whole number, and over the 1, 7, and 9 of the decimal fraction. It will, however, be unnecessary to insert the 0 of the whole number, if we commence pointing from the second figure of the decimal commencing from the decimal point.

$$\begin{array}{r|l} \dot{0}\cdot 0\dot{1}8\dot{7}6\dot{9} & (\cdot 137 \\ 1 & \\ \hline 23)\ 87 & \\ 69 & \\ \hline 267)\ 1869 & \\ 1869 & \\ \hline \end{array}$$

Since we commence by using figures of the decimal part, we begin by inserting a point in the answer.

EXAMPLES XCII.

Find the square root of

1. ·25.	**2.** ·64.	**3.** ·09.	**4.** ·0004.	**5.** ·0049.
6. ·0289.	**7.** 10·24.		**8.** 37·21.	**9.** 20·8849.
10. 8210·1721.	**11.** 18308·7961.		**12.** ·08450649.	**13.** 1·07101801.
14. ·02010724.	**15.** 1532·7225.		**16.** 170·3025.	**17.** 1200·6225.

166. Up to the present all the numbers with which we have dealt in this chapter have had a square root, that is, they have been perfect squares.

Suppose now that we try to find the square root of 3. By writing it in the form $\dot{3}\cdot\dot{0}0\dot{0}0\dot{0}0\dot{0}0$ ... we can continue the process of finding its square root to as many places of decimals as we please, but the operation will never terminate.

Thus the square root of 3 is not a terminating decimal.

167. In actual practical calculations, we never require the value of such a square root correct to more than a few decimal places, and these can always be found.

Ex. *Find to five places of decimals the value of* $\sqrt{5}$.

It will be here observed that after the 5 in the first line it is quite unnecessary to write down a decimal point and a number of zeros to its right merely for the purpose of putting a point over each alternate one. We can omit these zeros, and merely at each stage of the work annex two zeros as we have done.

$$\begin{array}{r|l} \dot{5}\cdot\dot{0}0\dot{0}0\dot{0}0\dot{0} \ldots & (2\cdot 23606 \ldots \\ 4 & \\ \hline 42)\ 1\ 00 & \\ 84 & \\ \hline 443)\ 1600 & \\ 1329 & \\ \hline 4466)\ 27100 & \\ 26796 & \\ \hline 447206)\ 3040000 & \\ 2683236 & \\ \hline 356764 & \end{array}$$

EXAMPLES XCIII.

Find the value, to five places of decimals, of the square roots of

1. 3.	**2.** 6.	**3.** 7.	**4.** 19.	**5.** 13.	**6.** 17.
7. ·1.	**8.** ·4.	**9.** ·005.	**10.** ·121.	**11.** ·169.	**12.** ·289.
13. 73.	**14.** 89.	**15.** ·029.	**16.** ·571.
17. ·0565.	**18.** 536·9.	**19.** 35·672.	**20.** 3·9726523.

SQUARE ROOTS OF VULGAR FRACTIONS AND MIXED NUMBERS.

168. Since the square of any vulgar fraction is obtained by squaring its numerator and also squaring its denominator, therefore, conversely, the square root of a vulgar fraction is obtained by finding the square root of its numerator and also the square root of its denominator.

A mixed number may be reduced to an improper fraction.

Thus $\sqrt{\frac{25}{36}} = \frac{\sqrt{25}}{\sqrt{36}} = \frac{5}{6}$,

$\sqrt{1\frac{1}{16}} = \sqrt{\frac{17}{16}} = \frac{\sqrt{17}}{\sqrt{16}} = \frac{4\cdot123\,1\ldots}{4} = 1\cdot0307\ldots$,

$\sqrt{\frac{3}{7}} = \sqrt{\frac{21}{49}} = \frac{\sqrt{21}}{7} = \frac{4\cdot582\,5\ldots}{7} = \cdot6546\ldots$.

It is often more easy to turn the vulgar fraction into a decimal fraction, and then to find an approximate value of the square root of the latter. Thus

$$\sqrt{4\tfrac{25}{37}} = \sqrt{4\cdot6943699\ldots} = 2\cdot1666\ldots.$$

EXAMPLES XCIV.

Find the square roots of

1. $\frac{36}{121}$.	**2.** $\frac{81}{169}$.	**3.** $5\frac{4}{9}$.	**4.** $4\frac{29}{49}$.	**5.** $9\frac{67}{121}$.
6. $17\frac{8}{49}$.	**7.** $4\frac{92}{121}$.	**8.** $65\frac{64}{81}$.	**9.** $210\frac{1}{4}$.	**10.** $2\frac{33439}{50625}$.
11. $514\frac{185}{256}$.	**12.** $\dot{4}$.	**13.** $65\cdot07\dot{1}$.	**14.** $6\cdot24\dot{9}$.	

Find the value to four places of decimals of the square roots of

15. $\frac{1}{8}$.	**16.** $\frac{1}{20}$.	**17.** $\frac{2}{5}$.	**18.** $\frac{7}{11}$.	**19.** $\frac{8}{13}$.
20. $\frac{22}{7}$.	**21.** $5\frac{1}{5}$.	**22.** $8\frac{3}{8}$.	**23.** $\frac{1}{7}$.	**24.** $7\frac{30}{81}$.

Find the value of the square roots of

25. the product of 50836900 and ·000001.

26. the sum and difference of 29347781 and 24983860.

27. the L.C.M. of $9\frac{1}{7}$, $19\frac{9}{25}$, and $20\frac{1}{4}$.

Find the value correct to two places of decimals of

28. $\sqrt{\frac{\cdot0864\times753}{\cdot00391}}$. **29.** $\sqrt{\frac{\cdot0132\times\cdot543}{7\cdot65\times\cdot0301}}$.

APPLICATIONS OF SQUARE ROOT.

169. **Ex. 1.** *The area of a square field is* 2 *ac.* 3 *r.* 4 *sq. po.* 25 *sq yds.; find the length of the side of the field.*

2 ac. 3 r. 4 sq. po. 25 sq. yds. = 11 r. 4 sq. po. 25 sq. yds.

= 444 sq. po. 25 sq. yds. = $(444 \times 30\frac{1}{4} + 25)$ sq. yds. = 13456 sq. yds.

Now, since the area of a square is found by finding the square of the length of its sides, therefore the length of the side is obtained by finding the square root of the area.

Thus the side of the field = $\sqrt{13456}$ yds. = 116 yds.

Ex. 2. *Find the length of the side of the square which is equal in area to the rectangle, whose length is* 15 *ft.* 7 *in., and whose breadth is* 28 *ft.* 3 *in.*

The area = length × breadth = 187 in. × 339 in. = 63393 sq. in.

Hence the length of the required side

= $\sqrt{63393}$ in. = 251·77 in. = 20 ft. 11·77 in.

MISCELLANEOUS EXAMPLES XCV.

Square Root.

Find the length of the side of a square field whose area is

1. 2 r. 1 sq. po.

2. 3 ac. 2 r. 16 sq. po.

3. 6,773,948,416 sq. inches.

4. 15·625 acres.

5. 2 acres, correct to one thousandth of the whole.

6. Find the length of the side of a square board whose area is 280·0209 sq. inches.

7. Find the length of the side of a square room whose area is 35 sq. yds. 9 sq. in.

Find the length of the side of a square whose area is exactly equal to the area of

8. a rectangle whose sides are 513 yds. 1 ft. 11 in. and 1628 yds. 11 in.

9. the sum of two squares whose sides are 225 and 120 yards.

How long will a man take to walk round the boundary of a square field containing

10. 90 acres at the rate of 5 miles an hour?

11. 160 acres at the rate of 4 miles an hour?

12. How many yards of fencing are required to enclose a square park containing 832 ac. 2 r. 25 sq. po.?

13. Find the cost of erecting a fence round a square field of 12 acres at 3*s.* 7½*d.* per yard.

14. Find the length of the side of a square lawn if the cost of making it, at 3*s.* 4½*d.* per sq. yard, be £351. 18*s.* 4½*d.*

15. The cost of levelling a square lawn-tennis ground, at 6*d.* a sq. yard, is £11. 0*s.* 6*d.*; find the cost of enclosing it with an iron railing at 2*s.* 6*d.* per yard.

16. What is the number whose fifth part multiplied by its seventh part gives 4235?

17. A society collected annually £30. 9*s.* 2¼*d.*, each member contributing as many farthings as there were members; find the number of members.

18. A man plants his orchard with 7225 trees, and arranges them so that there are as many rows as there are trees in a row; how many rows were there?

19. A man, after a tour, finds that he had spent every day half as many shillings as the number of days he had been away from home, and he spent in all £57. 12*s.*; how long did his tour last?

20. The area of a rectangular field is 975,744 sq. feet, and one side is 3½ times as long as the other; what is the length of each side?

21. The sides of a rectangular piece of ground are in the ratio of 2 to 7; what is the length of each side if the ground contains 341,373,816 sq. ft.?

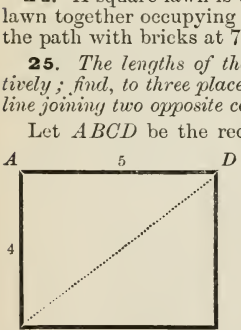
22. A cubical cistern open at the top costs 15 guineas to line with lead at 1*s.* 9*d.* per sq. foot; how many cubic feet will it contain?

23. A square garden occupies two acres. If a pathway go quite round the garden close to its edge, and occupy one-eighth of an acre, find its width.

24. A square lawn is bordered by a path 4½ feet wide, the path and lawn together occupying one-tenth of an acre; find the cost of paving the path with bricks at 7*s.* 6*d.* per sq. yard.

25. *The lengths of the sides of a rectangle are* 4 *ft. and* 5 *ft. respectively; find, to three places of decimals, the length of its diagonal, i.e. the line joining two opposite corners of the rectangle.*

Let $ABCD$ be the rectangle. By the 47th proposition of Euclid's first book we know that the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the sides. Hence



$$BD^2 = AB^2 + AD^2$$
$$= 4^2 + 5^2 = 16 + 25 = 41;$$
$$\therefore BD = \sqrt{41} \text{ ft.} = 6{\cdot}403\ldots \text{ ft.}$$

41 (6·403...
36
124) 500
496
12803) 40000
38409
1591

26. A ladder, whose length is 50 feet, is placed with its foot at a distance of 35 feet from a house: to what height on the house will the ladder reach?

27. Two persons start to walk from the junction of two roads which cross at right angles; if they walk at the rate of 3 and 4 miles an hour respectively, how far apart are they at the end of 3 hours?

28. The diagonal of a square court-yard is 90 feet; find the cost of gravelling the court at the rate of 1*s.* for every nine square yards.

CHAPTER XIX.

MISCELLANEOUS PROBLEMS.

170. Averages. In general, the average height (weight, cost, etc.) of any number of quantities is found by finding the total height (weight, cost, etc.) of the quantities and dividing the result by the number of quantities.

Ex. 1. *A man bought* 15 *horses at* £21. 10*s. each*, 12 *horses at* £24. 15*s. each, and* 25 *horses at* £18. 5*s. each; what is the average price paid per horse?*

Cost of 15 horses at £21. 10*s.* each $= £21\frac{1}{2} \times 15 = £322\frac{1}{2}$.
,, 12 ,, £24. 15*s.* ,, $= £24\frac{3}{4} \times 12 = £297$.
,, 25 ,, £18. 5*s.* ,, $= £18\frac{1}{4} \times 25 = £456\frac{1}{4}$.

$\therefore$ Cost of 52 horses $= £1075\frac{3}{4}$.

$\therefore$ Average cost $= £1075\frac{3}{4} \div 52 = £\frac{4\,3\,0\,3}{4 \times 5\,2} = £\frac{3\,3\,1}{1\,6} = £20.\ 13s.\ 9d.$
(4)

Ex. 2. *A train runs from King's Cross to Edinburgh, a distance of* 395 *miles, in* 10 *hours and* 10 *minutes; it stops* 15 *minutes at one station,* 9 *minutes at another,* 5 *minutes at each of* 6 *others, and* 2 *minutes at each of* 8 *other stations. Find its average speed while in motion.*

The total time occupied in stoppages $= (15+9+5\times6+8\times2)$ mins.
$= (15+9+30+16)$ mins. $= 70$ mins. $= 1$ hr. 10 mins.

Hence the total time during which the train was in motion
$= 10$ hrs. 10 mins. $- 1$ hr. 10 mins. $= 9$ hrs.

In this time the train goes 395 miles;

$\therefore$ average speed $= \frac{3\,9\,5}{9}$ miles per hour $= 43\frac{8}{9}$ miles per hour.

EXAMPLES XCVI.

1. The area of Bedfordshire is 294,983 acres, and the population 149,173; find to the nearest integer the average number of persons to the square mile.

2. The men forming a boat's crew weigh 10 st. 11 lbs., 11 st. 10 lbs., 12 st. $1\frac{1}{2}$ lbs., 13 st., 13 st. $2\frac{1}{2}$ lbs., 13 st. 6 lbs., 12 st. $10\frac{1}{2}$ lbs., and 10 st. $12\frac{1}{2}$ lbs. respectively; what is their average weight?

3. The weights of a University Eight are 10 st. 9 lbs., 12 st. $9\frac{3}{4}$ lbs., 12 st. 6 lbs., 11 st. $13\frac{1}{2}$ lbs., 12 st. $8\frac{1}{2}$ lbs., 12 st. $10\frac{1}{2}$ lbs., 12 st. $4\frac{1}{2}$ lbs., and 10 st. $12\frac{3}{4}$ lbs.; what is the average weight of the crew?

4. Of 5 boys the first is 5 ft. 4 in. high, the second is 5 ft. $1\frac{1}{2}$ in., the third is 4 ft. 10 in., the fourth is 5 ft., and the fifth is 5 ft. $2\frac{1}{4}$ in.; what is the average height of the boys?

5. In a mixed school, with 400 names on the rolls, the boys made 69,435 attendances, and the girls 65,783 at 419 openings of the school; find (1) the average attendance of each sex, and
(2) the average number of attendances made by each child.

6. The Flying Dutchman runs from London to Exeter, a distance of $193\frac{1}{3}$ miles in $4\frac{1}{4}$ hours making one stoppage of 10 mins., two of 5 mins. each, and one of 3 mins. on the way; what is its average speed when in motion?

7. A man's average expenditure for the years 1886 to 1895, both inclusive, was found to be £635. 11*s.* 6*d.*; he spent £467. 10*s.* in 1886, and £695. 15*s.* in 1895: what was his average expenditure for the years 1887 to 1894?

8. The average height of a regiment of 1000 men is 5 ft. 10 in.; the average height of the 100 tallest of these is 6 ft.: find the average height of the others.

9. The average of 17 results is 47; that of the first 6 is 40, and that of the next 8 is 50: find the average of the last 3.

10. The average of 25 results is 18, that of the first 12 being 14, and that of the last 12 being 17: what was the thirteenth result?

11. The mean temperature from the 9th to the 15th of January, both included, was 36·6°, and from the 10th to the 16th it was 35·9°; the mean temperature on the 9th was 30·5°: what was it on the 16th?

12. The average temperature for Monday, Tuesday, and Wednesday was 53°. The average for Tuesday, Wednesday, and Thursday was 56°, that for Thursday being 60°; what was the temperature on Monday?

171. Work Problems.

Ex. 1. *Three men A, B, and C can dig a certain piece of ground in* 10, 12, *and* 15 *days respectively; how long will they take to dig the ground if all three work together?*

A can dig the whole ground in 10 days.

$\therefore$ in one day he digs $\frac{1}{10}$th of the ground (since we suppose that in each day he does the same amount of work).

So in one day B digs $\frac{1}{12}$th and C $\frac{1}{15}$th.

$\therefore$ in one day A, B, and C together dig a fraction equal to $\left(\frac{1}{10}+\frac{1}{12}+\frac{1}{15}\right)$ of the ground, that is, $\frac{1}{4}$ of the ground. Hence when working together, they dig the whole ground in $1 \div \frac{1}{4}$, that is, 4 days.

Ex. 2. *If* 5 *men and* 6 *boys in* 9 *weeks earn* £67. 10*s. and* 4 *men and* 5 *boys in* 7 *weeks earn* £42. 14*s., find the weekly wages of a man and of a boy.*

Let a man earn x shillings per week and a boy y shillings per week:

then 5 men earn $5x$ shillings in 1 week, *i.e.* $45x$ shillings in 9 weeks;

and 6 boys ,, $6y$,, 1 ,, *i.e.* $54y$,, 9 ,,

but together they earn £67. 10*s.*, *i.e.* 1350 shillings in 9 weeks;

$$\therefore\ 45x+54y=1350. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Similarly 4 men will earn $28x$ shillings in 7 weeks;
and 5 boys ,, $35y$,, 7 ,,
and these together earn £42. 14*s.*, *i.e.* 854 shillings in 7 weeks;

$$\therefore\ 28x+35y=854. \quad \ldots\ldots(2)$$

From equations (1) and (2), by ordinary Algebra, we find

$$x=18 \text{ and } y=10;$$

$\therefore$ a man earns 18*s.* and a boy 10*s.* a week.

EXAMPLES XCVII.

1. A and B can mow a field in 10 hours and 15 hours respectively, how long will they take working together?

2. If A can do a piece of work in 6 days and B can do it in 7 days, how long will they take to do it together?

3. A man can dig a ditch in 5 days, a boy in 9 days; how long will it take 2 men and 3 boys working together to dig it?

4. A can do a piece of work in 6 days, and A and B in $2\frac{2}{5}$ days; how long would B alone take to do it?

5. A and B working together could mow a certain field in 126 hours, and with the help of C they could have mowed it in 77 hours; how long would C take by himself?

6. A can copy 150 pages in 30 hours. A and B together can copy 280 pages in 35 hours; in what time can B copy 200 pages?

7. A, B, and C could mow a field in 5 days, whilst A and B would take 7 days; how long would it take C to mow a field of twice the size?

8. If a man can do a piece of work in 77 hours which a boy wants 121 hours for, in how many hours can they do it working together?

9. If 7 men and 2 women earn £80. 8*s.* in 8 weeks, and 4 men and 3 women earn £83. 8*s.* in 12 weeks, what are the weekly wages of a man and a woman?

10. If 6 women and 2 boys earn £22. 10*s.* in 5 weeks and 4 women and 7 boys earn £16. 13*s.* in 3 weeks, find the weekly wages of a woman and of a boy.

11. A, B, and C together do a piece of work for £2. 13*s.* 6*d.* A working alone could do it in 7 days, B working alone could do it in 6 days, and C working alone could do it in 5 days; how should the money be divided between them?

12. One pipe can fill a cistern in 4 minutes and another in 5 minutes; how long would it take to fill the cistern if both are open?

13. One pipe can empty a cistern in 6 minutes and a second pipe in 12 minutes; how long would it take to empty half the cistern if both pipes are open?

14. *A cistern can be filled by one pipe in* 15 *minutes, and emptied by another in* 20 *minutes; when both are open together, in what time will the cistern be filled?*

The first pipe in one minute pours in $\frac{1}{15}$th of the cistern and the second pipe in the same time draws off $\frac{1}{20}$th.

Hence, when both are open together, the amount that enters per minute $= (\frac{1}{15} - \frac{1}{20}) = \frac{1}{60}$th of the cistern. Thus in 60 minutes the amount that enters the cistern will be sufficient to fill it.

15. A cistern can be filled by a tap in 3 hours; how long will the tap take to fill it if the cistern has a leak which would empty it in 8 hours?

16. Two pipes running together can fill a cistern in 8 minutes, and one of them alone in 24 minutes; how long would the other take alone?

17. Two pipes, A and B, can fill a cistern in 4 and 7 minutes respectively, and a third pipe, C, can empty it in 10 minutes; how long will it take to fill the cistern if all three pipes are open at the same time?

18. A hot-water pipe can fill a bath in 5 minutes and a cold-water pipe can fill it in 3 minutes, whilst the waste pipe empties it in $7\frac{1}{2}$ minutes; if all three pipes be open, how long does the bath take to fill?

19. An empty cistern has three pipes A, B, and C. A and B can fill it in 3 and 4 hours respectively and C can empty it in 1 hour; if these pipes are opened in order at one, two, and three o'clock, find when the cistern will be empty.

172. Speed and Velocity Problems. The speed, or velocity, of a body is the rate at which the body is moving, and is measured by the space which the body would move through in a given time.

We can easily convert a speed expressed in one denomination in distance and time into other denominations.

Ex. 1. *Express a speed of* 60 *miles per hour in feet per second.*

A body, which describes 60 miles in 1 hour,

describes $60 \times 1760 \times 3$ feet in 60×60 seconds,

that is, describes $\frac{\cancel{60} \times 1760 \times \cancel{3}}{\cancel{60} \times \cancel{60}}$,, 1 second.

,, ,, $\frac{88}{2}$,, ,,

Ex. 2. *Express a speed of* 1120 *feet per second in miles per hour.*

A body will go (if it travels at a constant rate) 60 times as far in a minute as in a second, and 60 times as far in an hour as in a minute.

$\therefore$ a speed of 1120 feet per second

$=$,, 1120×60 ,, minute

$=$,, $1120 \times 60 \times 60$,, hour.

Now $1120 \times 60 \times 60$ ft. $= \frac{\overset{140}{\cancel{1120}} \times \cancel{60} \times \cancel{60}}{\underset{11}{\cancel{1760}} \times \cancel{3}}$ miles $= \frac{8400}{11}$ miles $= 763\frac{7}{11}$ miles.

Hence the given speed is $763\frac{7}{11}$ miles per hour.

EXAMPLES XCVIII.

1. Walking at the rate of $3\frac{1}{2}$ miles per hour, how long do I take to accomplish $5\frac{1}{2}$ yards?

2. If I have walked for $4\frac{1}{2}$ hours at the rate of 80 yards per minute, how far am I from the end of a journey of 16 miles?

3. If I walk for $8\frac{1}{3}$ hours a day at the rate of $3\frac{3}{4}$ miles per hour, resting on Sundays, in how many weeks shall I have passed through a district which is 1500 miles long?

4. A railway train, which travels at the uniform rate of 66 feet per second, leaves London at 9 o'clock; when will it reach Edinburgh, the distance from London to Edinburgh being 405 miles?

5. Light travels at the rate of 186,000 miles per second; how many years will it be in coming from the nearest fixed star whose distance is 25,600,000 million miles?

6. In which of the following journeys does the train travel most quickly? Arrange the other trains in order of quickness.

Railway.	Time of leaving London.	Time of arriving at	Distance.
Great Northern -	10.0 A.M.	York, 1.45 P.M.	168 miles.
London and S.-W.	3.0 P.M.	Exeter, 6.46 P.M.	$171\frac{1}{2}$ miles.
London and N.-W.	4.10 P.M.	Crewe, 7.41 P.M.	158 miles.

7. A bicyclist calculates that he will make a certain journey in six hours by travelling at $12\frac{1}{2}$ miles an hour, but he is delayed an hour in starting; at what pace must he go to make up for the loss of time?

8. The circumference of the equator of the earth being 25,000 miles, find the rate per minute at which a point on the equator moves.

9. If sound travel at the rate of 1120 feet per second, find how far off is a thunder cloud, when the sound of the thunder is heard 7 seconds after the lightning is seen, the flash being supposed to be seen as soon as it is caused.

10. How long after seeing a gun discharged at a distance of $2\frac{1}{2}$ miles will it be before I hear the sound of the discharge, it being assumed that the velocity of sound is 1120 feet per second?

11. If a shot be fired from a ship moving at the rate of 15 miles per hour, how far has the ship moved before a person who is 5 miles from the ship hears the report, the velocity of sound being 1120 feet per second?

12. Find the number of gallons of water which pass in 10 minutes under a bridge 17 ft. 8 in. wide, the stream being 10 ft. 11 in. deep, and its velocity 8 miles per hour. [A gallon contains 277·72 cubic inches.]

13. A railway passenger counts the telegraph posts on the line as he passes them. If they are 58 yards apart, and the train is going at the rate of 48 miles per hour, how many will he pass per minute?

14. On a certain line of railway the telegraph posts are placed 58 yards apart, and a passenger counts, as he passes them, 37 in two minutes; at what rate is he travelling?

15. Two men start at the same time from two towns, A and B, 300 miles apart, and travel at the rate of 7 and 8 miles an hour respectively towards the other town; when and where will they meet?

16. A man walks at the rate of 5 miles an hour alongside a railway line, starting at the time that a train travelling at 60 miles per hour is just 40 miles behind him; how far apart are they at the end of 35 minutes?

17. A train leaves Brighton at 9 A.M. and travels towards London at a uniform rate of 15 miles per hour; an express train leaves London for Brighton at 10 A.M., and travels at 40 miles an hour: when do they pass each other, and at what distance from Brighton, the distance between the two towns being 50 miles?

18. At what distance from London will a train which leaves London for Rugby at 2.45 P.M., and goes at the rate of 41 miles an hour, meet a train which leaves Rugby for London at 1.45 P.M., and goes at the rate of 25 miles per hour, the distance between the two towns being 80 miles?

19. Two trains are moving in the same direction on parallel lines. They move at the rates of 45 and 20 miles per hour, and their lengths are 80 and 120 yards respectively; how long will they take to pass each other, and how long would they take if they were going in opposite directions?

20. In the previous question, how long would each take to pass completely through a station 75 yards long? How long would each take to completely pass a man at rest by the side of the railway?

21. In a bumping race two boats, of length 57 feet, start with their sterns opposite two posts, 175 feet apart. The first boat travels at the rate of $8\frac{1}{2}$ miles an hour, and the one behind at the rate of $8\frac{3}{4}$ miles an hour; find how soon the bump will be made.

22. *A* walks 120 yards in 2 minutes, and *B* walks 80 yards in a minute; how far is *A* behind when *B* has walked a mile?

23. A policeman endeavours to catch a thief; four of the policeman's steps are equal to five of the thief's, but the thief takes four steps whilst the policeman takes three; will the policeman catch the thief?

24. A goods train is 8 miles ahead of an express which travels at the rate of a mile in $1\frac{1}{4}$ minutes. It is run into by the express in 20 minutes; at how many miles an hour was the goods train running?

25. *A gives B* 49 *yards start in a race of one mile, and is beaten by* 20 *yards; how many yards ought A to have given B?*

A runs 1740 yards whilst *B* runs 1711 yards;

$\therefore$,, 1 yard ,, $\frac{17}{1740}\left(=\frac{59}{6060}\right)$ yard;

$\therefore$,, 1760 yards ,, $\frac{59}{6060}\times 1760$, that is, $1730\frac{2}{3}$ yards.

Hence the start that *A* can give $B=(1760-1730\frac{2}{3})=29\frac{1}{3}$ yards.

26. *A* gives *B* 10 yards in a 200 yards race and is beaten by 7 yards: how many should he give *B* to make a level race?

27. *A* gives *B* 21 points in a game of 100 and is beaten by 3 points; how many can he give him so as just to win?

28. At a game of billiards *A* can give *B* 15 points in 50, and he can give *C* 32 in 80; how many can *B* give *C* in a game of 70?

29. *A* gives *B* 10 yards and *C* 15 yards starts in a race of 100 yards; how much should *B* give *C* in a race of 150 yards?

30. A man gives a boy 15 yards start in 100 yards, and loses the race by 5 yards; what would have been a fair start to give?

173. Areas and Circumferences of Circles. It is shown in books on Higher Mathematics, and it may be here assumed, that

The length of the circumference of a circle
$= 3{\cdot}14159 \ldots \times$ Diameter of the Circle. .....(1)

The area of a circle
$= 3{\cdot}14159 \ldots \times$ Square of its Radius. .....(2)

Thus the length of the circumference of a circle of radius 3 ft.
$= 3{\cdot}14159 \ldots \times 6 = 18{\cdot}8495 \ldots$ ft.
Its area $= 3{\cdot}14159 \ldots \times 3^2 = 3{\cdot}14159 \ldots \times 9 = 28{\cdot}2743 \ldots$ sq. ft.

An approximation to the value $3{\cdot}14159 \ldots$ is the vulgar fraction $\frac{22}{7}$.

EXAMPLES XCIX.

[*N.B.*—$1 \div 3{\cdot}14159 = {\cdot}31831 \ldots$ and $1 \div \sqrt{3{\cdot}14159} = {\cdot}56418 \ldots$].

Find the circumference and area of a circle whose diameter is

1. 5 ft. **2.** 100 yds. **3.** 1 mile. **4.** 3 ft. 7 in. **5.** $16\frac{3}{4}$ in.

Find the radius of a circle whose circumference is

6. 5 ft. **7.** 27 yds. **8.** 1 mile. **9.** $7\frac{3}{4}$ miles.

Find the radius of a circle whose area is

10. 10 sq. ft. **11.** 50 sq. yds. **12.** 10 acres.

13. Find the radius of a circle whose area is $\frac{2}{3}$rds that of a circle of radius 8 in.

14. A circular pond has an area of $346\frac{1}{2}$ sq. yds.; find, to the nearest penny, the cost of fencing it at 4*s.* 6*d.* per linear yard.

15. How many trees can be planted, 4 yards apart, round the edge of a circular field of radius 100 yds.?

16. How many revolutions does a wheel of radius 3 ft. make in 1 mile?

17. The wheel of a locomotive engine is 5 ft. in diameter, and turned round 5000 times in going 12 miles; what distance was lost owing to the slipping of the wheel on the rail?

18. The cost of fencing a circular piece of land, at 1*s.* 3*d.* per yd., was £39. 5*s.* 6*d.*; find the length of a straight path running across it through the centre.

CHAPTER XX.

ABBREVIATIONS AND APPROXIMATIONS.

174. In the present chapter we shall consider some abbreviations of, and approximations to, some of the work in the previous chapters. In most practical examples of the use of decimal fractions we require only approximate results. The "tail," or end, of such a decimal is quite unnecessary.

175. When we speak of a decimal as being "correct" to 4 places, we mean that the decimal, as written down when "correct," shall represent the true value of the decimal as nearly as it is possible, so long as we only use four places.

Consider the decimal 3·1415926. .....(1)

If we wish to express this correct to six places, the result is 3·141593; here we change the 2 of the sixth place into 3. The reason is this: the last two places of (1) give the number 26, and 26 is nearer 30 than 20. Hence it is more accurate to replace 26 by 30 than by 20.

To five places the value of (1) is 3·14159. So to four places the value is 3·1416; for 59 is nearer to 60 than it is to 50.

Similarly the value to three places is 3·142; for 16 is nearer to 20 than it is to 10.

We therefore see that in writing down the value of a decimal correct to, say, four places, *we write down the first four figures of the decimal, but if the figure in the fifth place be* 5 *or a figure greater than* 5, *we increase by unity the figure in the fourth place.*

176. Contracted Multiplication.

Multiply 1·3976 *by* 3·4582 *correct to* 3 *decimal places.*

It is found that we must work to **2** places more than are required, *i.e.* to 5 places in this case.

Multiply by 3 and we get 1st line.

Multiply by 4 and we get 2nd line.

Multiply by 5. We cross through the '6': we then say 5 times 6 are 30, but do not write down the '**0**' because it would be in the 6th decimal place: we carry the '**3**.' Continuing, 5 times 7 are 35 and 3 carried are 38: put down 8, carry 3, and finish in the ordinary way.

Multiply by 8. Cross through the '7': 8 times 7 are 56: carry **6**

1·397	6
3·458	2
4·192	8
·559	04
·069	88
·011	18
·000	28
4·833	18

Ans. = 4·833.

(since 56 is nearer 60 than 50) : 8 times 9 are 72 and 6 carried are 78 : put down **8** in 5th place and finish in the ordinary way.

Multiply by 2. Cross through the 9 and multiply as in the previous line.

On adding we get 4·83318, and this becomes 4·833 correct to three places.

177. We give two other examples.

Multiply 3·8457895 *by* 639 *and* 3·14159 *by* ·31831 *each correct to two places.*

3·84 57895	
639	
2307·47	37
115·37	37
34·61	21
2457·45	95

Ans. = 2457·46

3·1415 9	
·3183	1
·9424	77
· 314	16
· 251	33
· 9	42
·	31
·9999	99

Ans. = ·9999 + ·0001 = 1.

EXAMPLES C.

Find the value, correct to the number of places stated, of

1. 7·34 × 3·654 ; 2 places.
2. 8·675 × 4·387 ; 2 places.
3. ·0234 × ·785 ; 3 places.
4. 29·053 × ·7065 ; 3 places.
5. 7·065 × 3·7481 ; 4 places.
6. ·3807 × ·5938 ; 4 places.
7. 637·834 × ·7375 ; 3 places.
8. 4·2387 × 7·638 ; 4 places.
9. ·0003857 × 783·54312 ; 4 places.
10. 5·3084 × 7·83459 ; 4 places.

178. Contracted Division.

Ex. *Divide, correct to six places of decimals,* 648935 *by* 859432.

859432) 6489350 (·755074	
6016024	
4733260	
4297160	
436100	0 .....(2)
429716	0
6384	000
6016	024
367	9760
343	7728
24	2032

85|9|4|3|2) 6489350 (·755074
6016024
4733260
4297160
436100
429716
6384
6016
368
344
24

On the left hand the work is performed in full. Cut off by a vertical line all the figures to the right of the figures 24 of the last remainder. We can obtain the quotient without writing down any of the figures to the right of this vertical line.

In line (2) the remainder was 436100 and we continue the work by annexing a cypher, that is, by multiplying the remainder

by 10. But this multiplication would have been performed if we had *divided the divisor by* 10, and this is approximately done if we cut off the right-hand digit of the divisor.

Thus, as in the right-hand piece of work, we divide 436100 by 85943. The corresponding figure in the quotient is 5.

Remembering that the figure cut off at the right hand of the 3 was 2, and that this 2 multiplied by 5 would give a "carried" figure 1, we set down 429716 instead of 429715.

We thus have a remainder 6384.

Cutting off the 3 of the divisor we see that 8594 is larger than 6384. Hence we set down a 0 in the quotient.

The 4 being cut off, the curtailed divisor 859 divides into 6384 seven times. Remembering that the figures cut off, when multiplied by 7, would have caused a "carried" figure 3, we set down 6016 instead of 6013.

The remainder is now 368.

Cutting off the 9 of the divisor and altering the 5 into a 6 (because 59 is nearer to 60 than 50), we have 86 as the final divisor. Also 86 divides into 368 four times. The whole process may be written as above.

179. We see that in the previous work we should arrange so that the last curtailed divisor should consist of two digits (such as the 86). We should therefore commence the curtailing of the divisor at that stage of the work *where the number of figures still to be found in the quotient is just two less than the number of the digits in the divisor.*

For, since we cut off one digit from the divisor for each further place found in the quotient, it is clear that if we begin the curtailing at the stage given by the above rule, we shall always have two figures left in the divisor at the last division.

Ex. *Divide* 1 *by* 3·14159 (*i.e. find the reciprocal of* 3·14159) *to* 5 *places of decimals.*

Here, after moving the decimal point six places to the right in both dividend and divisor, we find that the quotient begins with ·3.

As we have four more decimal places to find, and there are six figures in the divisor (*i.e.* two more than four), we can begin to curtail at once.

On making the best approximation to the figures that would be "carried" due to the omitted portion of the divisor, we have the working as in the margin.

```
31|4|1|5|9 ) 100000·0 ( ·31831
             94247 7
             -------
              5752 3
              3141 6
              ------
              2610 7
              2513 3
              ------
                 974
                 942
                 ---
                  32
                  31
                  --
```

EXAMPLES CI.

Find the value, correct to the number of places of decimals stated, of

1. $23{\cdot}459 \div 3{\cdot}6345$; 2 places.
2. $13{\cdot}784 \div 5{\cdot}9034$; 3 places.
3. $\cdot0736 \div 2{\cdot}8573$; 3 places.
4. $2{\cdot}85 \div \cdot0736$; 3 places.
5. $4873 \div 7539$; 4 places.
6. $37{\cdot}4236 \div \cdot008437$; 2 places.
7. $237{\cdot}438 \div \cdot073428$; 4 places.
8. $2 \div 23{\cdot}8342$; 4 places.
9. $73{\cdot}834 \div \cdot000738$; 3 places.
10. $3{\cdot}2\dot{7} \div 1{\cdot}\dot{7}$; 3 places.

Find the value, correct to three places of decimals, of

11. $\frac{3848}{4754}$.
12. $\frac{7853}{8465}$.

In the following pairs of quantities express the first as a decimal of the second correct to four places :

13. £3. 17*s.* 2½*d.*
14. 5 lbs. 6 oz. 11 drs. ; 1 ton 3 cwt.

Find, correct to four places of decimals, the value of

15. $\frac{1}{3}+\frac{1}{3\times2}+\frac{1}{3\times2\times3}+\frac{1}{3\times2\times3\times2}+\frac{1}{3\times2\times3\times2\times3}$.

16. $\frac{1}{2}+\frac{1}{2.4}+\frac{1}{2.4.6}+\frac{1}{2.4.6.8}+\frac{1}{2.4.6.8.10}+\frac{1}{2.4.6.8.10.12}$.

17. Given that a gallon contains 277·274 cubic inches, find, to five places of decimals, the number of cubic feet of water in 60 gallons.

18. An imperial gallon contains 277·274 cubic inches, and a cubic foot of water weighs 1000 ounces Av. ; find the weight of a pint of water correct to within the thousandth part of an ounce and thus prove that the popular rule, "a pint of water weighs a pound and a quarter," is nearly true.

19. Having given that the circumference of a sphere is 3·1416 times the diameter, find the diameter, to the nearest mile, of a sphere whose circumference is 25,000 miles.

DECIMALIZATION OF MONEY.

180. By the decimalization of money we mean the expressing the value of a sum of money (given in pounds, shillings, and pence) in £'s and a decimal of £1.

In questions involving money, the answer need never be carried to a greater extent of accuracy than to the nearest farthing. For, since there is no coin less than one farthing, fractions of a penny less than one farthing are neglected.

Thus, suppose a lady were to buy at a shop 2¼ yards of stuff at 1*s.* 11¾*d.* per yard, the cost accurately calculated would be 4*s.* $5\frac{7}{16}d.$ The price charged to her would be 4*s.* 5½*d.* For ½*d.* is the coin which most nearly approximates to $\frac{7}{16}d.$

181. Rule for the decimalization of money. *For each florin in the given sum put* £·1 ; *for any remaining shilling put* £·05 ; *for any remaining sixpence put* £·025, *and for each*

remaining farthing put £·001. But if the number of remaining farthings be 12, *or greater than* 12, *add an extra* ·001 *to the answer obtained as above.*

The result will be correct to three places of decimals.

[This rule is sometimes known as the Actuaries' Rule, and is said to be very old.]

Ex. 1. £13. 17*s.* 8¼*d.*
= £13 + 8 florins + 1 shilling
+ 1 sixpence + 9 farthings
= £13·884.

	£13
8 × ·1 =	·8
1 × ·05 =	·05
1 × ·025 =	·025
9 × ·001 =	·009
	£13·884

Ex. 2. £27. 15*s.* 11½*d.* = 27·798.

In actual practice the student will set down the sum as in the margin, the first line represents the pounds and shillings [for 15*s.* = 7 florins + 1 shilling = £·75].

£27·75
·025
·023
£27·798

The second line represents the one sixpence (= £·025) in the 11½*d.*, and we are left with 5½*d.*, that is, 22 farthings. Since 22 is greater than 11, therefore, by the rule, 22 farthings is 23 × £·001, and is therefore £·023. This is the third line.

Proof of the above rule. 2*s.* = £$\frac{1}{10}$ = £·1,

1*s.* = £$\frac{1}{20}$ = £·05, 6*d.* = $\frac{1}{2}$ shil. = £·025.

Also, on accurate division, we have

1 farthing = £$\frac{1}{960}$ = £·00104166 ....

Hence

11 farthings = £·011458 ... = £·011 correct to 3 places.

and 13 „ = £·01354 ... = £·014 „ „

The rule is therefore true; for up to 11 farthings it is easily seen that each farthing is represented by ·001; but for farthings over 12 in number we must add an extra 1 to the third place of decimals. Thus 17 farthings = 18 × (£·001) = £·018.

182. The converse of the above approximate rule will now be clear, namely:

To convert a decimal of a pound into shillings, pence, and farthings, write down the decimal correct to three places; for each £·1 *put* 2*s.; for a remaining* £·05 *put* 1*s.; for a remaining* £·025 *put* 6*d.; and for each remaining* £·001 *put one farthing, except that if we should by this method get a number of farthings exceeding* 12 *in number we must deduct* 1 *from that number.*

Ex. 1. £9·383 = £9 + £·3 + £·05 + £·025 + £·008
= £9 + 6*s.* + 1*s.* + 6*d.* + 2*d.* = £9. 7*s.* 8*d.*

Ex. 2. £17·791 = £17 + £·7 + £·05 + £·025 + £·016
= £17 + 14*s.* + 1*s.* + 6*d.* + **15** *far.* = £17. 15*s.* $9\frac{3}{4}d.$

Ex. 3. *Divide* £1835. 15*s.* $10\frac{1}{4}d.$ *by* 157, *the answer being correct to the nearest farthing.*

£1835. 15*s.* $10\frac{1}{4}d.$ = £1835·793

£.	£.
157) 1835·793 (	11·693
1727	
108 7	
94 2	
14 59	
14 13	
463	

The last figure of the quotient is 3 because 157 divides into 463 more nearly 3 than 2.

£1835·75
·025
·018
£1835·793

Hence the answer = £11 + £·6 + £·05 + £·025 + £·018 = £11. 13*s.* $10\frac{1}{4}d.$

183. Since the rule of Art. 181 is quite accurate as far as the nearest sixpence in the given sum, it is not difficult to express any given sum of money quite accurately as a decimal.

Ex. *Express* £1. 11*s.* $3\frac{3}{4}d.$ *as an accurate decimal of* £1.

£1. 11*s.* = £1·55
$3d. = \frac{1}{2}$ of 6*d.* = ·125
$\frac{3}{4}d. = \frac{1}{4}$ of 3*d.* = ·3125
Ans. £1·565625

EXAMPLES CII.

Write down as a decimal of £1, correct to three places, the value of

1. 4*s.* 6*d.* **2.** 5*s.* 7*d.* **3.** 18*s.* 2*d.* **4.** 8*s.* $3\frac{1}{2}d.$ **5.** 12*s.* $7\frac{3}{4}d.$
6. 9*s.* $11\frac{1}{4}d.$ **7.** £3. 14*s.* 3*d.* **8.** £2. 8*s.* 11*d.* **9.** £4. 7*s.* $5\frac{1}{2}d.$
10. £3. 13*s.* $3\frac{1}{2}d.$

Write down the values, correct to the nearest penny, of

11. £·625. **12.** £·785. **13.** £·545. **14.** £·365. **15.** £·406.
16. £·857. **17.** £1·628. **18.** £·531. **19.** £1·443. **20.** £·739.
21. £5·710. **22.** £5·879. **23.** £1·3895.

Find to the nearest farthing the value of

24. £37·8374 × 24·2895. **25.** £623·4392 × 31·3476.

26. Find, correct to a penny, the rent of 345·575 acres at £2. 11*s.* $6\frac{1}{2}d.$ per acre.

MISCELLANEOUS EXAMPLES.

1. Express in words 50780407003.

2. Multiply 784299 by 3627, and 7 cwt. 3 qrs. 22 lbs. by 327.

3. How many hours are there in 15 years of 365 days each?

4. How many packets, each containing $14\frac{1}{2}$ lbs., can be made from 3 cwts. 3 qrs. 15 lbs.?

5. Multiply £13. 15*s*. $11\frac{1}{4}d$. by 469.

6. Find the G.C.M. of 4176 and 5133.

---

7. From one million five hundred and sixty-seven thousand three hundred and ninety-four subtract eight hundred and ninety thousand five hundred and seventy.

8. Multiply 7401 by 3056, and divide 82749325 by 235.

9. A sum of £10. 19*s*. $9\frac{1}{4}d$. is to be divided amongst 137 people; how much does each receive?

10. Add $2\frac{1}{2}\frac{1}{1}$ to $8\frac{1}{3}\frac{3}{5}$, and subtract $3\frac{7}{2}\frac{}{4}$ from $8\frac{1}{1}\frac{1}{5}$.

11. Multiply $132\frac{4}{5}$ by $3\frac{1}{8}$, and divide $14\frac{4}{5}$ by $11\frac{2}{3}$.

12. Find the cost of 2 cwt. 3 qrs. 7 lbs. at £4. 15*s*. 8*d*. per cwt.

---

13. Multiply one million seven hundred thousand three hundred by one hundred thousand and thirty.

14. What is the value of 1296 articles at 16*s*. $10\frac{1}{2}d$. each?

15. Reduce 2 miles 6 fur. 4 yds. to feet, and 7625 grains to lbs. oz. etc. Troy.

16. If 6 tons 3 qrs. 12 lbs. be made up into 96 parcels of the same weight, how much will each weigh?

17. Subtract $22\frac{5}{7}$ from $31\frac{5}{8}$, and reduce $\frac{1}{1}\frac{8}{9}\frac{9}{3}\frac{2}{6}$ to its lowest terms.

18. Find by Practice the value of 3 miles 5 fur. 25 po. of telegraph cable at £198 per mile.

---

19. Multiply 85762 by 319, and prove your answer by Division.

20. Divide £22912. 9*s*. $0\frac{1}{2}d$. by 53.

21. Reduce £3478. 17*s*. 6*d*. to half-crowns, and 4789056 farthings to florins.

22. Find the difference between 12 cwt. 3 qrs. 17 lbs. 10 oz. and 5 cwt. 2 qrs. 23 lbs. 11 oz.

Find also the sum of 5 miles 7 fur. 37 po. 4 yds. 2 ft. 5 in. and 79 miles 4 fur. 21 po. 6 yds. 7 in.

23. Reduce $\frac{1}{2}\frac{9}{2}\frac{1}{2}\frac{9}{3}$ to its simplest form.

24. Add together $5\frac{4}{9}$, $\frac{3}{8}$, $\frac{7}{1}\frac{}{2}$, and $2\frac{5}{2}\frac{}{4}$.

---

25. Multiply £7850. 11*s.* $8\frac{3}{4}d.$ by 47, and divide £1027. 4*s.* $1\frac{1}{2}d.$ by 53.

26. How many rings, each weighing 4 dwt. 18 grs., can a goldsmith make from 2 lbs. 5 oz. 9 dwt. of gold?

27. How many coins worth $9\frac{3}{4}d.$ each are equal in value to 5421 others worth 2*s.* $10\frac{3}{4}d.$ each?

28. Find the L.C.M. of 48, 27, 81, 165 and 110, and the G.C.M. of 6003 and 14616.

29. Simplify $3\frac{1}{7} \times 3\frac{3}{10} \times 2\frac{3}{11} \times 2\frac{5}{22} \times \frac{2}{35}$.

30. Multiply ·0125 by 20·08 and divide 210720·6 by ·4206.

---

31. What is the value of 23 packages of cloth, each package containing 10 parcels, each parcel 16 pieces, and each piece 52 yards, at the rate of 4*s.* $7\frac{1}{4}d.$ per yard?

32. How many miles, furlongs, poles, etc., are there in 4,763,870 inches?

33. Simplify $\left(\frac{1}{2} \times \frac{5}{12} \times 1\frac{2}{5}\right) \div \left(\frac{3}{5} + \frac{1}{6} - \frac{1}{10}\right)$.

34. Reduce $\frac{5}{11}$ of £33. 13*s.* 9*d.* to the fraction of £26. 5*s.*

35. Express $\frac{57}{125}$ and $\frac{3}{7}$ as decimals.

36. Divide ·2332728 by 32·58, and find the value of ·45 of £11. 14*s* $4\frac{1}{2}d.$

---

37. The sum of £39. 18*s.* $8\frac{1}{2}d.$ is made up of equal numbers of sovereigns, crowns, florins, sixpences, and half-pennies; how many are there of each?

38. Find in tons, cwts. etc. the weight of 72 bars of iron, each 18 ft. long, at the rate of 20 oz. to the inch.

39. Find the G.C.M. of 929,181 and 1,012,891.

40. Simplify $\dfrac{3\frac{1}{4} - 2\frac{1}{3}}{3\frac{1}{6} + 1\frac{1}{2}} \div 1\frac{4}{7}$.

41. If $\frac{1}{3}$ of a stick be cut off and then $\frac{1}{4}$ of the remainder, what fraction of the stick is left?

42. Express $\frac{4}{5}$ of $\frac{7}{8}$ of $1\frac{1}{12}$ of £1 as a decimal of 3 guineas.

---

43. What number divided by 596 gives 624 as quotient and 245 as remainder?

44. How many times may the sum of £5. 6*s.* 7*d.* be paid from a sum of £327. 13*s.* 5*d.*, and how many pence will there be in the remainder?

45. Multiply 5 po. 3 yds. 2 ft. 3 in. by 47, and reduce 78912144 cub. in. to cub. yds.

46. By what amount does the sum of $2\frac{1}{9}$ of £1. 11*s.* $3\frac{3}{4}d.$ and $4\frac{6}{7}$ of £2. 6*s.* $9\frac{3}{4}d.$ exceed $\frac{1}{13}$ of £3. 11*s.* $9\frac{1}{4}d.$?

47. A person takes to market a certain sum of money, $\frac{2}{3}$ of which amounts to $1\frac{1}{5}$ guineas. He buys 15 score of eggs at $6\frac{1}{2}d$ a dozen. What money does he take home?

48. Find by Practice the value of 3 tons 17 cwt. 49 lbs. of tapioca at £65. 6*s.* 8*d.* per ton.

---

49. How many half-guineas are there in three hundred and four thousand and seventeen half-crowns?

50. Three bells ring simultaneously and afterwards at intervals of 20, 30, and 35 seconds respectively; how often will they all ring together?

51. If the product of $\frac{3}{7}$ and $5\frac{3}{4}$ be added to the difference between $34\frac{5}{7}$ and $3\frac{7}{8}$, by how much does the result differ from 50?

52. A wire 2·385 yards long is cut into pieces each ·013 of a yard long; how many such pieces are there, and how much is left over?

53. Find the cost of covering the floor of a hall, 120 ft. long by 80 ft. wide, with matting at 4*s.* 6*d.* per square yard.

54. If 32 horses eat 96 bushels of corn in 21 days, for how many days will 66 bushels feed 7 horses?

---

55. Find the L.C.M. and the G.C.M. of 14938, 23474, and 32010.

56. If 40 women can do a piece of work in 20 days, in how many days will 15 men do the same piece of work, the work of 5 women being equal to that of 3 men?

57. Divide 4·29 by ·0026, and reduce $\frac{1}{6}$ of 1 ac. 1 ro. 25 sq. po. to the decimal of 2 a. 1 r. 15 sq. po.

58. Find the sum of $1\cdot8$, $1\cdot\dot{9}1600\dot{6}$, $2\cdot8\dot{5}\dot{6}$, and $3\cdot\dot{4}2\dot{7}$.

59. Find by Practice the value of

(1) 1296 articles at 19*s.* $10\frac{1}{2}d.$ each.
(2) 21 pockets of hops, each weighing 1 cwt. 2 qrs. 6 lbs., at £5. 15*s.* 6*d.* per cwt.

60. If a school-room be 26 ft. long and 20 ft. wide, how many children will it accommodate, allowing for each of them 8 superficial feet? Also, if the room be 10 ft. 4 in. high, what cubical space is there for each child?

---

61. A merchant exchanged 600 yards of silk worth 11*s.* 3*d.* per yard for 480 yards of velvet. What was the price to him of velvet per yard?

62. Divide 1365 cub. yds. 13 cub. ft. 487 cub. in. by 79.

63. Add together $\frac{3}{7}$ of half a guinea, $\frac{5}{1\frac{1}{2}}$ of a half-crown, and $\frac{5}{8}$ of a shilling, and express the result as a fraction of £3. 15*s.*

64. Multiply ·003105 by 36·4, and divide the result by ·000161.

65. Express the sum of ·6075 of £3. 6*s.* 8*d.* and ·432 of £2. 7*s.* 6*d.* as a decimal of £16. 10*s.*

66. If it cost £59. 2*s.* $1\frac{1}{2}d.$ to keep 3 horses for 7 months, what will it cost to keep 2 horses for 11 months?

---

67. Supposing one million three hundred thousand French bronze coins to be exchanged for English pence, at the rate of thirteen for a shilling, what will be the loss to those who had taken them for pence?

68. At the end of a term two schools have respectively 105 and 80 boys. At the beginning of the next term each is found to have diminished by one-fifth of its number. How many has the one school more than the other then?

69. Find the value of $\frac{5}{7}$ of $\frac{7}{9}+\frac{5}{8}-\frac{7}{10}$ of $\frac{2}{6}\frac{5}{3}+7\frac{2}{3}-5\frac{4}{5}$.

70. Convert $\frac{4}{13}$ and $\frac{8}{37}$ into decimals.

71. A gentleman gives a gold snuff-box weighing 11 oz. 19 dwt. 16 grs., valued at £4. 3*s.* 9*d.* per oz., in part payment for 24 silver plates each weighing 7 oz. 11 dwt. 6 grs., valued at 6*s.* 8*d.* an ounce; what sum of money will he have to pay in order to discharge the bill?

72. If a person out of an income of £400 spends five times as much as he saves, find how much he saves.

---

73. Divide 3 guineas between five persons so that two of them may obtain 2*s.* 4*d.* more than each of the other three.

74. The *Daily Chronicle* newspaper of Saturday, September 22nd, 1900, bore the number 12031; what was the number borne by the issue of September 19th, 1899;

75. *A* and *B* respectively received $\frac{3}{19}$ and $\frac{4}{17}$ of a certain sum of money, and *C* obtained the remainder. *A* received £107. 14*s.* 9*d.*; what sum did *B* get, and what fractional part of *A*'s share was *C*'s share?

76. Find the value of

$$2\tfrac{2}{5} \text{ of £3. } 14s.\ 2d. + 4\tfrac{1}{7} \text{ of 3 guineas} + \tfrac{7}{19} \text{ of £1. } 5s.\ 4d.$$

77. One side of an oblong yard measures 12 ft. 3 ins., and the cost of paving it at 1*s.* a square foot is £12. 11*s.* $1\frac{1}{2}d.$ What is the length of the other side?

78. If 41 qrs. 2 bush. 3 pks. of wheat cost £86. 16*s.* $5\frac{1}{4}d.$, how much wheat can be bought for £136. 8*s.* $8\frac{1}{4}d.$?

---

79. At an election there were two candidates, and 4501 people voted. One candidate was returned by a majority of 397 votes. How many voted for him?

80. A person possessing $\frac{3}{14}$ of an estate sold $\frac{5}{56}$ of his share for £$120\frac{5}{8}$; what is the value of the estate?

81. Express $\cdot 01\dot{2}\dot{5}$ of 5 miles in feet and $\cdot 1136$ of a mile in yards.

82. Find by Practice the cost of 3 tons 5 cwt. 14 lbs. at £5. 16*s.* 8*d.* per ton.

83. A postman has 10 miles to walk in 3 hours, and he walks the first 4 miles at the rate of $3\frac{3}{4}$ miles per hour, the next 2 at the rate of $3\frac{1}{3}$ miles per hour, and the next mile at 3 miles per hour. At what rate must he walk the rest to finish the journey in the proper time?

84. Find the amount at simple interest of £45737. 10*s.* in $2\frac{1}{2}$ years at $4\frac{1}{2}\%$.

---

85. When the number 582167 is divided by a certain divisor the quotient is 761, and the remainder is 763; find the divisor.

86. A man leaves £10000 to his wife, and the remainder of his property to be equally divided between his five children; it is found that each child has $\frac{1}{4}$ of the whole. How much does each child get?

87. Change $\cdot 43\dot{1}\dot{8}$ and $\cdot 4\dot{5}5\dot{4}$ into vulgar fractions in their lowest terms.

88. If a room be 13·5 ft. long, and 12·5 ft. wide, what will be the expense of carpeting it at 10·625*s.* per yard, the carpet being 30 in. wide?

89. Two sheep and 3 lambs cost £9; I sell the lambs for 18*s.* 8*d.* each; for how much must I sell each sheep to gain 15% on the transaction?

90. At what rate per cent. simple interest will £440. 6*s.* 8*d.* amount to £511. 17*s.* 9*d.* in 5 years?

---

91. The quotient in a division is 479, the dividend is 3476418, and the remainder is 794: what is the divisor?

92. How many acres, roods, sq. poles, sq. yards, sq. feet and sq. inches are there in 40,253,798 square inches?

93. Find by Practice the value of 3 tons 3 cwt. 5 lbs. 8 oz. at £70 per ton.

94. Divide £89. 3*s.* 6*d.* between *A*, *B*, and *C* in the ratios of 2, 3, and 4.

95. A person possessing $\frac{3}{7}$ of an estate sold $\frac{2}{5}$ of his share for £533. 5*s.*; what is the total value of the estate?

96. After paying income tax at 7*d.* in the £, a person's net income was £699; find his gross income.

---

97. How many denarii, worth $8\frac{1}{2}d.$ each, were contained in a talent of gold worth £5464. 5*s.* 11*d.*?

98. A man paid $\frac{5}{7}$ of the money that he had, and $\frac{7}{11}$ of what he has left is £168. 16*s.* 4*d.*; what was his original money?

99. If a postage stamp be 1 inch long and $\frac{5}{6}$ inch wide, how many stamps would be required to cover the wall of a room which is 18 feet long, 14 feet wide, and 10 feet high?

100. If a man working for 9 hours per day receive 31*s.* 6*d.* per week, what ought a woman to receive per week who works 7 hours per day, when 6 men can do the work of 7 women?

101. Divide £90. 6*s.* among *A*, *B*, and *C*, so that *A* may receive $\frac{3}{4}$ as much as *B* receives, and *C* $\frac{1}{5}$ as much as *A* and *B* together.

102. By selling goods for £240 a tradesman gains 25 per cent.; how much would he have gained per cent. by selling them for £204?

---

103. The number of votes polled in all by two candidates at an election was 2,163, and the majority of the successful candidate was 195. What number of votes did each candidate receive?

104. Find the value of $11{\cdot}8\dot{3}$ of 8*s.* $+7{\cdot}15$ of 14*s.* $0\frac{3}{4}d.$

105. If by selling my stock of Irish cloth at $4\frac{1}{2}d.$ a yard profit, I gain £168. 15*s.* on the whole, and at $5\frac{1}{2}d.$ a yard profit gain £206. 5*s.* on the whole, what stock of cloth have I on hand?

106. Goods are marked at the credit price of 8*s.* 4*d.*, 9 months' credit being allowed; what should be the price for ready money, discount at $4\frac{1}{2}$ per cent. being allowed?

107. What is the rate of simple interest per cent. per annum when £277. 10*s.* amounts to £366. 6*s.* in 4 years?

108. Find the square root of 5196·9681.

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109. Find the G.C.M. of 46806 and 201579.

110. If the value of our copper coins were reduced so that 250 pence made £1, find the loss that would result in one year to a tramway company which carried each day 15000 people paying twopence each.

111. Given that the 7th of February, 1904, was a Sunday, find the day of the week on which fell March 16th, 1905.

112. A ship is worth £16000, and a person who owns $\frac{3}{8}$ of her sells $\frac{5}{16}$th of his share; how much is the remainder of his share worth?

113. What sum must be invested in the $3\frac{1}{2}$ per cents. at 94 to obtain an income of £329?

114. The area of an oblong field, whose length is twice its breadth, is 80 acres; find the length of its sides.

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115. What is the smallest number which when subtracted from 99099 will make it exactly divisible by 909?

116. How many times is £14. 17*s.* $3\frac{1}{4}d.$ contained in £1473. 8*s.* 6*d.*, and how much remains over?

117. Find the value of

$$1{\cdot}1\dot{6} \text{ of } 2s.\ 6d. + 2{\cdot}44\dot{9} \text{ of a crown} + 3{\cdot}5\dot{4}\dot{9} \text{ of } 9s.\ 3d.$$

118. A rectangular cistern, whose length is $13\frac{3}{8}$ feet and whose breadth is 6 feet, contains $294\frac{1}{4}$ cub. feet of water; what is the depth of the water and what is its weight if a cubic inch of water weighs 252·4 grains?

119. Divide £234. 10*s.* 7*d.* among 4 persons in the ratio of

$$\tfrac{1}{2} : \tfrac{1}{3} : \tfrac{1}{4} : \tfrac{1}{5}.$$

120. How many francs should be transmitted from Paris to New York in discharge of a debt of 420 dollars? [1 dollar = 4*s.*; 25 francs = £1.]

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121. After paying rates and taxes amounting to 3*s.* 11*d.* in the £ on his gross income a man has £490. 10*s.* 10*d.* left; what was his gross income?

122. If $\frac{13}{15}$ of a ship's cargo be worth £170. 6*s.*, what is the whole cargo worth? What fraction of another cargo, which is worth £235. 16*s.*, will have the same value as the whole of the first cargo?

123. Divide 36·287 by ·346 correct to 3 places of decimals.

124. Simplify $\frac{1}{8}$ of $(\frac{1}{10} - \frac{1}{11}) \div \frac{\frac{1}{7} - \frac{1}{9} \div (\frac{4}{9} + \frac{4}{11})}{\frac{1}{7} + \frac{1}{9} \div (\frac{4}{9} - \frac{4}{11})}$.

125. A stationer sold quills at 11*s.* per thousand, by which he cleared $\frac{3}{8}$ of the money he laid out; he raises the price to 13*s.* 6*d.* What does he clear per cent. at the latter price?

126. In a certain school 80 per cent. of the scholars examined pass in Arithmetic, one-tenth more in Writing than in Arithmetic, and one-eleventh more in Reading than in Writing. If the passes in Reading number 120, how many scholars were examined?

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127. Divide £34. 9*s.* between *A*, *B*, *C* and *D* so that *A* may have £2. 15*s.* more than *B*, *C* have £3. 7*s.* more than *B*, and *D* have £4. 8*s.* less than *C*.

128. What would be the dividend if in a division sum 902341 were the divisor, 978 the quotient, and 1857 the remainder?

129. Find the value of 1571 bales of wool at £5. 7*s.* $10\frac{1}{4}$*d.* per bale.

130. A farmer buys a number of cattle; 26 at £8. 15*s.*, 17 at £11. 5*s.* 6*d.*, and 10 at £10 19*s.* each; what is the average price per head?

131. The length of a rectangular piece of ground is twice its breadth, and its area is 30479·805 square feet; find its length and breadth.

132. Three graziers *A*, *B*, and *C* rent a piece of pasture-land for one month. *A* puts on 27 cattle for 21 days, *B* 19 for 24 days, and *C* 23 for 25 days. If the rent and other charges amount to £33. 5*s.* 10*d.*, how should this sum be divided?

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133. Reduce 1 mile 3 fur. 25 po. 2 yds. 1 ft. to inches.

134. How much land is there in 92 farms each containing 214 ac. 2 r. 23 sq. po.?

135. If $\frac{3}{5}$ of an estate is worth £6237. 10*s.* 6*d.*, what is the value of $\frac{2}{7}$ of it?

136. Simplify $\frac{1\frac{1}{2}}{4\frac{1}{3}} + \frac{3\frac{1}{3} - 2\frac{5}{6}}{4\frac{1}{2} + 5\frac{9}{10}} + \frac{3}{26}$ of $5\frac{1}{4}$.

137. Find the cost of 8 oz. 12 dwt. 18 grs. of gold at £3. 18*s.* 4*d.* per ounce.

138. If 9 men can build a wall in $30\frac{5}{6}$ days of $6\frac{2}{5}$ hours each, how long will it take 12 men at $12\frac{1}{3}$ hours per day to do $3\frac{3}{4}$ times as much?

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139. Find the L.C.M. of 18, 30, 35, 77, and 44, and the G.C.M. of 2268 and 3444.

140. Simplify $\{\frac{5}{7}$ of $\frac{2}{9}$ of $13\frac{1}{2}\} \div \{(\frac{1}{9}$ of $\frac{3}{7}) + 40\} + \frac{7\,9\,6}{8\,4\,1}$.

141. Find the difference between $\frac{5}{3}$ of £1. 13*s.* 9*d.* and $\frac{3}{7}$ of £5. 18*s.* 5*d.*, and reduce the result to a fraction of a guinea.

142. What will be the cost of the paving of a court, of length 58 ft. 6 in. and breadth 42 ft., at $4\frac{3}{4}d.$ per sq. yd.?

143. Find the simple interest on £433. 6*s.* 8*d.* when lent for $2\frac{1}{2}$ years at 4 %.

144. Find the discount on a bill for £657 drawn on March 16th for 3 months and discounted on April 14th at 4 %.

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145. Find the L.C.M. of 48, 72, 115, 256 and 864.

146. Simplify $\dfrac{7\frac{1}{5} \times 5\frac{1}{7}}{7\frac{1}{5} - 5\frac{1}{7}} \div \dfrac{5\frac{1}{3} \times 3\frac{1}{5}}{5\frac{1}{3} - 3\frac{1}{5}}$.

147. How many pence are there in $\cdot 52\dot{7}$ of a guinea?

148. A certain number of men, twice as many women, and three times as many boys earn £7. 15*s.* in 5 days; each man earned 3*s.*, each woman 1*s.* 8*d.*, and each boy 1*s.* 4*d.* per day. How many were there of each? [Use x.]

149. Two rooms contain equal quantities of air. The area of the floor of the one is 340 square feet and its height 12 feet; find the area of the floor of the other whose height is 17 feet.

150. If railway fares in Prussia are only two-thirds of what they are in England, and it costs £1. 2*s.* 6*d.* to travel 180 miles in England, what will it cost to travel 160 miles in Prussia?

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151. A man died in the year 1903 at the age of 75; his son died in 1884 aged 29; how old was the man when his son was born?

152. Standard gold being coined at the rate of £3. 17*s.* $10\frac{1}{2}d.$ per oz., what is the least number of ozs. that can be coined into an exact number of sovereigns?

153. Find in tons what weight of water falls upon $85\frac{1}{3}$ acres of ground during a rainfall of $1\frac{3}{4}$ inches, a cubic foot of water being taken to weigh 1000 oz.

154. *A* and *B* join in giving a picnic, each paying in proportion to the number of guests he invites, *A* inviting 15 and *B* 12. The whole expense of the picnic is £11. 18*s.* 6*d.* *A* pays for the luncheon of the party at 3*s.* 6*d.* a head, and *B* pays all the other expenses. How much must *A* pay to *B* to settle the account?

155. Which is the greater, $\cdot 2\dot{7}$ of a league or $\cdot 7\dot{2}$ of a mile? Express the difference as the decimal of a furlong.

156. What sum of money will amount to £350. 19*s.* $4\frac{1}{2}d.$ in $6\frac{1}{2}$ years at $4\frac{1}{4}$ per cent. per annum simple interest?

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157. The remainder of a division sum is 423, the quotient is larger than it by 113, and the divisor is greater than the sum of both by 19; find the dividend.

158. Simplify $\left[\dfrac{2\frac{1}{4} - \frac{2}{3} \text{ of } 1\frac{5}{6}}{\frac{1}{5} \text{ of } 3\frac{1}{3} + \frac{1}{3}\frac{3}{6}}\right] \div \dfrac{15\frac{3}{4}}{26\frac{1}{4}}$.

159. Find by Practice, the cost of 2420 lbs. of tea at 2*s.* 1½*d.* per lb. If 20 lbs. be spoiled, what amount of profit will be made by selling the remainder at 2*s.* 6*d.* per lb. ?

160. Find the number of yards in the side of a square field whose area is 780,978,916 square yards.

161. A merchant buys 1134 cwt. of sugar at 19*s.* 11¼*d.* per cwt., and mixes it with 2322 cwt. at £1. 1*s.* 3¼*d.* ; at what price per cwt. must he sell the mixture in order to realize a profit of 24 per cent. ?

162. How much water must be added to 40 gallons of spirit worth 22*s.* per gallon so that the value of the mixture may be £1 per gallon ? [Use x].

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163. What number indicating three successive years, when added together and divided by 27, will give the quotient 208 and remainder 3?

164. In a book on Arithmetic an example was printed thus : "Add together $\frac{1}{14\frac{2}{3}}, \frac{1}{19\frac{1}{4}}, \frac{1}{\quad}, \frac{1}{13\frac{3}{4}}$," the denominator of one fraction being accidentally omitted. The answer given at the end of the book was $\frac{1}{2}\frac{1}{8}$. Required the missing denominator.

165. Find the recurring decimal which is equal in value to the difference between ·369 and ·32$\dot{4}$.

166. If the carriage of 3½ tons for a distance of 39 miles cost 14*s.* 7*d.* what will be the cost of the carriage of 25 tons for a distance of 156 miles at half the former rate ?

167. A Christmas dole is given to the inhabitants of certain alms houses consisting of as many shillings as there were almsfolk. The sum so expended being in all £9. 16*s.* Find the number of almsfolk.

168. Ammonia contains Nitrogen, Hydrogen, and Chlorine in the proportions of 14 : 4 : 35·5 ; find the weight of each in 51·895 Kilogrammes of Ammonia.

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169. How many half-guineas are there in three hundred and four millions twenty thousand and thirty-four farthings ?

170. Find the smallest integral number of pounds sterling which can be divided into an equal number of guineas, sovereigns, half-sovereigns, crowns, half-crowns, florins, shillings, sixpences, and four-penny pieces.

171. *A* gives away in charity one-eighth of his income, and pays one-tenth of it in rates and taxes ; with these deductions he has £473. 13*s.* 1*d.* left. What is his gross income ?

172. A person ordered by his will that $\frac{1}{2}$ of his property was to be given to *A*, $\frac{1}{3}$ to *B*, and $\frac{1}{4}$ to *C* ; show that this cannot be done. If his property amount to £1226. 4*s.* 6*d.*, divide it so that the shares of *A*, *B*, and *C* may be to one another in the ratios that he intended them to be.

173. A grocer sells tea at 2*s.* 6*d.* per lb., of which 10 per cent. is profit. How much per cent. profit will he make if the duty of 6*d.* per lb. is taken off, and he sells the tea at 2*s.* per lb. ?

174. A can do a piece of work in 16 days, A and B can do it in 12 days, and A, B and C in 8 days; in how many days can A and C do it?

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175. Find the value of $23{\cdot}2837 \times 1{\cdot}4635$ correct to 4 places of decimals.

176. A man spent ·3 of his money, then ·7 of what was left, and afterwards ·37 of the remainder, and had £5. 8*s.* 6*d.* left; what had he to start with?

177. The content of a cistern is the sum of two cubes whose edges are 10 inches and 2 inches; the area of its base is the difference of two squares whose sides are $1\frac{1}{9}$ and $1\frac{2}{9}$ feet. Find its depth.

178. The cost price of a certain article was 25 shillings, the expenses connected with its sale 10 per cent. on the cost price, and the profit 20 per cent. on the whole outlay; what is the selling price?

179. What sum of money must I invest in a 4 per cent. stock at $95\frac{1}{5}$ to have an income of £700 after deducting income-tax at 8*d.* in the £?

180. Simplify $\dfrac{4\frac{2}{3} \text{ of } 7\frac{8}{9} - 15\frac{4}{7} \text{ of } \frac{2}{3}}{2\frac{3}{5} \text{ of } \frac{7}{9} + 14\frac{6}{7} \text{ of } \frac{4}{15}} \times \left(\frac{3}{8} \text{ of } \frac{13}{17} \text{ of } \frac{29}{37}\right)$.

181. A and B can do a piece of work in 20 and 24 days respectively; they work together for 5 days when B leaves; A works by himself for 5 days more, and is then joined by C, and together they finish the work in 4 more days, ; in how many days would C have done the work?

182. The $2\frac{1}{2}$ per cent. Consols are at $90\frac{1}{8}$; taking brokerage at $\frac{1}{4}$th per cent., what sum must be invested to produce an income of £100 a year after deducting an income tax of 1*s.* in the £?

183. Find the ratio of the simple and compound interest on £510. 8*s.* at the end of 3 years, reckoning interest at 4 per cent. per annum.

184. A bill for £15000 at 3 months was discounted at $4\frac{1}{2}\%$ per annum and the proceeds invested in $2\frac{1}{2}$ per cent. Consols at $90\frac{7}{8}$, brokerage being $\frac{1}{8}$ per cent., and income tax 10*d.* in the £; find the net quarterly income.

185. A contractor undertakes to complete a building in 4 years. He puts 400 men on to the work, and at the end of 3 years finds that he has only performed $\frac{5}{7}$ths of his contract. Find how many additional men he must employ to finish the contract.

186. In a large institution where there is always the same number of inmates to be fed, the contract price of meat rises 20 per cent., and the daily allowance of meat is reduced from 9 oz. to 8 oz. If the yearly charge for meat is thenceforward £597, what was it before the changes were made?

187. If a metre be 3·2809 feet, and the length of a line drawn on the earth from the North Pole to the Equator be 10,000,000 metres, find the length of the circumference of the earth to the nearest mile.

188. A person invests £1842. 15*s.* in a 4 per cent. stock at $102\frac{3}{8}$; he afterwards sells out at 105, and re-invests at 126 in a 5 per cent. stock: find the change in his income.

189. A kilometre being 1093·638 yards, find, to two places of decimals, how many kilometres there are in 100 English miles.

190. How many hours will a train going at the average rate of 25 miles per hour occupy in travelling from Paris to Madrid, a distance of 1450 kilometres?

191. Find the length in decametres of each side of a square plot containing 34741·2321 sq. decametres.

192. An acre is ·40467 hectares, and £1 is equal to 25·25 francs. An estate measuring 1927 hectares is sold for 10,100,000 francs. What is this per acre?

193. If a man working for 10 hours per day receive 35*s.* per week, what ought a woman to receive per week who works 8 hours per day, when 3 men can do the work of 4 women?

194. Divide £21. 10*s.* between 5 men, 6 women, 7 boys, and 8 girls, giving each woman a seventh, each boy two-sevenths, and each girl one-third, less than each man.

195. If $2\frac{1}{2}$ per cent. more be gained by selling a house for £1743. 15*s.* than by selling it for £1705, what did the house cost?

196. Gunpowder is composed of 2 parts of sulphur, 3 parts of charcoal and 15 parts of nitre; how much of each is required for $8\frac{3}{4}$ cwts. of gunpowder?

197. A room 22 ft. by 19 ft. has in it a Turkey carpet 19 ft. 6 in. by 15 ft. 9 in., costing 12*s.* per square yard; the rest of the floor is covered with felt at 4*d.* per square foot; find the cost of the carpet and of the felt.

198. Find how many flagstones, each 5·76 ft. long and 4·15 ft. wide, are requisite for paving a cloister which encloses a rectangular court 45·77 yds. long and 41·93 yds. wide, the cloister being 12·45 ft. wide.

199. My agent has sold goods for me to the value of £3257. 15*s.* 9*d.* What is his commission at 3 per cent.?

200. A man borrows £100, and at the end of each year he pays £25 to reduce the debt and to pay interest at the rate of 4 per cent. on the sum that has been standing against him during the year; how much will he owe at the end of three years?

201. A shot is fired from a ship which is moving at the rate of 10 miles an hour. Assuming the velocity of sound to be 1120 feet per second, find how far the ship will have moved before the report is heard at a place which is $14\frac{1}{4}$ miles off.

202. Two persons buy certain articles at the rate of twelve per shilling; one sells them at 11 per shilling and the other at 13 pence for a dozen; compare their gains on selling the same number of articles.

203. The external length, breadth, and depth, of a box with a lid are 4 ft. 2 in., 3 ft. 8 in., and 2 ft. respectively, and it is made of wood 1 in. thick. Find the quantity used.

204. If an American dollar be worth 4*s.* $3\frac{3}{5}d.$, and be also worth 5·42 francs, what is the value in francs of an English sovereign?

205. What number of men, women, and children, might be fed for 52 weeks at a cost of £11,000,000, supposing each child to cost 3*s.* 6*d.*, each woman 8*s.* 9*d.*, and each man 12*s.* 3*d.*, per week, the number of men, women, and children being equal?

206. If a certain MS. fill 6 printed sheets, each containing 32 pages and each page 24 lines, with 9 words in each line; how many lines of 12 words each must there be in a page, in order that another similar MS., twice as long, may fill 3 sheets, each containing 48 pages?

207. Three partners in a trading firm divide their year's profits into shares which are as the numbers 6, 7, 8. If the whole profits amount to 1678 guineas, what is the value of each share?

208. A draper buys cloth and reckons that by selling it at 11*s.* 8*d.* a yard he will gain 5 per cent. on his outlay; he is, however, obliged to sell it at 10*s.* 4*d.*; how much per cent. does he gain or lose, and how much per yard?

209. A man commenced business with a certain capital; at the end of each of four successive years he adds to his capital $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ respectively of his capital at the beginning of each such year. What was his original capital if his capital finally is £5474?

210. A gravel walk 6 ft. wide runs round a grass plot 60 feet long and 40 feet wide. If gravel cost 3*s.* per cubic yard, find the cost of a coat of gravel on the path 3 inches deep.

211. It costs £643. 10*s.* 9*d.* to level and turf a square cricket ground at 9*d.* per square yard. What will it cost to enclose it with an iron railing at 7*s.* 6*d.* per yard?

212. In what proportions ought a merchant to mix teas costing him respectively 2*s.* and 2*s.* 5*d.* per lb. so that by selling the mixture at 2*s.* 4*d.* per lb. he may gain $6\frac{2}{3}$ per cent?

213. A and B can do a piece of work alone in sixteen and twenty days respectively; they work together at it for four days, when B leaves. After one day A is joined by C and they finish it in three days more. How long would C take alone?

214. A boy loses one-quarter of his marbles and then gains 15. He then loses $\frac{1}{5}$ of what he now has and then gains 15. He afterwards loses $\frac{5}{9}$ of what he now has, and has 60 left. How many had he at first?

215. A rectangular field of $4\frac{1}{4}$ ac., whose length is 170 yards., is planted with strawberries in rows perpendicular to the length, and 1 ft. 9 in. apart. If the width of a yard all round the field remain unplanted, find the number of rows, and the total length of the rows.

216. If the population of a certain country be 10 per cent. more than it was ten years ago, and 5 per cent. more than it was 4 years ago, what is the increase in the last 10 years, given that the population 4 years ago was twenty-two millions?

217. The incomes of A, B and C are as 2 : 5 : 7, and C's exceeds A's by £725. Determine the several amounts. [Use x.]

218. A dairyman buys milk at $2\frac{1}{2}d.$ a quart, dilutes it with water, and sells the mixture at $3d.$ per quart. He thus makes a profit of 60%. How much water does he mix with each quart of milk? [Use $x.$]

219. How much is the difference between the interest and true discount on £82. 10*s.* for two years at 5 per cent?

220. Of three sisters, the youngest is three years younger than the second and the second is five years younger than the eldest. Their united ages amount to 47 years; find the age of each. [Use $x.$]

221. A has 9 sheep for every 5 that B has. A's sheep are worth 25 shillings each and B's £1. 12*s.* 6*d.* If A's flock is worth £53. 2*s.* 6*d.* more than B's, how many sheep do they each possess?

222. Simplify $\left\{\dfrac{\frac{5}{9}\text{ of }\frac{7}{8}\text{ of }2\frac{6}{15}}{\frac{3}{4}+1\frac{5}{7}-1\frac{1}{8}}\times\dfrac{4\frac{3}{10}\text{ of }2\frac{1}{9}}{7\frac{2}{9}-3\frac{1}{5}}\right\}\div\left(\frac{1}{2}\frac{7}{0}-\frac{4}{5}\right)$.

223. A person walks at the rate of $3\frac{1}{2}$ miles an hour. Three hours after he is followed by another person walking at the rate of 5 miles an hour. In what time will the latter overtake the former?

224. How much money must be invested in a 3 per cent. stock at $92\frac{1}{2}$ to produce the same income as would be produced by £1520 invested in a $3\frac{1}{2}$ per cent. stock at 95? [Use $x.$]

225. If there were an error of one-sixth of an inch in the yard measure, what would the whole error amount to in measuring a race-course of 1 m. 543 yds. 2 ft. 11 in.? Does it make any difference whether the error be in excess or defect? If so, what difference?

226. A and B are partners, A's capital being £450 and B's £750. A is to have 10% of the profits for managing the business, and the rest of the profits is to be divided between them in proportion to their capitals. If B's share of the profits is £157. 10*s.*, how much does A receive?

227. A shopkeeper marks his goods with a price from which he can deduct 15 per cent. for ready money and still have $10\frac{1}{2}$ per cent. of profit. What is the marked price of an article which cost him £3. 19*s.* 2*d.*? [Use $x.$]

228. If 2 turkeys and 9 fowls cost £2. 18*s.* 6*d.*, and 7 turkeys and 3 fowls cost £6. 13*s.* 6*d.*; find the cost of 1 turkey and 1 fowl.

229. A and B running a race for 100 yds., A wins by 17 yds.; when A and C run the same distance A wins by 8 yds. How many yards' start ought C to give B, that, racing for 200 yds., they may make a dead heat (supposing A, B, and C to run respectively at the same speed throughout)?

230. A wire 1509·85 feet in length is cut into portions each 45·6 feet long. How many such portions can be cut off and what is the length of the remainder in inches?

231. Find the cost of papering a room 22 ft. 9 in. long, 18 ft. 6 in. wide, and 10 ft. 6 in. high with paper 21 in. wide at 2*s.* 6*d.* per piece of 12 yards.

232. A room, three times as long as it is broad, is carpeted at a cost of 4*s.* 6*d.* per sq. yd., and the walls are coloured at 9*d.* per sq. yd., the respective costs being £8. 5*s.* $4\frac{1}{2}d.$ and 4 guineas. Find the dimensions of the room.

233. The supply pipe to a cistern of 400 gallons can fill it in 40 minutes, and the discharge pipe empty it in 25 minutes; if, when the cistern is three parts full the two pipes are set in action together, how much water will be in the cistern at the end of 15 minutes?

234. If silk is bought at 3 francs per metre, at what price per yard, to the nearest penny, must it be sold so that the gain is 10%? [£1 = 25 francs; 1 metre = 39·4 inches.]

235. *A* sold goods to *B* at a profit of 25%, and *B* sold them to *C* at a profit of 15%. If *A* had sold to *C* at the same price that *B* sold him the goods, find what profit % *A* would have made.

236. I drive from *A* to *B* at the rate of $12\frac{1}{2}$ miles per hour, and return walking at $3\frac{3}{4}$ miles per hour. If the whole journey occupies $6\frac{1}{2}$ hours, what is the distance from *A* to *B*? [Use x.]

237. A silver cup was sold for 12 guineas at a loss of $5\frac{1}{2}$%; at what price ought it to have been sold to gain 26%?

238. Find the value of $\cdot\dot{4}2857\dot{1}$ of a pole + $\cdot\dot{1}42857\dot{1}$ of a mile.

239. In a cricket match *A* made $\frac{3}{4}$ as many runs as *B*, *B* $\frac{1}{4}$ as many as *C*, *C* $\frac{2}{5}$ as many as *D*, *D* $\frac{4}{3}$ as many as *E*, and *E* obtained $\frac{1}{3}$ as many as the rest of the eleven. If *E* made 60, what was the total score?

240. Find, to the nearest penny, the simple interest on £865. 3*s.* 6*d.* from April 14th to September 23rd, at 4% per annum.

241. I invest £7100 partly in a 3 per cent. stock at 98, and partly in a 4 per cent. stock at 106; find how much must be invested in each so that the income from each stock may be the same. [Use x.]

242. Find how much water must be mixed with 40 gallons of spirit which cost 15*s.* a gallon, so that by selling the mixture at 12*s.* a gallon there may be a gain of 10 per cent. on the outlay. [Use x.]

243. *A* starts 3 minutes after *B* for a place $4\frac{1}{2}$ miles distant. *B*, on reaching his destination, immediately returns, and after walking a mile meets *A*. If *A*'s speed be a mile in 18 minutes, what is *B*'s speed?

244. A man owes three bills, of which one could be paid by a certain number of florins, another by twice that number of half-crowns, and the third by six times that number of shillings. The bills amount in all to £7. 3*s.* 0*d.* What are the several amounts?

245. A bill is drawn for £378. 15*s.* on August 12th at 90 days' sight; what will a banker pay for it on September 10th, if he charge 5% discount?

246. A man has 6 hours to spare for an excursion; how far can he ride out in a carriage which travels 8 miles an hour so as to return home in time if he walk back at the rate of 4 miles per hour? [Use x.]

247. *A* pays £9. 3*s.* 4*d.* more rates than *B*, the value of their houses being equal; living in different towns they are rated at 2*s.* and 1*s.* 4*d.* in the £ respectively; what is the rent of each house?

248. A reservoir, containing 1200 cubic feet of water, is emptied by means of a pipe 3 inches in diameter. If the water flows out with a velocity of 2 miles an hour, how long will it take to empty it?

249. Two cogged wheels work together, there being 32 cogs on one and 36 on the other. The larger wheel makes 64 revolutions per second. How many times will the same cogs come in contact during 6 working days of 10 hours each?

250. What sum of money will amount to £299. 15*s.* 9*d.* in 9 months at $4\frac{1}{2}\%$?

251. A mixture of tea is formed by mixing one chest of 2 qrs. 17 lbs. at 3*s.* $1\frac{1}{2}d.$ per lb. with two chests each containing 3 qrs. 7 lbs. at 3*s.* $5\frac{1}{2}d.$ If the mixture be sold at 4*s.* per lb., what will be the gain per cent.?

252. The price of 26 horses and 22 cows is £980, and that of 14 horses and 66 cows is £1,340. Find the values of a horse and of a cow.

253. In a race of 100 yards A allows B 4 yards and C 6 yards start in order that they may reach the goal at the same time. How much start should B allow C when B has to run the whole distance?

254. Two clocks point to 8 o'clock at the same instant on the morning of New Year's Day; one loses six seconds and the other gains 10 seconds in 24 hours; when will one be half an hour before the other and what time will each clock then show?

255. £8234 is invested partly in a 3 per cent. stock at 98 and partly in a 4 per cent. stock at 108 so that the income derived from the two is the same; is the total income greater or less than it would be if the £8234 were divided equally between the two stocks and by how much to the nearest penny? [Use x.]

256. I row against a stream flowing $1\frac{1}{2}$ miles an hour to a certain point and then turn back, stopping two miles short of the place at which I originally started. If the whole time occupied in rowing be 2 hrs. 10 min. and my speed in still water be $4\frac{1}{2}$ miles an hour, find how far upstream I went.

257. By selling oranges at 10*d.* per dozen a man gains 12 per cent. How much would he gain per cent. if he were to sell them at the rate of 14 for a shilling?

258. A man invests £1995 in a $4\frac{1}{2}\%$ stock at 84; what income does he receive? If he sell his stock at the end of one year at the price of 90, what rate of interest will he have obtained by the whole transaction?

259. Find a number between 300,000 and 310,000 which is exactly divisible by both 79 and by 97.

260. A strip of building land on the side of a straight road has a depth of 180 feet. It is sold at the rate of £5. 5*s.* per foot of road-frontage. What price is this per acre?

261. A runner starts in a mile race at the rate of 6 yards per second. How far from the end must he quicken to 7 yards per second in order to complete the mile in 4 min. 45 sec.?

262. Find the cost of papering a room 12 metres long, 6·2 metres broad, and 4·25 metres high, with paper of which the breadth is 70 centimetres and the price is 2 francs per piece of 8 metres, 21 square metres being allowed for the windows and doors.

263. Potassium Cyanide is composed of Potassium, Carbon and Nitrogen in the ratio of 39 : 12 : 14. Find its percentage composition correct to 2 places of decimals.

264. Three tramps meet together for a meal; the first has 5 loaves, the second has 3, and the third who has his share of the loaves pays the others 8 half-pence; how ought they to divide the money?

265. A merchant buys two kinds of tea at 1*s.* $11\frac{3}{4}d.$ and 1*s.* 5*d.* per lb. respectively. In what proportions must he mix them so as to gain $37\frac{1}{2}$ per cent. by selling the mixture at 2*s.* $3\frac{1}{2}d.$ per lb.? [Consider a mixture of x lbs. of the first tea with one lb. of the second.]

266. A wine merchant mixed 45 gallons of one kind of wine with $22\frac{1}{2}$ gallons of another kind and sold the mixture at 14*s.* 8*d.* a gallon, thus making a profit of $33\frac{1}{3}$ per cent. If a gallon of the first kind of wine cost 12*s.*, how much did a gallon of the second kind cost?

267. The front seats of a concert cost half-a-crown each, and the back seats a shilling. If 400 tickets are sold and the receipts are £30. 10*s.*, how many front seats are taken? [Use x.]

268. In consequence of the reduction of duty by 1*s.* per cwt. on goods, the consumption of the goods increases by 25 per cent., but the revenue derived from the duty decreases by 5 per cent.; find the original duty per cwt. on the goods.

269. A manufacturer sold an article to a retailer at a profit of 20 per cent. on the cost of manufacture, but deducted 10*s.* from his bill; the retailer sold at a profit of 25 per cent. on the money which he had paid, but allowed a discount of 8 per cent. on his bill; if the retailer received £5. 3*s.* 6*d.* net cash, what did the article cost to make?

270. A train whose length is 176 feet is moving at the rate of 45 miles an hour, and overtakes a second train going at the rate of 30 miles an hour. How long does the first train take to pass a passenger in the second train? And if the first train takes a quarter of a minute to pass the second train, find the length of the second train.

271. A profit of 20 per cent. is made on goods when a discount of 10 per cent. is given on the list price. What profit per cent. will be made when a discount of 20 per cent. is given on the list price?

272. A and B can run at the rate of $12\frac{1}{2}$ and $12\frac{7}{16}$ miles an hour respectively. If A give B ten yards start, in what time will he overtake him? When A has run a mile will he be in front of or behind B?

273. A legacy is divided between three brothers, so that the eldest has $\frac{2}{5}$ of it, the second $\frac{3}{8}$, and the youngest the remainder, which is £35 less than the share of the eldest. Find the value of the legacy.

274. In a certain country the birth-rate per annum is 41, and the death-rate 25, for every 1000 inhabitants at the beginning of the year. If the population at the beginning of a certain year is four millions, find to the nearest thousand what it was three years earlier, assuming no immigration or emigration.

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